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Brief paper

Switch observability for switched linear systems ☆

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ABSTRACT

Mode observability of switched systems requires observability of each individual mode. We consider other concepts of observability that do not have this requirement: Switching time observability and switch observability. The latter notion is based on the assumption that at least one switch occurs. These concepts are analyzed and characterized both for homogeneous and inhomogeneous systems.

1. Introduction

Mode observability of switched systems is concerned with recovering the initial state as well as the switching signal from the output (and the input) and has been widely studied, see e.g. Vidal, Chiuso, Soatto, and Sastry (2003) for homogeneous systems, Elhamifar, Petreczky, and Vidal (2009) for inhomogeneous discrete-time systems, Babaali and Pappas (2005) for a generic observability notion of inhomogeneous systems and Lou and Si (2009) for inhomogeneous systems. For a recent overview of observability for general hybrid systems, see De Santis and Di Benedetto (2016).

Since for mode observable systems it is in particular possible to recover the state for constant switching signals, each mode necessarily has to be observable. In the context of fault-detection (or diagnosis) the different modes of a switched system describe faulty and non-faulty variants of the system and a switch represents a fault. Requiring observability of each mode, in particular of each faulty mode, might be a too strong assumption. Instead of mode observability, it would be sufficient to compute the switching signal and the state *if an error occurs*. This idea is formalized in the novel notion of switch observability, (x, σ_1)-observability for short.

Before characterizing (x, σ_1)-observability, we first have to consider the problem of detecting switches (switching time observability or t_S-observability). This has been done in Vidal et al. (2003) in the homogeneous case, but the generalization to inhomogeneous systems is not straightforward as the switch might occur in an interval where the state is zero. This difficulty has been avoided so far, e.g. in Elhamifar et al. (2009) by assuming mode observability. We are able to relax this assumption and to fully characterize t_S-observability without any additional assumptions.

Similar to the classical observability of linear systems, we derive characterizations of the observability notions based on rank-conditions on the Kalman observability matrices. Our results are summarized in Fig. 1, where $\mathcal{O}_i$ and Γ_i are the Kalman observability matrix and Hankel matrix of mode i, respectively. These notions are defined in Sections 2 and 3; $\text{rk}(A)$ denotes the rank of A.

The first column in Fig. 1 gives the result for the homogeneous case: The strongest notion considered here is (x, σ)-observability, which coincides with switching signal observability (σ-observability). It implies (x, σ_1)-observability and t_S-observability. The reverse implications are false in general, we will show this by some examples. For the inhomogeneous case, we consider two different setups. First we restrict our attention to systems with analytic input and with some restriction on the input matrices (assumption (A2)). Then we drop (A2) and require only smooth input. This makes it necessary to consider equivalence classes of switching signals, but gives observability notions with the same characterizations as in the more restrictive setup

Our main contribution is the concept of (strong) (x, σ_1)-observability and its characterization. Also the characterization of strong switching time observability for inhomogeneous systems is new.

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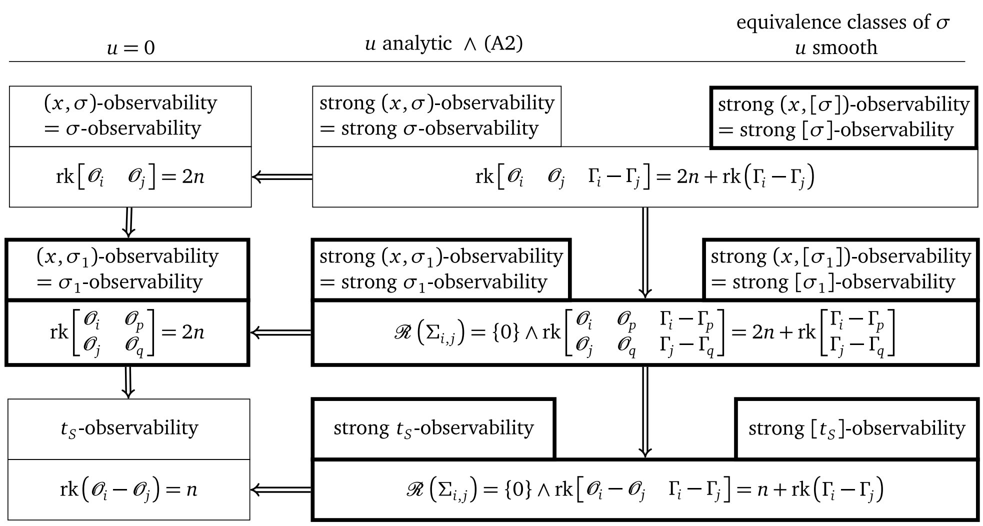

Fig. 1. Brief characterizations of the observability notions and their relations. Novel results are indicated by bold boxes.

2. Homogeneous systems

2.1. System class and preliminaries

A *switching signal* is a piecewise constant, right-continuous function $\sigma\colon \mathbb{R} \to \mathscr{P} := \{1,\dots,N\}$, $N \in \mathbb{N}$, with locally finitely many discontinuities. The discontinuities of σ are also called *switching times*:

$$T_\sigma := \{\, t_S \in \mathbb{R} \mid t_S \text{ is a discontinuity of } \sigma \,\}.$$

We assume that all switches occur for $t > 0$, i.e. $T_\sigma \subset \mathbb{R}_{>0}$. Consider switched linear systems of the form

$$\dot{x} = A_\sigma x, \quad x(0) = x_0, \tag{1a}$$
$$y = C_\sigma x, \tag{1b}$$

with switching signal σ and $A_i \in \mathbb{R}^{n\times n}$, $C_i \in \mathbb{R}^{p\times n}$ for all $i \in \mathscr{P}$ and denote its solution and output by $x_{(x_0,\sigma)}$ and $y_{(x_0,\sigma)}$, respectively.

Furthermore, let $\mathcal{O}_i^{[\nu]}$ be the Kalman observability matrix for mode i with ν row blocks, i.e.

$$\mathcal{O}_i^{[\nu]} = \begin{bmatrix} C_i^\top & (C_iA_i)^\top & (C_iA_i^2)^\top & \cdots & (C_iA_i^{\nu-1})^\top \end{bmatrix}^\top$$

and let $\mathcal{O}_i^{[\infty]}$ be the corresponding infinite Kalman observability matrix. For observability of unswitched systems, it suffices to consider $\nu = n$. In our setting, the required size increases as we have to compare the output from different modes.

For any sufficiently smooth function $y : \mathbb{R} \to \mathbb{R}^p$ denote by $y^{[\nu]} : \mathbb{R} \to \mathbb{R}^{\nu p}$ the vector of y and its first $\nu - 1$ derivatives and by $y^{[\infty]}$ the (countably) infinite vector of y and its derivatives. The same can be done for piecewise-smooth functions, where $y(t^-)$ and $y(t^+)$ denote the left-hand side and right-hand side limit at t, respectively. Then the output $y_{(x_0,\sigma)}$ of (1) satisfies for all $t \in \mathbb{R}$:

$$y^{[\nu]}_{(x_0,\sigma)}(t^+) = \mathcal{O}^{[\nu]}_{\sigma(t^+)} x_{(x_0,\sigma)}(t), \quad \nu \in \mathbb{N}\cup\{\infty\},$$
$$y^{[\nu]}_{(x_0,\sigma)}(t^-) = \mathcal{O}^{[\nu]}_{\sigma(t^-)} x_{(x_0,\sigma)}(t), \quad \nu \in \mathbb{N}\cup\{\infty\}.$$

2.2. Known results and definitions

Definition 1. The switched system (1) is called

- (x,σ)-*observable* iff for all $(x_0, \widetilde{x}_0) \neq (0,0)$ the following implication holds:

  $$(x_0 \neq \widetilde{x}_0 \ \vee\ \sigma \not\equiv \widetilde{\sigma}) \quad\Rightarrow\quad y_{(x_0,\sigma)} \not\equiv y_{(\widetilde{x}_0,\widetilde{\sigma})}, \tag{2}$$

  i.e., iff it is possible to determine simultaneously the state and current mode from the output;
- σ-*observable* iff for all $(x_0, \widetilde{x}_0) \neq (0,0)$

  $$\sigma \not\equiv \widetilde{\sigma} \quad\Rightarrow\quad y_{(x_0,\sigma)} \not\equiv y_{(\widetilde{x}_0,\widetilde{\sigma})}, \tag{3}$$

  i.e., iff it is possible to determine the current mode from the output;
- t_S-*observable* (or *switching time observable*) iff for all $x_0 \neq 0$, σ nonconstant and all $\widetilde{x}_0, \widetilde{\sigma}$:

  $$T_\sigma \neq T_{\widetilde{\sigma}} \quad\Rightarrow\quad y_{(x_0,\sigma)} \not\equiv y_{(\widetilde{x}_0,\widetilde{\sigma})},$$

  i.e., iff it is possible to determine the switching times from the output.

Clearly, (x,σ)-observability implies σ-observability which in turn implies t_S-observability. Furthermore, it seems quite obvious that it is much harder to determine both the state and the switching signal compared to just determining the current mode from the output. However, this intuition is wrong:

Lemma 2. *For the switched system (1) it holds that*

$$(x,\sigma)-\textit{observability} \quad\Leftrightarrow\quad \sigma-\textit{observability}.$$

Proof. The implication "⇒" is clear. Now let the system be σ-observable, but not (x,σ)-observable. This means that there exist $(x_0, \widetilde{x}_0) \neq (0,0)$ and $\sigma, \widetilde{\sigma}$ with

$$(x_0 \neq \widetilde{x}_0 \ \vee\ \sigma \not\equiv \widetilde{\sigma}) \ \wedge\ y_{(x_0,\sigma)} \equiv y_{(\widetilde{x}_0,\widetilde{\sigma})}.$$

$\sigma \not\equiv \widetilde{\sigma}$ would contradict σ-observability. Hence we have $\sigma \equiv \widetilde{\sigma}$ and $x_0 \neq \widetilde{x}_0$. This means that $y_{(x_0,\sigma)} \equiv y_{(\widetilde{x}_0,\sigma)}$ and, by linearity, $y_{(x_0-\widetilde{x}_0,\sigma)} \equiv 0$. This contradicts σ-observability, as it implies $y_{(x_0-\widetilde{x}_0,\sigma)} \equiv 0 \equiv y_{(0,\hat{\sigma})}$ for all $\hat{\sigma}$. □

This relation was already implicitly stated in Elhamifar et al. (2009) for discrete-time systems. Note that observability of the (continuous) state in each mode is necessary for (x,σ)-observability (just consider the constant switching signals). However, state-observability in each mode is not sufficient for (x,σ)-observability (c.f. Babaali and Pappas, 2005). A trivial counterexample for the latter is a system for which each mode describes the same observable system.

The next example shows that t_S-observability is indeed weaker than (x,σ)-observability: