Automatica 87 (2018) 251–257

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Dynamics of delayed switched nonlinear systems with applications to cascade systems[☆]

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ARTICLE INFO

Article history:
Received 23 December 2016
Received in revised form 5 May 2017
Accepted 15 September 2017
Available online 6 November 2017

Keywords:
Asymptotic stability
Cascade systems
Delays
Exponential stability
Switched systems

ABSTRACT

This paper addresses the dynamic properties of a class of continuous-time switched nonlinear systems with perturbations and delays. With the assumption that the nominal system is exponentially stable, it is shown that the trajectory of the perturbed system exponentially decays to or asymptotically approaches origin provided that the perturbation exponentially decays to or asymptotically approaches origin. These properties are then applied to cascade systems for their stability analysis. It is proven that a delayed switched nonlinear cascade system is exponentially stable if and only if all subsystems obtained from the cascade system by deleting the coupling terms are exponentially stable. A sufficient condition ensuring asymptotic stability of a cascade system is also proposed.

1. Introduction

Many systems in the real world can be modeled by switched systems with combined continuous and discrete states (Fu, Ma, & Chai, 2015; Liberzon, 2003). A switched system consists of a family of (finitely or infinitely many) dynamic subsystems with a rule, called a switching signal, that orchestrates the switching behavior among the subsystems. Researchers from the control and system theory communities have been working with switched systems due to their abilities to capture the dynamics of various physical systems (Deaecto, Souza, & Geromel, 2015; Li, Soh, & Wen, 2005; Sun & Ge, 2011; Xiang & Xiao, 2011). Since nonlinearities, perturbations, and delays widely exist in various engineering systems and have complicated impacts on their dynamics (Hale & Verduyn Lunel, 1993; Yang, Zhong, Li, & Luo, 2009; Zamani, Shafiee, & Ibeas, 2015), delayed switched nonlinear systems subject to perturbations have been intensively investigated (Sun & Wang, 2013; Zamani, Shafiee, & Ibeas, 2014) and will be further discussed in the present paper.

Switched systems have many complex dynamic properties. Stabilities in the sense of Lyapunov were investigated in Mahmoud and AL-Sunni (2010), controllability and observability were studied in Baştuğ, Petreczky, Wisniewski, and Leth (2016) and Pequito and Pappas (2017), and the issue of stabilization was discussed in Fu, Li, Chai, and Su (2016). Convergence characteristics were deeply discussed in Serres, Vivalda, and Riedinger (2011) for switched linear systems with different switching signals, without considering the effects of perturbations. It is observed that there exist different perturbations and each of them has different influence on system dynamics (Anh, Son, & Thanh, 2009). In Li, Zhao, Dimirovski, and Liu (2010), the authors pointed out that if a nominal delayed switched linear system is exponentially stable, and if the exogenous perturbation asymptotically decays (exponentially decay to or asymptotically approach zero), then the perturbed system behaves as the perturbation. This paper mainly focuses on the dynamics of systems subject to decaying perturbations.

Cascade systems have broad applications in engineering (Chen, 2009; Ding, Li, & Zheng, 2012). It is interesting and helpful to treat cascade systems from the following viewpoint: Suppose that the state $\boldsymbol{x}$ can be partitioned into two parts $\boldsymbol{x} = \left[\boldsymbol{x}_1^{\mathrm{T}}\ \boldsymbol{x}_2^{\mathrm{T}}\right]^{\mathrm{T}}$ such that the evolution of $\boldsymbol{x}_1$ is not affected by $\boldsymbol{x}_2$ and $\boldsymbol{x}_1$ does impact on the evolution of $\boldsymbol{x}_2$. In this situation, if $\boldsymbol{x}_1$ asymptotically approaches zero, then it may be viewed as a decaying external perturbation of $\boldsymbol{x}_2$. For nonlinear cascade systems, it is desirable to establish stability condition by exploring the stability properties of all subsystems obtained by deleting coupling terms, because each subsystem has lower dimension and is easier to study. Inspired by the thought, it was shown that a block triangular system with time-varying delays (a special class of cascade systems) is exponentially stable if and

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Luca Zaccarian under the direction of Editor Daniel Liberzon.

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https://doi.org/10.1016/j.automatica.2017.10.012

only if each system described by the diagonal blocks of the original system matrices is exponentially stable (Liu, 2014). This paper tries to extend the results in Liu (2014) to cascade switched nonlinear system with delays.

Based on the above observations, it is seen that exploring the dynamics of switched nonlinear systems with delays and perturbation is the key task in the present paper. There are different methods to analyze and synthesize a system subject to perturbations. The frequently used methods include H_∞, H_2 approaches (Li & Xiang, 2016; Luan, Zhao, & Liu, 2013), where the Lyapunov method was employed. This method is inapplicable here, since the nominal system may be a delayed, perturbed, switched nonlinear system, and in this situation a converse Lyapunov theorem has not been established for exponential stability. The method employed in Li et al. (2010) cannot be used here since the model in Li et al. (2010) is linear. Therefore, some new approach is required.

The basic idea employed in the present paper lies in: a covering function is constructed guaranteeing the converge of trajectory of the perturbed systems. We call this method as "covering method". The main contribution of the paper lies in the following two aspects: (1) it is shown that, with the assumption that the nominal switched nonlinear system is exponentially stable, the perturbed system will decay at an exponential rate if the perturbation decays at an exponential rate, and will asymptotically approach zero if so does the perturbation, both global and local cases are investigated. (2) the previous findings are applied to cascade switched nonlinear systems, showing that the stability property of a cascade system can be analyzed in a decomposition manner.

Notation: $\mathbb{R}_{0,+} = [0, \infty)$, $\mathbb{R}_+ = (0, \infty)$. $\mathbb{N}_0$ denotes the set of nonnegative integers and $\mathbb{N} = \mathbb{N}_0 \backslash \{0\}$. For any $m \in \mathbb{N}$, $\underline{m} = \{1, \ldots, m\}$. $|a|$ is the absolute value of a real number a. $\lceil a \rceil$ is the minimum integer not less than real number a. $\|\boldsymbol{x}\|_\infty = \max\{|x_1|, \ldots, |x_n|\}$ is the l_∞ norm of vector $\boldsymbol{x} \in \mathbb{R}^n$, and for simplicity, is denoted by $\|\boldsymbol{x}\|$. For any continuous function $\boldsymbol{x}(s)$ on $[-d, a)$ with scalars $a > 0$, $d > 0$ and any $t \in [0, a)$, $\boldsymbol{x}_t$ denotes a continuous function on $[t-d, t]$ defined by $\boldsymbol{x}_t(\theta) = \boldsymbol{x}(t+\theta)$ for each $\theta \in [-d, 0]$. $\|\boldsymbol{x}_t\| = \sup_{t-d\le s\le t}\{\|\boldsymbol{x}(s)\|\}$. $\mathcal{B}_a = \{\boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x}\| < a\}$. $\mathcal{C}([a, b], \mathbb{R}^n)$ is the set of continuous functions from interval $[a, b]$ to $\mathbb{R}^n$. $\mathcal{C}_\delta([a, b], \mathbb{R}^n) = \{\boldsymbol{x} \in \mathcal{C}([a, b], \mathbb{R}^n) : \|\boldsymbol{x}\| < \delta\}$.

2. Preliminaries and problem statements

Consider the following switched nonlinear system:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}_{\sigma(t)}(t, \boldsymbol{x}_t), \quad t \ge t_0 \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t \in [t_0 - d, t_0] \end{aligned} \tag{1}$$

where $t_0 \ge 0$, $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state, $\sigma : [t_0, \infty) \to \underline{m}$ is a switching signal with m being the number of subsystems. It is assumed that $\sigma(t)$ is with switching sequence $\{t_i\}_{i=0}^\infty$ satisfying $t_i > t_{i-1}(\forall i \in \mathbb{N})$ and $\lim_{i\to\infty} t_i = \infty$ and that $\sigma(t)$ is piecewise constant and continuous from the right. For each $l \in \underline{m}$, $\boldsymbol{f}_l$ maps $[t_0, \infty) \times \mathcal{C}([-d_{2l}, -d_{1l}], \mathbb{R}^n)$ into $\mathbb{R}^n$ with d_{1l} and d_{2l} being constants, $0 \le d_{1l} \le d_{2l}$, $d = \max_{l\in\underline{m}}\{d_{2l}\}$. $\boldsymbol{\phi}(t) \in \mathcal{C}([t_0 - d, t_0], \mathbb{R}^n)$ is an initial vector-valued function.

The following assumption is made for system (1).

Assumption 1. $\boldsymbol{f}_l$ is continuous on $[t_0, \infty) \times \mathcal{C}([-d_{2l}, -d_{1l}], \mathbb{R}^n)$. Moreover, there exist positive scalars L, δ such that

$$\begin{aligned} &\|\boldsymbol{f}_l(\cdot, \boldsymbol{x}) - \boldsymbol{f}_l(\cdot, \boldsymbol{y})\| \le L\|\boldsymbol{x} - \boldsymbol{y}\| \\ &\qquad \boldsymbol{x}, \boldsymbol{y} \in \mathcal{C}_\delta\left([-d_{2l}, -d_{1l}], \mathbb{R}^n\right) \end{aligned} \tag{2}$$

If $\delta = \infty$, then $\boldsymbol{f}_l$ is globally Lipschitz.

There are different kinds of switching signals some of which are defined below.

Definition 2 (*Liberzon, 2003*). For switching signal σ, $\tau_d \in \mathbb{R}_+$ is said to be the dwell time of σ if $t_{i+1} - t_i \ge \tau_d$, $\forall i \in \mathbb{N}_0$. For any $T > t \ge t_0$, let $N_\sigma(T, t)$ be the switching numbers of σ on the open interval (t, T). σ is said to have average dwell time τ_a and "chatter-bound" N_0 if there exist two positive numbers N_0 and τ_a such that $N_\sigma(T, t) \le N_0 + \frac{T-t}{\tau_a}$. A switching signal σ is said to be periodic if there exists a scalar $\kappa > 0$ such that $\sigma(t+\kappa) = \sigma(t)$ holds for any $t \ge t_0$, such a minimal κ is called the period of σ.

The following four sets of switching signals are frequently encountered in literature (Liu & Liu, 2016) and will be considered in the sequel: (i) $\mathbb{S}_1 = \{\sigma : \sigma$ has finite many discontinuities on any finite interval$\}$; (ii) $\mathbb{S}_2(N_0, \tau_a) = \{\sigma : \sigma$ has average dwell time τ_a and chatter-bound $N_0\}$; (iii) $\mathbb{S}_3(\tau_d) = \{\sigma : t_{i+1} - t_i \ge \tau_d > 0, \forall i \in \mathbb{N}_0\}$; and (vi) $\mathbb{S}_4(\kappa) = \{\sigma : \sigma$ has a period $\kappa\}$. A system has a certain property over a given set $\mathbb{S}$ of switching signals if the property holds for all switching signals in $\mathbb{S}$. In what follows, it is assumed that the switching signal set $\mathbb{S}$ is any one of the four sets: $\mathbb{S}_1$, $\mathbb{S}_2(N_0, \tau_a)$, $\mathbb{S}_3(\tau_d)$, $\mathbb{S}_4(\kappa)$. In order to avoid any ambiguity, we will explicitly point out a system has a property "over $\mathbb{S}$″ when the underlying set $\mathbb{S}$ needs to be mentioned, otherwise "over $\mathbb{S}$″ will be omitted.

Definition 3 (*Khalil, 2002*). A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$, where a is any positive real number or ∞, and is said to belong to class $\mathcal{K}_\infty$ if it belongs to class $\mathcal{K}$ and $\alpha(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

Definition 4 (*Mahmoud, 2010*). Denote $\boldsymbol{x}(t; t_0, \boldsymbol{\phi})$ the solution to (1) with starting time t_0 and initial function $\boldsymbol{\phi}$. System (1) is locally uniformly exponentially stable (LUES) if there exist positive scalars α, γ, δ such that $\|\boldsymbol{x}(t; t_0, \boldsymbol{\phi})\| \le \alpha\exp(-\gamma(t - t_0))\|\boldsymbol{\phi}\|$, $\forall t_0 \ge 0$, $t \ge t_0$, $\|\boldsymbol{\phi}\| \le \delta$; if δ can be arbitrarily large, then system (1) is globally uniformly exponentially stable (GUES). System (1) is locally uniformly asymptotically stable (LUAS) if there exists a class $\mathcal{KL}$ function β such that $\|\boldsymbol{x}(t; t_0, \boldsymbol{\phi})\| \le \beta(\|\boldsymbol{\phi}\|, t - t_0)$, $\forall t_0 \ge 0$, $t \ge t_0$, $\|\boldsymbol{\phi}\| \le \delta$; if δ can be arbitrarily large, then (1) is globally uniformly asymptotically stable (GUAS).

Consider the following perturbed system of (1)

$$\begin{aligned} &\dot{\boldsymbol{y}}(t) = \boldsymbol{f}_{\sigma(t)}(t, \boldsymbol{y}_t) + \boldsymbol{u}(t), \quad t \ge t_0 \\ &\boldsymbol{y}(t) = \boldsymbol{\phi}(t), \quad t \in [t_0 - d, t_0]. \end{aligned} \tag{3}$$

For system (3), two assumptions are imposed on perturbation $\boldsymbol{u}(t)$ in different situations.

Assumption 5. There exist $\alpha_1 > 0, \gamma_1 > 0$ such that $\|\boldsymbol{u}(t)\| \le \alpha_1\exp(-\gamma_1(t - t_0))$, $t \ge t_0$.

Assumption 6. There exists a function $\rho : \mathbb{R}_{0,+} \to \mathbb{R}_{0,+}$ which is monotonically decreasing and converges to zero such that $\|\boldsymbol{u}(t)\| \le \rho(t - t_0)$, $t \ge t_0$.

Objectives: With the assumption that system (1) is exponentially stable, we will first explore the dynamic properties of system (3) with perturbation $\boldsymbol{u}(t)$ satisfying Assumptions 5 or 6, and then apply these properties to establish stability criteria for cascade systems.

3. Dynamics of delayed switched systems with perturbations

This section investigates the dynamics of switched systems with perturbations. To fulfill the task, we need to estimate the