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Brief paper

Delay estimation via sliding mode for nonlinear time-delay systems ☆

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ABSTRACT

This paper deals with delay identifiability and delay estimation for a class of nonlinear time-delay systems. The theory of non-commutative rings is used to analyze the identifiability. In order to estimate the delay, a sliding mode technique and a classical Newton method are combined to show the possibility to have a local (or global) delay estimation for periodic (or aperiodic) delay signals.

1. Introduction

Time-delay systems are widely used to model concrete systems in engineering sciences, such as biology, chemistry, mechanics and so on Kolmanovskii and Myshkis (1999) and Niculescu (2001). Many results have been reported for the purpose of stability and observability analysis, by assuming that the delay of the studied systems is known. It makes the delay identification one of the most important topics in the field of time-delay systems.

Delay identifiability has been widely studied in the literature for linear time-delay system. However, for a nonlinear time-delay system, this issue is not trivial, and we will borrow the concept of non-commutative rings to analyze it. The theory of non-commutative rings was firstly proposed by Moog, Castro-Linares, Velasco-Villa, and Marque-Martinez (2000) for the disturbance decoupling problem of nonlinear time-delay systems. Then this method was applied to study observability of nonlinear time-delay systems with known inputs in Xia, Marquez, Zagalak, and Moog (2002), to analyze parameter identifiability for nonlinear time-delay systems in Zhang, Xia, and Moog (2006), and to study state elimination and delay identification of nonlinear time-delay systems in Anguelova and Wennberg (2008), Zheng, Barbot, Floquet, Boutat, and Richard (2011) and Zheng, Barbot, and Boutat (2013).

Concerning the techniques to identify the delay, up to now, various methods have been proposed, and a nice survey can be found in Bjorklund and Ljung (2003). Briefly, Diop, Kolmanovsky, Moraal, and van Nieuwstadt (2001), Ren, Rad, Chan, and Lo (2005) and Tuch, Feuer, and Palmor (1994) used the least squares method or its variation to estimate the delay for linear time-delay systems. An adaptive identification method was proposed in Orlov, Belkoura, Richard, and Dambrine (2003) for identification of both parameters and delay. By using a convolution approach, Belkoura (2005) and Belkoura, Richard, and Fliess (2009) proposed an algebraic method to identify the delay involved in a linear time-delay system. In Drakunov, Perruquetti, Richard, and Belkoura (2006), a variable structure observer was proposed to estimate the delay. More recently, a recursive gradient method was proposed in Barbot, Zheng, Floquet, Boutat, and Richard (2012) to estimate the delay for a nonlinear time-delay system.

Comparing with the existing identification methods, the main contributions of this paper are twofold. Firstly, this paper investigates the delay identification problem for a class of nonlinear time-delay systems while most of existing works in the literature are for a linear time-delay system. Secondly, compared with the existing results which assume the delay should be involved in aperiodic trajectory, our method combines the high order sliding mode technique Levant (2003) and classical Newton method Saupe (1988) to identify the delay, and the proposed method can also treat periodic case.

This paper is an extension of Zheng, Polyakov, and Levant (2016), and is organized as follows. Section 2 states the systems

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and the problem which will be studied in this paper. The delay identifiability property will be analyzed in Section 3. This property is based on the algebraic framework proposed in Xia et al. (2002). By using a high order sliding mode technique, an estimator is proposed in Section 4 to identify the delay. This identification is local if the delay signal is periodic. Finally the proposed result is applied to analyze the identifiability for an illustrative example in Section 5.

2. Problem statement

It is assumed that the delays are constant and commensurate, that is all of them are multiples of an *elementary unknown delay* τ. Under this assumption, the considered nonlinear time-delay system is described as follows:

$$\begin{cases} \dot{x} = f(x(t), x(t-\tau), \dots, x(t-s\tau)), \\ y = h(x(t), x(t-\tau), \dots, x(t-s\tau)), \\ x(t) = \psi(t), \ t \in [-s\tau, 0], \end{cases} \tag{1}$$

$x \in \mathbb{R}^n$, $y \in \mathbb{R}$ is the measurable output; f and h are meromorphic functions[1] which are functions of variables $\{x, x(t-\tau), \dots, x(t-s\tau)\}$; $\psi : [-s\tau, 0] \to \mathbb{R}^n$ denotes unknown continuous initial functions.

Assumption 1. For the initial function ψ, the system (1) admits a unique smooth solution, which is bounded on the compact set W, i.e. $x \in W \subset \mathbb{R}^n$.

Unlike conventional control systems, the studied system (1) does not contain an input (control) u. The choice of such a form (1) is motivated by the fact that the input signal is normally known and it is function of x. Therefore, the closed-loop system might be written as (1) for which this paper focuses on the identification of the unknown but constant elementary delay τ.

3. Identifiability analysis

For a nonlinear time-delay system described in (1), the analysis of delay identifiability is not trivial as that for the linear case, where commutative algebra can be applied. For nonlinear case, we have to use the non-commutative algebraic framework introduced in Xia et al. (2002), which will be firstly recalled in the following section.

3.1. Algebraic framework

Denote by $\mathscr{K}$ the field of meromorphic functions of a finite number of the variables from $\{x_j(t-i\tau), j \in [1, n], i \in [0, s]\}$. For the sake of simplicity, introduce the delay operator δ, which means, for $i \in \mathbb{Z}^+$:

$$\delta^i \xi(t) = \xi(t - i\tau), \ \ \xi(t) \in \mathscr{K}, \tag{2}$$

$$\begin{aligned} \delta^i \left(a(t)\xi(t)\right) &= \delta^i a(t) \delta^i \xi(t) \\ &= a(t-i\tau)\xi(t-i\tau). \end{aligned} \tag{3}$$

Let $\mathscr{K}(\delta]$ denote the set of polynomials in δ over $\mathscr{K}$ of the form

$$a(\delta] = a_0(t) + a_1(t)\delta + \dots + a_{r_a}(t)\delta^{r_a} \tag{4}$$

where $a_i(t) \in \mathscr{K}$ and $r_a \in \mathbb{Z}^+$. The addition in $\mathscr{K}(\delta]$ is defined as usual, but the multiplication is given as:

$$a(\delta]b(\delta] = \sum_{k=0}^{r_a+r_b} \sum_{i+j=k}^{i \le r_a, j \le r_b} a_i(t) b_j(t-i\tau)\delta^k. \tag{5}$$

[1] Means quotients of convergent power series with real coefficients Conte, Moog, & Perdon (1999) and Xia et al. (2002).

Thanks to the definition of $\mathscr{K}(\delta]$, (1) can be rewritten in a more compact form:

$$\begin{cases} \dot{x} = f(x, \delta) \\ y = h(x, \delta) \\ x(t) = \psi(t), \ t \in [-s\tau, 0], \end{cases} \tag{6}$$

where $f(x, \delta) = f(x(t), x(t-\tau), \dots, x(t-s\tau))$ and $h(x, \delta) = h(x(t), x(t-\tau), \dots, x(t-s\tau))$, with entries belonging to $\mathscr{K}$.

With the standard differential operator d, denote by $\mathscr{M}$ the left module over $\mathscr{K}(\delta]$:

$$\mathscr{M} = span_{\mathscr{K}(\delta]}\{d\xi, \ \xi \in \mathscr{K}\} \tag{7}$$

where $\mathscr{K}(\delta]$ acts on $d\xi$ according to (2) and (3). Note that $\mathscr{K}(\delta]$ is a non-commutative ring, however it is proved that it is a left Ore ring Ježek (1996) and Xia et al. (2002), which enables to define the rank of a left module over $\mathscr{K}(\delta]$.

3.2. Output delay equation

In order to study the delay identifiability of (6), we need to first deduce a certain output delay dependent equation, based on which the identifiability can be then analyzed.

Definition 1 (*Zheng & Richard, 2016*)**.** For system (6), an equation with delays, containing only the output and a finite number of its derivatives:

$$\alpha(h, \dot{h} \dots, h^{(k)}, \delta) = 0, k \in \mathbb{Z}^+$$

is said to be an *output delay equation* (of order k). Moreover, this equation is said to involve the delay in an essential way for (6) if it cannot be written as $\alpha(h, \dot{h} \dots, h^{(k)}, \delta) = a(\delta]\tilde{\alpha}(h, \dot{h} \dots, h^{(k)})$ with $a(\delta] \in \mathscr{K}(\delta]$.

If there exists an output equation involving the delay in an essential way, then the delay might be identifiable if certain sufficient conditions are satisfied. Thus, the first task is to seek such an output delay equation. In what follows, we define the derivative and Lie derivative for nonlinear time-delay systems from the non-commutative point of view.

For $0 \le j \le s$, let $f(x(t-j\tau))$ with $f_r \in \mathscr{K}$ for $1 \le r \le n$ and $h(x(t-j\tau)) \in \mathscr{K}$, define

$$\frac{\partial h}{\partial x_r} = \sum_{j=0}^{s} \frac{\partial h}{\partial x_r(t-j\tau)} \delta^j \in \mathscr{K}(\delta]$$

then the Lie derivative for nonlinear systems without delays can be extended to nonlinear time-delay systems in the framework of Xia et al. (2002) as follows

$$\frac{dh}{dt} = L_f h = \frac{\partial h}{\partial x}(f) = \sum_{r=1}^{n} \sum_{j=0}^{s} \frac{\partial h}{\partial x_r(t-j\tau)} \delta^j (f_r). \tag{8}$$

Based on the above notations, one can define the observability indices introduced in Krener (1985) over non-commutative rings. For $1 \le k \le n$, let $\mathscr{F}_k$ be the following left module over $\mathscr{K}(\delta]$:

$$\mathscr{F}_k := span_{\mathscr{K}(\delta]} \left\{dh, dL_f h, \dots, dL_f^{k-1} h\right\}.$$

It was shown in Zhang et al. (2006) that the filtration of $\mathscr{K}(\delta]$-module satisfies $\mathscr{F}_1 \subset \mathscr{F}_2 \subset \dots \subset \mathscr{F}_\rho$, and it is stationary for a certain $\rho \le n$, thus ρ is called the observability index of the output y.

Define

$$\pounds = span_{\mathbb{R}[\delta]} \left\{h, \dots, L_f^{\rho-1} h\right\}, \tag{9}$$