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Brief paper

Disturbance attenuation and rejection for stochastic Markovian jump system with partially known transition probabilities☆

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ABSTRACT

In this paper, the problem of disturbance attenuation and rejection is investigated for stochastic Markovian jump system with multiple disturbances, which include white noises and disturbances with partially known information. A disturbance observer is designed to estimate the disturbances with partially known information. Based on the estimation value, a disturbance observer based attenuation and rejection controller is constructed such that the closed-loop system is asymptotically bounded in mean square or asymptotically stable in probability under different conditions. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

1. Introduction

Disturbances widely exist in actual control systems, which may degrade the performance of the system. In order to enhance the control performance of system with disturbances, many advanced control schemes have been proposed to deal with disturbances for systems, such as adaptive control scheme (Khalil, 2012), H_∞ control theory (Liu & Lin, 2015), and sliding mode control (Ding, Leant, & Li, 2016; Ding, Wang, & Zheng, 2015), and so on. Recently, disturbance compensation control methods, including disturbance-observer-based-control (DOBC) (Li, Yang, Chen, & Chen, 2014) and active disturbance rejection control (ADRC) (Han, 2009), have been widely investigated to achieve disturbance rejection performance and robustness against uncertainties (Chen, 2004; Chen & Chen, 2010; Han, 2009; Li et al., 2014; Sun, Hou, & Zong, 2016; Sun, Hou, Zong, & Guo, 2017; Sun, Li, & Sun, 2013; Yang, Li, & Yu, 2013).

Although many significant developments have been obtained in the field of disturbance compensation control, the disturbances are regarded as a single disturbance (Guo & Cao, 2014). However, in practical engineering systems, the disturbances come from a variety of sources and may exhibit distinct features, and should be described by different models and signals (Guo & Cao, 2014). Clearly, the above disturbance compensation control schemes do not have the ability to achieve higher control performance in the presence of multiple disturbances (Guo & Chen, 2005). In order to handle the multiple disturbances, a composite hierarchical anti-disturbance control (CHADC) approach has been developed. The basic idea behind CHADC approach is to combine DOBC with H_∞ control (Wei & Guo, 2010), sliding mode control (Wei & Guo, 2009) or adaptive control (Wei, Chen, & Li, 2013) for disturbance attenuation and rejection. Further result has been proposed to handle intelligent control problem for system with mismatched disturbances and unknown nonlinear functions (Sun & Guo, 2017). Very recently, the problem of disturbance observer-based disturbance attenuation and rejection control has been developed for a class of stochastic systems with multiple disturbances in Wei, Wu, and Karimi (2016).

As a kind of special hybrid system, Markovian jump system is widespread in actual control systems, whose switching law depends on the Markovian jump parameters, i.e., the transition rate matrix of the continuous-time system or the transition

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probability matrix of the discrete-time system. In recent years, many scholars pay attention to studying the stability analysis and controller design for Markovian jump system and a large amount of meaningful results have been proposed (Boukas, 2005; Costa, Fragoso, & Marques, 2005). Because it is difficult to obtain all the information of Markovian jump parameters, it is necessary to investigate the control problem for Markovian jump system with partially known Markovian jump parameters. In Zhang and Boukas (2009b), a new framework is established for a class of continuous-time and discrete-time Markovian jump linear system with partly unknown transition probabilities and the relationship between the stability criteria currently obtained for the usual Markovian jump linear system and switched linear system under arbitrary switching, is exposed. Inspired by Zhang and Boukas (2009b), many results are reported on Markovian jump system with partly unknown transition probabilities (Zhang & Boukas, 2009a; Zhang, He, Wu, & Zhang, 2011; Zhang, Lou, Ge, & Zhang, 2017; Zong, Yang, Hou, & Wang, 2013). Meanwhile, along with the development of stochastic differential equations, stochastic Markovian jump systems have received more and more attention, and many meaningful results on stability and control problems have been investigated (Ma, Wang, Ding, & Dong, 2016; Niu, Ho Daniel, & Wang, 2007; Wang, Liu, & Liu, 2010). However, in the above references, the results have been presented for the systems with no disturbances or single disturbance. As mentioned earlier, the system always subjects to multiple disturbances. For Markovian jump systems with multiple disturbances, the CHADC method has been developed in Li, Sun, Zong, and Hou (2016), Li, Sun, Zong, and Hou (2017), Yao and Guo (2013) and Yao, Zhu, and Guo (2014). To the best of our knowledge, the disturbance attenuation and rejection problem for stochastic Markovian jump system with multiple disturbances, including the white noises and the nonrandom disturbances, and partially known Markovian jump parameters is seldom considered so far.

In this paper, the problem of disturbance attenuation and rejection is addressed for stochastic Markovian jump system with multiple disturbances and partially known Markovian jump parameters, where the disturbances contain the white noises and the nonrandom disturbances with partially known information. By constructing a disturbance observer, the disturbances described by the exogenous system is estimated and the estimation value is applied to feedforward compensation. Then, the disturbance observer based attenuation and rejection controller is built such that the closed-loop system is asymptotically bounded in mean square and asymptotically stable in probability for stochastic Markovian jump systems with different conditions. Inspired by Zhang and Boukas (2009b), some solvable sufficient conditions are developed for stochastic Markovian jump system with partially known Markovian jump parameters. Furthermore, some results are obtained for stochastic Markovian jump system with completely known or unknown Markovian jump parameters. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

Notations. The notations of this paper are standard. $E\{\cdot\}$ denotes the expectation operator with respect to probability measure $\mathcal{P}$; $C^{2,1}(\mathbb{S}_h \times \mathbb{R}_+ \times S; \mathbb{R}_+)$ denotes the family of all real-valued functions $V(x, t, i)$ defined on $\mathbb{S}_h \times \mathbb{R}_+ \times S$ which are continuously twice differentiable in $x \in \mathbb{S}_h$ and once differentiable in $t \in \mathbb{R}_+$; $\mathcal{K}$ is the family of all continuous increasing functions $k : \mathbb{R}_+ \to \mathbb{R}_+$ such that $k(0) = 0$ while $k(u) > 0$ for $u > 0$; k^{-1} means the inverse function of $k \in \mathcal{K}$; $K_{+\infty}$ denotes the family of all functions $k \in \mathcal{K}$ with property that $k(\infty) = \infty$; $\mathcal{K}_\vee$ is the family of all convex functions $k \in \mathcal{K}$; matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

2.1. Problem formulation

Fix the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following stochastic Markovian jump systems with multiple disturbances

$$\begin{aligned}\dot{x}(t) &= A(r_t)x(t) + B_0(r_t)(u(t) + v(t)) \\ &\quad + B_1(r_t)\xi_1(t) + B_2(r_t)x(t)\xi_2(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are system states and control inputs, respectively; the additional noise $\xi_1(t)$ and multiplicative noise $\xi_2(t)$ are white noises; $v(t)$ is supposed to be described by an exogenous system

$$\begin{aligned}\dot{m}(t) &= M(r_t)m(t) + D(r_t)\delta(t), \\ v(t) &= V(r_t)m(t) + H(r_t)\xi_3(t),\end{aligned} \tag{2}$$

where $\xi_3(t)$ is a white noise, and $\delta(t) \in \mathbb{R}^r$ is a bounded nonrandom disturbance. $\xi_1(t), \xi_2(t), \xi_3(t)$ are independent. $\{r_t\}$ is a continuous-time Markovian process with right continuous trajectories taking values in a finite set $S = \{1, 2, \ldots, N\}$. The transition probability is given by

$$\mathcal{P}\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } j \neq i, \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{if } j = i, \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$, π_{ij} is the transition rate from mode i at time t to mode j at time $t + \Delta$, $\pi_{ij} \geq 0$ when $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$. For simplicity, let $A_i \triangleq A(r_t = i)$, $B_{0i} \triangleq B_0(r_t = i)$, $B_{1i} \triangleq B_1(r_t = i)$, $B_{2i} \triangleq B_2(r_t = i)$, $M_i \triangleq M(r_t = i)$, $D_i \triangleq D(r_t = i)$, $V_i \triangleq V(r_t = i)$, $H_i \triangleq H(r_t = i)$ be known matrices.

In this paper, the anti-disturbance control problem is considered for stochastic Markovian jump systems with partially known transition rate matrix. Taking the system with four modes as an example, the transition rate matrix is written as follows

$$\Pi = \begin{pmatrix} ? & \pi_{12} & \pi_{13} & ? \\ ? & \pi_{22} & ? & \pi_{24} \\ ? & ? & ? & \pi_{34} \\ \pi_{41} & \pi_{42} & ? & ? \end{pmatrix},$$

where "?" represents the unknown element. For convenience, we denote $S = S_i^k + S_i^{uk}$, $i \in S$, and

$$S_i^k = \{j \mid \pi_{ij} \text{ is known}\}, \; S_i^{uk} = \{j \mid \pi_{ij} \text{ is unknown}\}.$$

If $S_i^k \neq \emptyset$, it is described as $S_i^k = \{k_i^1, \ldots, k_i^m\}$, for $\forall 1 \leq m \leq N$, where $k_i^m \in N^+$, and it represents the mth known element with the index k_i^m in the ith row of matrix Π.

Substituting (2) into (1), we have

$$\begin{aligned}\dot{x}(t) &= A(r_t)x(t) + B_0(r_t)u(t) + B_0(r_t)V(r_t)m(t) \\ &\quad + B_1(r_t)\xi_1(t) + B_0(r_t)H(r_t)\xi_3(t) + B_2(r_t)x(t)\xi_2(t).\end{aligned} \tag{3}$$

Defining $F(r_t) = (B_1(r_t), B_0(r_t)H(r_t))$ and $\eta(t) = (\xi_1^T(t), \xi_3^T(t))^T$, (3) can be denoted as

$$\begin{aligned}\dot{x}(t) &= A(r_t)x(t) + B_0(r_t)u(t) + B_0(r_t)V(r_t)m(t) \\ &\quad + F(r_t)\eta(t) + B_2(r_t)x(t)\xi_2(t).\end{aligned} \tag{4}$$

Based on Øksendal (2003), by replacing $\eta(t)$ with $\frac{dw_1(t)}{dt}$ and $\xi_2(t)$ with $\frac{dw_2(t)}{dt}$, we obtain

$$\begin{aligned}dx(t) &= (A(r_t)x(t) + B_0(r_t)u(t) + B_0(r_t)V(r_t)m(t))dt \\ &\quad + F(r_t)dw_1(t) + B_2(r_t)x(t)dw_2(t),\end{aligned} \tag{5}$$

$$dm(t) = (M(r_t)m(t) + D(r_t)\delta(t))dt, \tag{6}$$