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Brief paper

Robust stability of uncertain linear systems with input and output quantization and packet loss☆

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ABSTRACT

This paper investigates the robust stability of uncertain discrete-time linear systems subject to input and output quantization and packet loss. First, a necessary and sufficient condition in terms of LMIs is proposed for the quadratic stability of the closed-loop system with double quantization and norm bounded uncertainty in the plant. Moreover, it is shown that the proposed condition can be exploited to derive the coarsest logarithmic quantization density under which the uncertain plant can be quadratically stabilized via quantized state feedback. Second, a new class of Lyapunov function which depends on the quantization errors in a multilinear way is developed to obtain less conservative results. Lastly, the case with input and output packet-loss channels is investigated.

1. Introduction

Motivated by finite network resource, quantized feedback control has been one of the most popular research trends in the field of networked control systems (see, e.g., Zhang, Gao, and Kaynak, 2013). It naturally becomes significant that how the quantization error will influence the stability and performance of the feedback systems. In the meanwhile, a great deal of effort has gone into establishing the minimum feedback information needed to stabilize an open-loop unstable system. Perhaps the most important results in recent years on the quantized feedback control should be traced back to Elia and Mitter (2001) where the logarithmic quantization was proposed and shown to be the coarsest quantizer to quadratically stabilize discrete-time linear time-invariant systems. The logarithmic quantizer was further investigated by Fu and Xie (2005) in which the sector bound approach was exploited to relate the design problem for quantized feedback control to the optimal H_∞ control problem. Besides, the quantized feedback control problem has been studied in different scenarios. For instance, Gu and Qiu (2014) put forward the polar logarithmic quantization for multi-input systems; Gu, Wan, and Qiu (2015) studied the mean-square stabilization for networked control systems with both fading channels and logarithmic quantization; Coutinho, Fu, and de Souza (2010) and Xia, Yan, Shi, and Fu (2013) considered feedback control systems with input and output quantization. On the other hand, packet loss is also a widely studied topic as one of the main communication constraints, see, e.g., Rich and Elia (2015). Among the works considering both the effect of quantization and packet loss, one should mention Ishido, Takaba, & Quevedo (2011) which investigated the digital channel subject to packet loss and finite-level quantization, Tsumura, Ishii, and Hoshina (2009) which analyzed the tradeoffs between the coarsest quantizer, packet-loss rate and the instability of the plant.

More recently, another research aspect that researchers have started to deal with is the effect of plant uncertainty. See, e.g., Su and Chesi (2017a) which considered robust stability of uncertain system over fading channels, Fu and Xie (2010) where sufficient condition was proposed for robust stabilization for linear uncertain systems via logarithmic quantized feedback, Liu, Fridman, and Johansson (2015) which studied the stability analysis of continuous-time uncertain system with dynamic quantization and communication delays, Hayakawa, Ishii, and Tsumura (2009) in which adaptive quantized control was designed for nonlinear uncertain system, Kang and Ishii (2015) which considered coarsest quantization for a class of finite-order uncertain autoregressive plant.

In this paper, we first consider the model of double quantization as studied in Coutinho et al. (2010) with the plant affected by unstructured uncertainty and then further integrate the effect of input and output packet loss. Specifically, the controller output and the plant output are transmitted through input and output packet-loss channels respectively after being quantized via two independent logarithmic quantizers. First, a necessary and sufficient condition in terms of LMIs is proposed for the quadratic stability

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of the closed-loop system with double quantization and norm bounded uncertainty in the plant. Moreover, it is shown that the proposed condition can be exploited to derive the coarsest logarithmic quantization density under which the uncertain plant can be robustly quadratically stabilized via quantized state feedback. Second, a new class of Lyapunov function which depends on the quantization errors in a multilinear way is developed to obtain less conservative result. Lastly, a sufficient condition is established to ensure the robust stability in the mean square sense for the uncertain closed-loop systems with input and output quantization and packet-loss channels. A conference version of this paper (without Section 4 and part of Section 5) is reported in Su and Chesi (2017b).

2. Quadratic stability of uncertain systems with input and output quantization

In this section, we focus on the robust quadratic stability of uncertain systems with input and output quantization. Let us first consider the single-input single-output plant affected by uncertainty described as

$$\begin{cases} x_p(k+1) = (A+A_1)x_p(k) + (B+B_1)u(k) \\ y(k) = Cx_p(k) \end{cases} \tag{1}$$

where $x_p(k) \in \mathbb{R}^n$ is the plant state, $u(k) \in \mathbb{R}$ is the plant input and $y(k) \in \mathbb{R}$ is the plant output, $(A,\ B)$ is the nominal system and the time-varying uncertainty $(A_1,\ B_1)$ is assumed to be norm bounded satisfying

$$[A_1\ B_1] = HF(k)[E_1\ E_2],\ F(k)F(k)^T \le I. \tag{2}$$

The controller is assumed to be dynamic, described as

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c v(k) \\ w(k) = C_c x_c(k) + D_c v(k) \end{cases} \tag{3}$$

where $x_c(k) \in \mathbb{R}^{n_c}$ is the controller state, $v(k) \in \mathbb{R}$ is the controller input and $w(k) \in \mathbb{R}$ is the controller output.

Following the works Elia & Mitter (2001) and Fu & Xie (2005), we utilize the logarithmic quantization defined as

$$Q(v) = \begin{cases} \rho^i & \text{if } \dfrac{1}{1+\delta}\rho^i < v \le \dfrac{1}{1-\delta}\rho^i \\ & v > 0,\ i = \pm 1, \pm 2, \ldots \\ 0 & \text{if } v = 0 \\ -Q(-v) & \text{if } v < 0 \end{cases} \tag{4}$$

where $0 < \rho < 1$ is the quantization density and $\delta = \frac{1-\rho}{1+\rho}$. It is assumed that the output of the plant $y(k)$ is quantized before being sent to the input of the controller $v(k)$ and the output of the controller $w(k)$ is quantized before being sent to the input of the plant $u(k)$. The two quantizers are modeled as

$$v(k) = Q_1(y(k)),\ u(k) = Q_2(w(k)) \tag{5}$$

where $Q_1(\cdot)$ and $Q_2(\cdot)$ are static logarithmic quantizers with quantization density ρ_1 and ρ_2.

Let $x(k) = [x_p(k)^T\ x_c(k)^T]^T$ be the state of the closed-loop system. Comprising the plant, the controller and the quantizers, such a closed-loop system is given by

$$\begin{aligned} x(k+1) &= \begin{pmatrix} x_p(k+1) \\ x_c(k+1) \end{pmatrix} = \begin{pmatrix} (A+A_1)x_p(k) \\ A_c x_c(k) \end{pmatrix} \\ &+ \begin{pmatrix} (B+B_1)Q_2(C_c x_c(k) + D_c Q_1(Cx_p(k))) \\ B_c Q_1(Cx_p(k)) \end{pmatrix}. \end{aligned} \tag{6}$$

When there is no uncertainty in the plant, i.e., $A_1 = 0$ and $B_1 = 0$, it is shown in Theorem 2 of Coutinho et al. (2010) that the closed-loop system (6) is quadratically stable if and only if there exists $P > 0$ such that

$$\begin{aligned} &\bar{A}(\Delta_1, \Delta_2)^T P \bar{A}(\Delta_1, \Delta_2) - P < 0 \\ &\quad \forall |\Delta_1| \le \delta_1, |\Delta_2| \le \delta_2 \end{aligned} \tag{7}$$

where

$$\bar{A}(\Delta_1, \Delta_2) = \begin{pmatrix} A + B(1+\Delta_2)D_c(1+\Delta_1)C & B(1+\Delta_2)C_c \\ B_c(1+\Delta_1)C & A_c \end{pmatrix}. \tag{8}$$

Lemma 1 (*Amato, Garofalo, Glielmo, and Pironti, 1996; Garofalo, Celentano, and Glielmo, 1993*)**.** *Consider the matrix-valued function $M(p) : \mathcal{P} \to \mathbb{R}^{n\times n}$, where $p \in \mathcal{P} \subset \mathbb{R}^q$ and the set $\mathcal{P}$ is a hyper-box, i.e., $\mathcal{P} := [\underline{p_1}, \overline{p_1}] \times [\underline{p_2}, \overline{p_2}] \times \cdots \times [\underline{p_q}, \overline{p_q}]$. Let us assume*

$$M(p) = \frac{N(p)}{d(p)}, \tag{9}$$

with $N(\cdot)$ a multi-affine matrix-valued function of p, $d(\cdot)$ a multi-affine polynomial of p and $d(p) \ne 0$ for all $p \in \mathcal{P}$. Then $M(p) > 0, \forall p \in \mathcal{P}$ if and only if $M(p_{(i)}) > 0, i = 1, \ldots, 2^q$ where $p_{(i)}$ is the i-th vertex of $\mathcal{P}$.

Therefore, it is necessary and sufficient to check the quadratic stability of an uncertain system depending multi-affinely on uncertain parameters constrained into a hyper-box on the vertices of the hyper-box under the same Lyapunov function $v(x(k)) = x(k)^T P x(k)$.

Next, let us take the uncertainty (A_1, B_1) into consideration. By treating the quantization errors as sector bounded time-varying uncertainties, let us define the auxiliary system for (6) as

$$\begin{cases} x(k+1) = \hat{A}(\Delta_1(k), \Delta_2(k))x(k) \\ \hat{A}(\Delta_1, \Delta_2) = \begin{pmatrix} A+A_1 & 0 \\ 0 & A_c \end{pmatrix} + \\ \begin{pmatrix} (B+B_1)(1+\Delta_2)([0\ \ C_c] + D_c(1+\Delta_1)[C\ \ 0]) \\ B_c(1+\Delta_1)[C\ \ 0] \end{pmatrix} \\ \forall |\Delta_1(k)| \le \delta_1, |\Delta_2(k)| \le \delta_2. \end{cases} \tag{10}$$

Before proceeding to our main result, let us report the following result (see, e.g., Xie, 1996).

Lemma 2. *Given real matrices $\mathcal{S} = \mathcal{S}^T, \mathcal{U}, \mathcal{V}$ with appropriate dimension, then*

$$\mathcal{S} + \mathcal{U}F(k)\mathcal{V} + \mathcal{V}^T F(k)^T \mathcal{U}^T > 0 \tag{11}$$

holds for all $F(k)$ satisfying $F(k)F(k)^T \le I$ if and only if there exists a scalar $\sigma > 0$ such that

$$\mathcal{S} - \sigma\mathcal{U}\mathcal{U}^T - \sigma^{-1}\mathcal{V}^T\mathcal{V} > 0. \tag{12}$$

Theorem 3. *The closed-loop system (6) is robustly quadratically stable if and only if there exist $Q > 0$ and a scalar $\sigma(\Delta_1, \Delta_2) > 0$*[1] *such that*

$$\begin{aligned} &\begin{pmatrix} Q & Q\bar{A}(\Delta_1, \Delta_2)^T & Q\bar{E}(\Delta_1, \Delta_2)^T \\ * & Q - \sigma(\Delta_1, \Delta_2)\bar{H}\bar{H}^T & 0 \\ * & * & \sigma(\Delta_1, \Delta_2)I \end{pmatrix} > 0 \\ &\quad \forall \Delta_1 \in \{-\delta_1, \delta_1\}, \Delta_2 \in \{-\delta_2, \delta_2\} \end{aligned} \tag{13}$$

where $\bar{A}(\Delta_1, \Delta_2)$ is defined in (8), and

$$\begin{cases} \bar{E}(\Delta_1, \Delta_2) = \\ \quad \begin{pmatrix} E_1 + E_2(1+\Delta_2)D_c(1+\Delta_1)C & E_2(1+\Delta_2)C_c \end{pmatrix} \\ \bar{H} = \begin{pmatrix} H^T & 0 \end{pmatrix}^T. \end{cases} \tag{14}$$

[1] Since (Δ_1, Δ_2) takes value only at the vertices of $[-\delta_1, \delta_1] \times [-\delta_2, \delta_2]$ in (13), $\sigma(\Delta_1, \Delta_2)$ amounts to 4 scalar variables corresponding to the 4 vertices, i.e., the variable σ in the LMI (13) is allowed to vary with different vertex.