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Brief paper

Network-based practical set consensus of multi-agent systems subject to input saturation ☆

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ABSTRACT

This paper is concerned with the network-based practical set consensus problem of multi-agent systems subject to input saturation constraints. Considering network-induced delays, data quantization and aperiodic sampling intervals, a network-based framework which allows each agent to be remotely controlled over the communication network is established. Under this framework, a new network-based consensus protocol with input saturation constraints is proposed. With this protocol, the consensus problem can be transformed into the stabilization problem of time-delay systems with bounded perturbations. By using the Lyapunov–Krasovskii approach, a stability condition guaranteeing that the error system can exponentially converge to a bounded set is derived, where the region of initial conditions can be estimated by considering the effect of the first delay interval. Based on this stability condition, the network-based consensus controller gain matrix can be obtained. An optimization algorithm is introduced for simultaneously designing the consensus controller gain and estimating the region of attraction as small as possible and the region of initial conditions as large as possible. A numerical example is given to illustrate the efficiency of all the results derived in this paper.

1. Introduction

In recent years, consensus of multi-agent systems (MASs), which means that all agents' states converge to the same vector under agents' interaction, have drawn considerable attention due to its wide applications such as formation control, distributed sensor networks, attitude of spacecraft alignment and so on (Olfati-Saber & Murray, 2004). There have been a large number of results available on the consensus problem of MASs from various viewpoints, see, e.g., Li, Duan, Chen, and Huang (2010), Li, Ren, Liu, and Xie (2013), Olfati-Saber and Murray (2004), Qin, Ma, Shi, and Wang (2017), Ren (2008) and Yu, Chen, and Cao (2010). In many applications, since there exist some physical restrictions and communication constraints, MASs may only reach a bounded set region compassing the equilibrium, which is usually named practical consensus of MASs. Some related research work can be found in Ceragioli, De Persis, and Frasca (2011), Li, Ho, and Lu (2013) and Shi and Hong (2009).

The rapid advances of digital technologies in sensing, computation and communication make it feasible that MASs are controlled using digital controllers. Hence, recently, consensus of MASs based on sampled-data control has been widely investigated (Cao & Ren, 2010; Chen, Li, & Jiao, 2013; Ding & Zheng, 2016; Guo, Ding, & Han, 2014; Qin & Gao, 2012; Yu, Zheng, Chen, Ren, & Cao, 2011). Note that most of these literatures focus on MASs with a point-to-point control structure. However, in some special working environments such as deep oceans, unmanned zones and primitive forests, it is more desirable to remotely operate the entire agent system through a communication network. Such control implementations can bring about benefits such as low cost, reduced system wiring, and simple installation and maintenance. As a result, a network-based framework for cooperative control of MASs was proposed in Ding, Han, and Guo (2013) and Ding and Zheng (2017), where a waiting mechanism was employed to handle the asynchronous effect of network-induced delays. Besides, both the limited capacity of communication channels and data quantization are two important practical communication constraints that should be carefully dealt with. Many research works on quantized consensus problem for MASs have been done, such as Ceragioli et al. (2011), Chen et al. (2013), Li, Fu, Xie, and Zhang (2011) and

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Li, Ho et al. (2013). A precise mathematical treatment for first-order integrator systems with quantization and time delay was provided in Li, Ho et al. (2013). However, because of complicated network environments, it is difficult to directly apply these results to networked control multi-agent systems. It should be pointed out that the network-based framework in Ding et al. (2013) and Ding and Zheng (2017) considered network-induced delays but ignored quantization. Therefore, this motivated us to further investigate the networked-based consensus of MASs with both network-induced delays and quantization.

On the other hand, in practical applications, MASs may be inevitably subject to magnitude limitation of control input due to their physical properties. Such limitation may degrade the system performance, or even cause instability. Thus, it is of great significance to study MASs with input saturation. So far, some results on consensus problems of MASs subject to input saturation constraints have been reported in the literature (Li, Xiang, & Wei, 2011; Meng, Zhao, & Lin, 2013; Qin, Fu, Zheng, & Gao, 2017; Ren, 2008; Su, Chen, Lam, & Lin, 2013; Su, Chen, Wang, & Lam, 2014; Wang, Yu, & Gao, 2014; Yang, Meng, Dimarogonas, & Johansson, 2014). In multi-agent consensus, input saturation constraints were considered for the single integrator case (Li, Xiang et al., 2011) and the double integrator case (Ren, 2008). By using the low gain feedback method, semi-global leader-following consensus of MASs with switching topology was studied in Su et al. (2013), which was extended to the cases of output feedback in Su et al. (2014). The global leader-following consensus problem for identical linear MASs subject to actuator saturation under fixed undirected network topologies and time varying network topologies was addressed in Meng et al. (2013). These results were then extended in Yang et al. (2014) to the discrete-time case. Some conditions for achieving semi-global synchronization for the case of unstable eigenvalues on the imaginary axis were provided in Wang et al. (2014). However, these existing results may be no longer applicable, when it comes to the network-based consensus of MASs, especially, in the presence of network-induced delays and quantization simultaneously. Therefore, it remains an open problem to find an effective approach to the network-based consensus of MASs with input saturation.

This paper mainly focuses on network-based practical set consensus tracking of linear MASs subject to input saturation. Different from Ding et al. (2013) and Ding and Zheng (2017) where only network-induced delays were considered, we further explore a comprehensive network-based control framework of MASs with network-induced delays, uniform quantization and input saturation constraints. *Under this framework, our concerning issues fall into three aspects: (1) how to study the impacts of network-induced delays, data quantization and input saturation constraints on consensus performance of MASs, i.e., both regions of attraction and initial conditions, respectively? (2) how to design an appropriate consensus controller under such constraints? (3) how to optimize both regions of attraction and initial conditions?* In order to address these issues, regarding uniform quantization and input saturation as 'perturbations', the consensus problem is transformed into the stabilization problem of time-delay systems with bounded perturbations. The contributions of the paper are summarized as follows:

(i) A sufficient condition for practical set exponential consensus is derived, which reveals the comprehensive effects of network-induced delays, uniform quantization and input saturation on the consensus performance;
(ii) Different from Su et al. (2013, 2014) and Wang et al. (2014), an estimate on the region of initial conditions can be computed by taking into account the first delay interval and using the Lyapunov–Krasovskii (LK) method;
(iii) Compared with the existing results in Meng et al. (2013), Su et al. (2013), Wang et al. (2014) and Yang et al. (2014) based on the requirement that the form of consensus controller gain is given a priori, an effective approach is provided to design the network-based consensus controller gain;
(iv) A two-step optimization algorithm is developed for simultaneously obtaining the consensus controller matrix and the estimates of the smallest region of attraction and the largest initial region.

2. Notation and preliminary

2.1. Notation

$\mathbb{R}^n$ represents the n-dimensional Euclidean space. I_N is an identity matrix with dimension N and diag$\{\cdot\}$ stands for a block-diagonal matrix. The superscript T for matrix denotes matrix transposition, and the symbol $\otimes$ represents the matrix Kronecker product. $|\cdot|$ and $\|\cdot\|$ stand for the absolute value of a scalar and for the Euclidean norm of a vector, respectively. $A_{(i)}$ and $A_{(i,i)}$ denotes the ith row and the diagonal element of matrix A, respectively. For symmetric matrices P and Q, $P \succ Q$ and $P \prec Q$ ($P \succeq Q$ and $P \preceq Q$) means that matrix $P-Q$ is positive and negative definite, respectively (positive and negative semi-definite, respectively). Sym$\{A\}$ represents $A+A^T$. The symmetric elements in a symmetric matrix are denoted by $*$. $\lfloor a \rfloor$ is a round floor function denoting the greatest integer that is less than or equal to a. For a scalar $a > 0$ and b, $\text{sat}_a(b) : \mathbb{R} \to \mathbb{R}$ is a saturation function defined as $\text{sat}_a(b) \triangleq \text{sign}(b)\min\{|b|, a\}$. For a vector $b = [b_1, b_2, \ldots, b_n]^T \in \mathbb{R}^n$, $\text{sat}_a(b) \triangleq [\text{sat}_a(b_1), \text{sat}_a(b_2), \ldots, \text{sat}_a(b_n)]^T$. Let $d(x, \mathcal{E}) = \inf_{y\in\mathcal{E}} \|x-y\|$ represents the Hausdorff distance from x to $\mathcal{E}$. Denote by $\mathcal{I}_i$ a block entry matrix with an identity matrix I and let $\mathcal{I}_{i,j} \triangleq \mathcal{I}_i - \mathcal{I}_j$.

2.2. Graph theory

Let $G = \{\Delta, E, \mathcal{W}\}$ denote a directed weighted graph of N order, where $\Delta = \{v_1, v_2, \ldots, v_N\}$ and $E \subseteq \Delta \times \Delta$ are the set of nodes and edges, respectively. $\mathcal{W} = [w_{ij}] \in \mathbb{R}^{N\times N}$ represents a weighted adjacency matrix with $w_{ii} = 0$ for any i. Node v_j is considered as a neighbor of node v_i if node v_i can receive information from node v_j, and let N_i be the neighbor set of node v_i. It is assumed that $w_{ij} > 0$ if $j \in N_i$, otherwise, $w_{ij} = 0$. The degree matrix of graph G is denoted by $\Lambda = \text{diag}\{\varpi_1, \varpi_2, \ldots, \varpi_N\}$, where the diagonal element is given by $\varpi_i = \sum w_{ij}$. Correspondingly, the Laplacian matrix of graph G is defined by $L = \Lambda - \mathcal{W}$.

Denote $\tilde{G}$ by a graph which contains N follower nodes and a leader node. It is assumed that the leader does not receive any information from the followers. A diagonal matrix $M = diag\{m_1, m_2, \ldots, m_N\} \in \mathbb{R}^{N\times N}$ is referred to as the leader adjacency matrix. If the leader is a neighbor of node v_i, then $m_i > 0$; otherwise, $m_i = 0$. A path is a sequence of connected edges in a graph. If there is a path in $\tilde{G}$ from each follower node i in G to the leader node, then the leader node is globally reachable in $\tilde{G}$. For the connectivity of graph, the following assumption is given.

Assumption 1. The leader is globally reachable in graph $\tilde{G}$.

2.3. Uniform quantizer

Here, a uniform quantizer can be described in Ceragioli et al. (2011) and Li, Ho et al. (2013) as a map $q : \mathbb{R} \to \mathbb{R}$ such that $q(x) = \Theta\left\lfloor \frac{x}{\Theta} + \frac{1}{2}\right\rfloor$, where $\Theta > 0$ is a constant number. For a uniform quantizer, the quantization error is always bounded by $\frac{\Theta}{2}$, i.e., $|x - q(x)| \leq \frac{\Theta}{2}$, $\forall x \in \mathbb{R}$. And it holds that $xq(x) \geq 0$, $\forall x \in \mathbb{R}$,