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Brief paper

Tuning function design for nonlinear adaptive control systems with multiple unknown control directions☆

Chao Huang, Changbin (Brad) Yu *

School of Automation, Hangzhou Dianzi University, Zhejiang 310018, China
Research School of Engineering, The Australian National University, Canberra ACT 2601 Australia

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ABSTRACT

To overcome the drawback of overparametrization in existing nonlinear adaptive control design with multiple unknown control directions, we propose a new algorithm which combines nonlinear integrator backstepping, tuning function design and a logic-based switching mechanism that tunes the control directions online in a switching manner. Global asymptotic tracking control is achieved for parametric-strict-feedback systems without overparametrization. The logic-based switching criterion is based on monitoring incremental errors caused during two consecutive switching moments, and thus can identify the true control direction quickly.

1. Introduction

Nonlinear integrator backstepping was first introduced in the seminal paper (Kanellakopoulos, Kokotović, & Morse, 1991) to systematically solve the adaptive control problem for a class of uncertain nonlinear systems in the parametric-pure-feedback form. The drawback of overparametrization in Kanellakopoulos et al. (1991) was successfully overcome by Krstić, Kanellakopoulos, and Kokotović (1992), with a combination of backstepping and the technique of "tuning function design". Now, many important results involving nonlinear and adaptive control are well documented in Krstic, Kanellakopoulos, and Kokotovic (1995) and Marino and Tomei (1996).

Adaptive control design for nonlinear systems in the parametric-strict-feedback (PSF) form with multiple unknown control directions was solved in Ye and Jiang (1998), where the smooth Nussbaum-type gain (Nussbaum, 1983) was successfully incorporated into backstepping, to overcome the lack of information on control directions. This new design technique was further applied to nonlinear robust regulation control (Ye, 1999) and nonlinear output feedback robust and adaptive control with unknown control directions (Ding & Ye, 2002; Ye, 2001), etc. However, in contrast to (Krstić et al., 1992), tuning function design cannot be incorporated into the Nussbaum-gain-based design (Ye & Jiang, 1998) to remove overparametrization.

Therefore, this paper develops a new adaptive control scheme for nonlinear PSF systems with multiple unknown control directions. The first contribution of the proposed scheme is that the drawback of overparametrization in Ye and Jiang (1998) is overcome. That is, for nonlinear PSF systems of order n with $n+q$ unknown parameters (including n unknown virtue control coefficients), Ye and Jiang (1998) require a total number of $n(q-1)+\frac{1}{2}n(n+1)$ estimators, whereas using the algorithm of this paper the total number is $n+q-1$, which is free of overparametrization.

The proposed adaptive control scheme is based on a combination of backstepping design, tuning function design and a logic-based switching mechanism which tunes the control directions online in a switching manner. It was recognized early in Ilchmann (1993) that, for MIMO linear adaptive control systems with unknown high frequency gain matrix (i.e., multiple unknown control directions), one can incorporate a switching-type Nussbaum function (SNF) or use a switching decision function (SDF) to achieve asymptotic control. Therefore it is not a surprise that in this paper we adopt a switching-type control law when there are multiple (virtue) control inputs in a nonlinear system. Our switching mechanism is more like the SDF-based approach in Ilchmann (1993).

The second contribution of the proposed adaptive control scheme is that it achieves asymptotic tracking control. It is worth pointing out that for nonlinear uncertain systems, there are essential differences between stabilization and tracking problems (Yan & Liu, 2010). Therefore, for uncertain nonlinear systems with

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* Corresponding author at: School of Automation, Hangzhou Dianzi University, Zhejiang 310018, China.
E-mail addresses: zjuhc2011@gmail.com (C. Huang), brad.yu@anu.edu.au (C. Yu).

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multiple unknown control directions, although the problems of stabilization (Ortega, Astolfi, & Barabanov, 2002; Psillakis, 2016; Wu, Chen, & Li, 2016) and practical tracking (Yan & Liu, 2010) have been tackled, very few methods are capable of tackling asymptotic tracking problems except the adaptive control schemes of Ye and Jiang (1998), Tiago et al. (2010) and our paper.

The third but not the least contribution is that the logic-based switching mechanism in this paper is novel in the sense that it is based on monitoring the incremental error caused during two consecutive switching moments, rather than monitoring total errors as many previous related works did (e.g. Ye, 2005, 2012). The advantage of the incremental-error based switching criterion is that it is more sensitive to mismatch between true control direction and the estimated ones. As is well known, a major disadvantage of Nussbaum-type control law is its large control overshoot, with the new switching criterion of this paper, instead, correct estimation of the control direction can be identified quickly, so that the drawback of overshoot is often effectively suppressed.

The rest of the paper is organized as follows: the problem under consideration is formulated in Section 2. Section 3 presents the controller structure design based on backstepping and tuning functions, and the switching mechanism design responsible for tuning control directions. The main result is presented in Section 4 with some remarks. A numerical example is provided to show the effectiveness of the proposed method in Section 5. Section 6 concludes this paper.

2. Problem formulation

We revisit the problem of global adaptive control of the following parametric-strict-feedback system with unknown control directions (Ye & Jiang, 1998):

$$\begin{aligned} &\dot{x}_i = \mu_i x_{i+1} + \theta^T \phi_i (x_1, \dots, x_i), \\ &i = 1, \dots, n-1, \\ &\dot{x}_n = \mu_n u + \theta^T \phi_n (x), \\ &y = x_1, \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the output. $\mu_i \neq 0, i = 1, \dots, n$, are unknown (virtue) control coefficients, particularly, whose signs representing the control directions, are unknown. $\theta = \left[\theta_1, \dots, \theta_q, \mu_1, \dots, \mu_{n-1}\right]^T \in \mathbb{R}^{q+n-1}$ represents the unknown parameter vector. Note that μ_n does not need to be estimated using our approach. $\phi_i \in \mathbb{R}^{q+n-1}, i = 1, \dots, n$, are known smooth functions. The control objective is to force the output asymptotically tracking the reference signal $r_0(t)$ whose up to nth time derivatives are assumed to be known, bounded and piecewise continuous, i.e., we require $\lim_{t\to\infty} y(t) - r_0(t) = 0$. Let r_0 be produced by the following known system:

$$\begin{aligned} &\dot{r}_i = r_{i+1}, \\ &i = 0, \dots, n-1. \end{aligned} \tag{2}$$

3. Adaptive controller design

3.1. controller structure

As mentioned in Section 1, the adaptive controller design is based on backstepping and tuning functions. Before the step-by-step design procedure, let us first define the change of coordinates:

$$\begin{aligned} &z_1 = x_1 - r_0, \\ &z_i = x_i - \alpha_{i-1}\Big(x_1, \dots, x_{i-1}, \rho_1, \dots, \rho_{i-1}, \\ &K_1, \dots, K_{i-1}, r_0, \dots, r_{i-1}, \hat{\theta}\Big), \\ &i = 1, 2, \dots, n, \end{aligned} \tag{3}$$

where $\rho_i(t) \in \mathbb{R}, i = 1, \dots, n$ are continuous signals to be designed, $\hat{\theta}(t) \in \mathbb{R}^{q+n-1}$ is a vector that serves as the estimate of θ, $K_i(t), i = 1, 2, \dots, n$ are switching signals which serve as the estimate of the signs of μ_i, and take values in the set $\{1, -1\}$, that is, either $K_i(t) = 1$ or $K_i(t) = -1$. The switching moments are recorded by a sequence of strictly increasing numbers $0 = T_0 < T_1 < T_2 < \cdots$ and the switching mechanism for $K_i(t)$ will be presented in Section 3.2. $\alpha_i, i = 1, \dots, n-1$ are smooth functions with respect to the variables, serving as the virtue control inputs. $K_i(t)$ is discontinuous at $T_0, T_1, \dots$, and so is α_i, but α_i is smooth with respect to t between any two consecutive switching moments. Throughout the paper we assume $K_i(t)$, as well as α_i, is *continuous from the right* at the switching moments. Now we start the design procedure.

Step 1. Define

$$V_1(t) = \frac{1}{2}z_1^2 + \frac{1}{2}\mu_1 K_1\left(\rho_1 + \frac{K_1}{\mu_1}\right)^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta},$$

where $\Gamma \in \mathbb{R}^{(n+q-1)\times(n+q-1)}$ is a positive definite matrix that can be selected freely by designers. $\tilde{\theta} = \theta - \hat{\theta}$ is the estimate error. Computing the time derivative of V_1 for all t except the switching moments $T_k, k = 1, 2, \dots$, we have

$$\begin{aligned} &\dot{V}_1 = z_1\left(\mu_1\alpha_1 + \theta^T\phi_1 - r_1\right) + \dot{\rho}_1 \\ &+ \mu_1 K_1 \rho_1 \dot{\rho}_1 - \tilde{\theta}^T \Gamma^{-1}\left(\dot{\hat{\theta}} - \Gamma z_1 \phi_1\right) + \mu_1 z_1 z_2 \\ &= -c_1 z_1^2 + \left(\dot{\rho}_1 + c_1 z_1^2 + z_1\hat{\theta}^T\phi_1 - z_1 r_1\right) \\ &+ \mu_1\left(z_1\alpha_1 + K_1\rho_1\dot{\rho}_1\right) - \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \Gamma z_1\phi_1\right) + \mu_1 z_1 z_2. \end{aligned}$$

Defining $\tilde{\rho}_1 = c_1 z_1 + \hat{\theta}^T\phi_1 - r_1$ and $\tau_1 = \Gamma z_1 \phi_1$, taking

$$\begin{aligned} &\dot{\rho}_1 = -z_1\tilde{\rho}_1, \\ &\alpha_1 = K_1\rho_1\tilde{\rho}_1, \end{aligned}$$

we obtain

$$\dot{V}_1 = -c_1 z_1^2 - \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_1\right) + \mu_1 z_1 z_2.$$

Step 2. Define

$$V_2(t) = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\mu_2 K_2\left(\rho_2 + \frac{K_2}{\mu_2}\right)^2.$$

Computing the time derivative of V_2 for all t except the switching moments, we have

$$\begin{aligned} &\dot{V}_2 = -c_1 z_1^2 - \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_1\right) \\ &+ z_2\left[\mu_1 z_1 + \mu_2\alpha_2 + \theta^T\phi_2 - \frac{\partial\alpha_1}{\partial x_1}\left(\mu_1 x_2 + \theta^T\phi_1\right)\right. \\ &\left. + \frac{\partial\alpha_1}{\partial\rho_1}z_1\tilde{\rho}_1 - \frac{\partial\alpha_1}{\partial r_0}r_1 - \frac{\partial\alpha_1}{\partial r_1}r_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right] \\ &+ \dot{\rho}_2 + \mu_2 K_2\rho_2\dot{\rho}_2 + \mu_2 z_2 z_3 \\ &= -c_1 z_1^2 - \tilde{\theta}^T\Gamma^{-1}\left\{\dot{\hat{\theta}} - \tau_1 - \Gamma z_2\left[z_1 e_{q+1} + \phi_2\right.\right. \\ &\left.\left. - \frac{\partial\alpha_1}{\partial x_1}\left(x_2 e_{q+1} + \phi_1\right)\right]\right\} \\ &+ z_2\left\{\mu_2\alpha_2 + \hat{\theta}^T\left[z_1 e_{q+1} + \phi_2 - \frac{\partial\alpha_1}{\partial x_1}\left(x_2 e_{q+1} + \phi_1\right)\right]\right. \\ &\left. + \frac{\partial\alpha_1}{\partial\rho_1}z_1\tilde{\rho}_1 - \frac{\partial\alpha_1}{\partial r_0}r_1 - \frac{\partial\alpha_1}{\partial r_1}r_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right\} \\ &+ \dot{\rho}_2 + \mu_2 K_2\rho_2\dot{\rho}_2 + \mu_2 z_2 z_3 \end{aligned}$$

where $e_j \in \mathbb{R}^{n+q-1}$ is a vector whose jth entry equals 1 while other entries equal 0. Taking

$$\tau_2 = \tau_1 + \Gamma z_2\left[z_1 e_{q+1} + \phi_2 - \frac{\partial\alpha_1}{\partial x_1}\left(x_2 e_{q+1} + \phi_1\right)\right]$$