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Brief paper

Robust adaptive output feedback control to a class of non-triangular stochastic nonlinear systems ☆

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ABSTRACT

In this paper, the robust adaptive control design problem is studied for a class of non-triangular nonlinear systems with unmodeled dynamics and stochastic disturbances. It is assumed that the states of the systems to be controlled are unmeasurable, and thus an adaptive state observer is first developed. By utilizing the stochastic small-gain theorem and the backstepping recursive design procedure, a robust adaptive output feedback control scheme is then proposed. It is shown that all the signals in the resulting closed-loop system are bounded in probability, and the system output converges to a small residual set of the equilibrium in probability.

1. Introduction

In the past decades, some effective robust adaptive control methods have been proposed for uncertain nonlinear systems (Chen, Ge, & Ren, 2011; Ge, Hong, & Lee, 2005; Ge & Tee, 2007; Li, Chen, Fu, & Sun, 2016; Li, Yang, Su, Deng, Sun, & Zhang, 2015; Zhang, Cui, & Luo, 2013; Zhang, Cui, Zhang, & Luo, 2011). It should be noted that all the aforementioned results are developed under a restrictive assumption that the nonlinear systems under consideration are in a strict-feedback form, in other words, in triangular structure, and these control methods for strict-feedback nonlinear systems cannot be directly used to stabilize non-triangular nonlinear systems. It is well known that for a strict-feedback nonlinear system, if applying the backstepping control design procedure, the state variable x_{i+1} $(i = 1, \ldots, n-1)$ is regarded as the control input for the ith subsystem, and a virtual control function α_i is designed to stabilize the ith subsystem. To guarantee the existence of virtual control function α_i, α_i should be the function of partial state vector $\bar{x}_i = [x_1, \ldots, x_i]^T$, which is exactly the case for a strict-feedback nonlinear system. However, for a non-triangular nonlinear system, its nonlinear function $f_i(\cdot)$ in each subsystem includes the whole state vector $x = [x_1, \ldots, x_n]^T$ instead of partial state vector $\bar{x}_i = [x_1, \cdots, x_i]^T$. In this case, if the traditional backstepping control design procedure is adopted, virtual control function α_i would be the function of whole state vector $x = [x_1, \ldots, x_n]^T$, which would lead to the so-called algebraic loop problem and thus the traditional backstepping control design procedure would fail. By utilizing the monotonously increasing property of the bounding functions, Chen, Liu, and Ge (2012) and Wang, Liu, Liu, and Karimi (2015) proposed a so-called variable separation technique, and developed robust adaptive control design methods for SISO non-triangular nonlinear systems. Chen, Zhang, and Lin (2016), Li and Tong (2017) and Tong, Li, and Sui (2016) further developed the observer-based robust adaptive control schemes for non-triangular nonlinear systems with unmeasurable states. However, it should be noted that the above mentioned results do not consider the issue of unmodeled dynamics.

It is well known that unmodeled dynamics widely exist in biological systems, economical systems, and other various engineering applications. It is one of the main sources leading to the instability or poor performance of systems (Jiang & Praly, 1998). By utilizing the input-to-state practical stability (ISpS) method and small-gain theory, a robust adaptive control approach was developed for deterministic SISO nonlinear systems with unmodeled dynamics in Jiang (1999). Robust adaptive control strategies were then proposed for stochastic nonlinear systems with unmodeled dynamics in Tong, Wang, Li, and Zhang (2013) and Wu, Xie, and Zhang (2007). However, it should be noted that the nonlinear

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systems under consideration in Jiang (1999), Tong et al. (2013) and Wu et al. (2007) are all in triangular form. As argued before for deterministic nonlinear systems, those control approaches for triangular stochastic nonlinear systems cannot be directly applied to control non-triangular stochastic nonlinear systems with unmodeled dynamics. In fact, to our best knowledge, there is no result in open literature on robust adaptive control of non-triangular stochastic nonlinear systems with unmodeled dynamics.

Motivated by the observation, this paper investigates the robust adaptive output feedback control problem for non-triangular stochastic nonlinear systems with unmodeled dynamics. By utilizing the backstepping technique and the stochastic small-gain theory, a new robust adaptive fuzzy output feedback control scheme is proposed. It is shown that all the signals in the resulting closed-loop system are bounded in probability, and the system output converges to a small neighborhood of the equilibrium in probability. Compared to existing works, our main contributions can be described in two aspects: (i) this is the first work on robust adaptive control for uncertain non-triangular stochastic nonlinear systems with unmodeled dynamics; and (ii) the algebraic loop problem is solved without assumption that the unknown nonlinear property or functions satisfy monotonically increasing or global Lipschitz conditions (Chen et al., 2012, 2016; Li & Tong, 2017; Wang et al., 2015).

2. Problem formulations and some preliminaries

2.1. Problem formulation

Consider a class of non-triangular stochastic nonlinear systems with unmodeled dynamics described as follows:

$$\begin{cases} d\zeta = q_1(x,\zeta)dt + q_2(x,\zeta)dw \\ dx_i = [f_i(x) + \Delta_i(x,\zeta) + x_{i+1}]dt + g_i(x)dw, \\ \qquad 1 \le i \le n-1 \\ dx_n = [f_n(x) + \Delta_n(x,\zeta) + u]dt + g_n(x)dw \\ y = x_1 \end{cases} \tag{1}$$

where $y \in R$ and $u \in R$ are system output and input respectively, and $x = [x_1, \dots, x_n]^T$ is the state of the system. $\zeta \in R^{n_0}$ is the state of unmodeled dynamics and $\Delta_i(x,\zeta)$ is a disturbance. $f_i(x)$ is an unknown smooth nonlinear function. $q_1(x,\zeta)$, $q_2(x,\zeta)$, $g_i(x)$ and $\Delta_i(x,\zeta)$ are uncertain functions, and satisfy the locally Lipschitz condition. $w \in R$ is an independent standard Wiener process defined on a complete probability space. In this paper, $x_2(t), \dots, x_n(t)$ are assumed to be unmeasurable, and y is the only measurable variable. The system under consideration is assumed to be completely controllable and observable, and the origin is the equilibrium point.

Control Objective: our control objective is to design an observer-based robust adaptive controller for the system (1) such that: (i) all the signals in the resulting closed-loop system are bounded in probability; and (ii) the output y converges to a small neighborhood of the equilibrium in probability.

In order to achieve the control objective, the following assumptions are needed.

Assumption 1 (*Jiang, 1999; Tong et al., 2013; Wu et al., 2007*)**.** Disturbance $\Delta_i(x,\zeta)$ $(1 \le i \le n)$ and uncertain function $g_i(x)$ respectively satisfy the following inequalities

$$\begin{aligned} |\Delta_i(x,\zeta)| &\le p_i^* y \bar{\psi}_{i1}(y) + p_i^* \psi_{i2}(|\zeta|) \\ |g_i(x)| &\le p_i^* y \bar{\psi}_{i3}(y) \end{aligned} \tag{2}$$

where p_i^* is an unknown positive constant, and $\bar{\psi}_{i1}$ and $\bar{\psi}_{i3}$ are known smooth functions; ψ_{i2} is a nonnegative known smooth function, and satisfies $\psi_{i2}(0) = 0$.

Assumption 2 (*Tong et al., 2013; Wu et al., 2007*)**.** For unmodeled dynamic ζ, there is a Lyapunov function $\bar{V}_0(\zeta)$ satisfying

$$\begin{aligned} &\underline{\alpha}_0(|\zeta|) \le \bar{V}_0(\zeta) \le \bar{\alpha}_0(|\zeta|) \\ &\ell \bar{V}_0 \le -\alpha_0(|\zeta|) + \gamma_0(|y|) + \bar{d}_0 \end{aligned}$$

where $\bar{d}_0$ is a positive constant; $\alpha_0, \gamma_0, \underline{\alpha}_0$ and $\bar{\alpha}_0$ are κ_∞-functions; and ℓ denotes the infinitesimal generator.

Remark 1. It is worth pointing out that Assumptions 1 and 2 are standard assumptions for nonlinear systems with unmodeled dynamic, and similar assumptions can be found in literatures like (Jiang, 1999; Tong et al., 2013; Wu et al., 2007).

2.2. Input-to-state practical stability in probability (ISpSiP)

Consider the following stochastic nonlinear system

$$dx = f(x,u)dt + g(x,u)dw(t) \tag{3}$$

where w is a r-dimensional independent standard Wiener process, $u \in R^m$ is the input and $x \in R^n$ is the state. $g(\cdot)$: $R^{m+n} \to R^{n+r}$ and $f(\cdot)$: $R^{m+n} \to R^n$ satisfy the locally Lipschitz condition respectively, and $g(0,0) = 0, f(0,0) = 0$.

Define the infinitesimal generator $\ell V(x)$ of C^2 positive function $V(x)$: $R^n \to R$ along with (3) as follows,

$$\ell V(x) = \frac{1}{2} Tr\{g^T(x,u) \frac{\partial^2 V}{\partial x^2} g(x,u)\} + \frac{\partial V(x)}{\partial x} f(x) \tag{4}$$

where $Tr(X)$ denotes the trace of matrix X.

Definition 1 (*Wu et al., 2007*)**.** If for any $\varepsilon > 0$ and $t \ge 0$, there exist a nonnegative constant d, a κ_∞-function γ and a $\kappa\ell$-function β such that

$$\begin{aligned} &P\{|x(t)| < d + \gamma(\|u_t\|) + \beta(|x(0)|, t)\} \ge 1 - \varepsilon, \\ &\text{for any } x_0 \in R^n \setminus \{0\}, \end{aligned} \tag{5}$$

where $\|u_t\| = \sup_{t \ge s \ge t_0} \|u(s)\|$ and $P(\cdot)$ denotes probability, then system (3) is said to be ISpSiP.

Lemma 1 (*Wu et al., 2007*)**.** *For stochastic nonlinear system (3), if there are a non-negative constant d, a C^2 function $V(x)$, a κ-function α, κ_∞-functions χ, $\underline{\alpha}$ and $\bar{\alpha}$ satisfying the following inequalities*

$$\bar{\alpha}(|x|) \ge V(x) \ge \underline{\alpha}(|x|) \tag{6}$$

$$\ell V(x) \le d + \chi(|u|) - \alpha(|x|) \tag{7}$$

then the stochastic nonlinear system (3) is ISpSiP.

2.3. Stochastic small-gain theorem

Consider the following stochastic interconnected nonlinear system,

$$\begin{aligned} dx_1 &= g_1(x_1, x_2, \Sigma_1)dw_{1t} + f_1(x_1, x_2, \Sigma_1)dt \\ dx_2 &= g_2(x_1, x_2, \Sigma_2)dw_{2t} + f_2(x_1, x_2, \Sigma_2)dt \end{aligned} \tag{8}$$

where w_{1t} and w_{2t} are independent standard Wiener processes; $\Sigma_i(i = 1, 2)$ denote uncertainties, and $x = [x_1, x_2]^T \in R^{n_1+n_2}$ is the state of the interconnected nonlinear system (8). The following stability result is available.

Lemma 2 (*Stochastic Small-Gain Theorem*)**.** (Wu et al., 2007). *Assume that subsystems x_1-system and x_2-system of (8) are ISpSiP with (Σ_1, x_2) as input and x_1 as state, and (Σ_2, x_1) as input and x_2 as state, respectively, i.e., for any positive constants ε_1 and ε_2, there exist*