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Brief Paper

Markov inequality rule for switching among time optimal controllers in a multiple vehicle intercept problem ☆

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ABSTRACT

In this paper, a Markov inequality based switching rule is proposed to switch among numerically computed, time optimal controllers in a multiple vehicle intercept problem. Each controller is optimal for the intercept of a single vehicle, i.e., for the *segment* of the complete time varying multiple vehicle target set. The switching rule guarantees that after every switch the time to the target set is shorter with a certain predefined probability. Furthermore, the rule guarantees that the target set is reached after a finite number of switches and the rule scales well with the number of vehicles, i.e., the segments covering the target set. The problem and results are illustrated by a numerical example.

1. Introduction

Numerical methods of stochastic optimal control (Kushner & Dupuis, 2001) are attractive for designing state feedback controllers for nonlinear dynamical systems described by stochastic differential equations (Øksendal, 2010). Generally, these computed controllers are in the form of a lookup table and can be executed fast in real-time applications for reaching a target set in a minimum expected time (Anderson, Efstathios, Milutinović, & Panagiotis, 2012). However, the computed controller is specific to the target set, and in the case of a time varying target set, the controller must be recomputed, which in most cases is too slow for real time implementations.

This paper is motivated by a multiple vehicle intercept problem presented in Section 2. While we know how to compute a minimum time optimal control to intercept a single vehicle by using its relative position coordinates, in the case of multiple vehicles, the number of necessary relative coordinates increases linearly, which quickly exhausts the computational power for computing an optimal solution due to the so-called *curse of dimensionality*. In addition, let us assume that we can compute the controller for N vehicles. That controller will be valid only for the specific number N. Adding or removing one vehicle would require that the optimal control be changed. It would mean that we need to compute not one, but multiple optimal controllers, one for each possible number of vehicles up to a maximum number of vehicles, and select or switch to the controller that matches the current number of vehicles. Since it is known (Branicky, 1998) that switching among multiple satisfactory controllers may lead to unsatisfactory outcomes, this is unacceptable and an additional level of analysis is necessary.

For the case of multiple vehicles, we propose to use the minimum time optimal control to intercept a single vehicle and a switching rule to select which vehicle to intercept. The proposed rule guarantees that after every switch, the time to intercept is reduced with a certain predefined probability. The rule is based on the computed state dependent expected time to intercept and the optimal control; therefore, the switching is also state dependent. The resulting switching rule based navigation scales well with the number of vehicles.

The dynamics of the motivating problem (Section 2) with switchings among the vehicles can be modeled with *deterministic transitions* over a finite number of discrete states (one per vehicle) and stochastic differential equation dynamics in each state, the so-called stochastic switched system (see Table 1 in Teel, Subbaramana, and Sferlazza (2014)). The stability of deterministic versions of such systems has been studied using Lyapunov function and multiple Lyapunov function approaches (Branicky, 1998) and a stability analysis of stochastic versions used similar tools applied to statistical estimates of Lyapunov functions (Chatterjee & Liberzon, 2004) and a comparison principle (Chatterjee & Liberzon, 2006). The moment stability for such systems has also been analyzed in Boukas (2006), Feng, Jieand, and Ping (2011), Feng and Zhang

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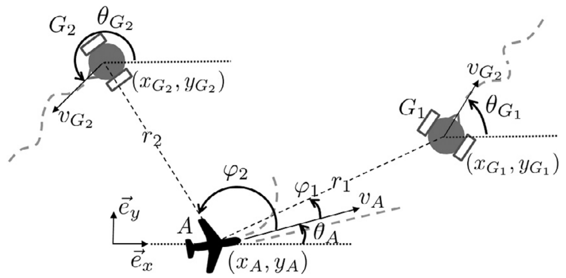

Fig. 1. Geometry and state-space representation of an aerial vehicle A at position (x_A, y_A) and two ground vehicles (G_i), at (x_{G_i}, y_{G_i}), $i = 1, 2$; θ_{EGi}, θ_A are heading angles; φ_i and r_i are bearing angles and ranges from A to G_i; v_A is the aerial vehicle velocity and v_{G_i} are ground vehicle velocities.

(2006) and Filipović (2009). However, in all these references, including (Chatterjee & Liberzon, 2004, 2006), the analysis is focused on a time-dependent switching signal and not on a state-dependent switching (Zhang, Wu, & Xia, 2014). A hysteresis-based switching in the context of supervisory control of uncertain systems has been proposed in Hespanha, Liberzon, and Morse (2003), but only recently has the state-dependent switching law for the stochastic switched systems been considered in Wu, Cui, Shi, and Karimi (2013) and Zhang et al. (2014). Furthermore, the stability of the sliding mode control for semi-Markovian jump systems has been considered in Li, Wua, Shi, and Lim (2015). To the best of our knowledge, a *state-dependent* switching policy that reduces the time to reach a target set and prevents an infinite number of switchings has not been considered so far. Another novelty of the work presented here is that the policy is scalable, which makes it suitable for the navigation of an autonomous vehicle surrounded by a number of other vehicles.

While the problem motivation in Section 2 suggests the application of the switching rule to navigation in multi-agent systems (Dimarogonas & Kyriakopoulos, 2004), in Section 3 we discuss the switching in the context of a *target set* which is a union of *target segment sets*, where each segment set corresponds to a single vehicle. Therefore, the switching rule is potentially relevant to other robotics applications that appear in the similar form, for example, those in which segments can be associated with sets corresponding to robot end effector grasping configurations. Section 4 illustrates our results using the multiple vehicle problem and statistical analysis, and Section 5 gives conclusions.

2. Problem motivation

Let us consider a scenario with three agents depicted in Fig. 1. Two of the agents are ground vehicles G_1 and G_2 with equal speeds $v_{G_1} = v_{G_2} = v_G$. The third agent is an aerial vehicle (A) flying at a constant altitude. The kinematic model of A is a deterministic Dubins vehicle model describing the vehicle's position x_A, y_A and heading angle θ_A as

$$dx_A = v_A \cos(\theta_A)dt \quad (1)$$
$$dy_A = v_A \sin(\theta_A)dt \quad (2)$$
$$d\theta_A = u_A dt \quad (3)$$

where the A's velocity, $v_A > v_G$, is a known constant, and its control input is the bounded heading rate $u_A \in [-1, 1]$. The agent A's goal is to navigate in a *minimum time* into the vulnerable tail sector $\mathcal{T}_i(t)$, $i = 1, 2$ of one or the other ground vehicle for inspection purposes. Therefore, the target *set for A* is $\mathcal{T}(t) = \mathcal{T}_1(t) \cup \mathcal{T}_2(t)$ and the time dependence is the consequence of ground vehicle motion. However, *A has no knowledge of ground vehicles' navigation strategy*. To anticipate that uncertainty, the kinematics of each ground vehicle is modeled by the stochastic dynamics

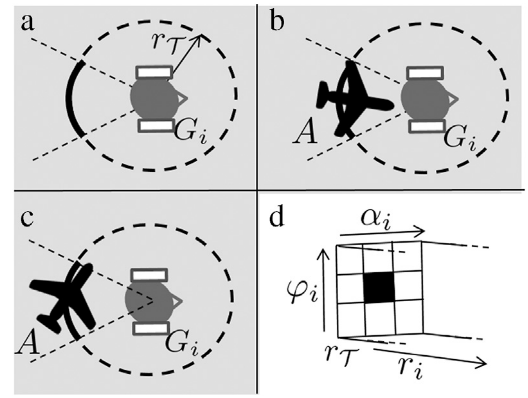

Fig. 2. (a) The target segment set $\mathcal{T}_i(t)$, which is the circular sector behind the moving G_i . (b) Agent A is inside the target set, while in (c) agent A is not in the target set, because its heading is not aligned with G_i. (d) The time invariant set $\mathcal{S}_i^{\mathcal{T}}$ is shown in a 3D-space, r_i, φ_i, α_i.

$$dx_{G_i} = v_G \cos(\theta_{G_i})dt, \quad i = 1, 2 \quad (4)$$
$$dy_{G_i} = v_G \sin(\theta_{G_i})dt \quad (5)$$
$$d\theta_{G_i} = \sigma_G dw_i, \quad (6)$$

where the vehicle positions are given by x_{G_i}, y_{G_i} and the heading angles are $\theta_{G_i}(t) = \int_0^t \sigma_E dw_i$, which are continuous time random walks since dw_i denotes the Wiener process increments. The scaling parameter σ_G is identical for both vehicles.

The vulnerable tail $\mathcal{T}_i(t)$ sector is a circular sector attached to the back of the ground vehicle G_i, see Fig. 2, and we also refer to it as a target segment. The relative position between A and G_i is uniquely defined based on the triple of relative coordinates $(r_i, \varphi_i, \alpha_i)$, where r_i is the distance between A and G_i, φ_i is the bearing angle from A to G_i, and α_i is the difference between the A's and G_i's heading angles. Therefore, the time varying target segment $\mathcal{T}_i(t)$ in the Cartesian space can be represented as a time invariant set $\mathcal{S}_i^{\mathcal{T}} \in \mathcal{R}$ in the space of relative coordinates $\mathcal{R} \subset \mathbb{R}^3$

$$\mathcal{S}_i^{\mathcal{T}} = \left\{[\underline{r}, \overline{r}] \times [-\overline{\varphi}, \overline{\varphi}] \times [-\overline{\alpha}, \overline{\alpha}]\right\}, \quad (7)$$

with $0 < \underline{r} < \overline{r}$, $\overline{\varphi} > 0$, and $\overline{\alpha} > 0$ (see Fig. 2). The time invariant $\mathcal{S}_i^{\mathcal{T}}$ is a box in the state space defined by r_i, φ_i, and α_i, and the facet of this cube for $r_i = \overline{r}$ is depicted in Fig. 2d. For the time invariant set $\mathcal{S}_i^{\mathcal{T}}$, we can formulate and compute the Hamilton–Jacobi–Bellman (HJB) equation solution for the minimum time optimal controller to reach $\mathcal{T}_i(t)$. The result is the optimal control $u_i(r_i, \varphi_i, \alpha_i)$ which is state dependent and defines the value of control variable for a given relative position $(r_i, \varphi_i, \alpha_i)$ between A and G_i, and for every *i*. If the aerial vehicle *A is tasked to* reach the tail sector $\mathcal{T}_i$ of a specific G_i, then the computed optimal control u_i defines the optimal way to do it and a further analysis of the problem is unnecessary.

However, computations for a minimum time optimal controller for the target set $\mathcal{T}(t) = \mathcal{T}_1(t) \cup \mathcal{T}_2(t)$ require the time invariant set $\mathcal{S}^{\mathcal{T}} \subset \mathbb{R}^6$ since the configuration of agents requires 6 relative coordinates, i.e., three per the target segment. The same number of dimensions is also necessary to describe the state feedback control. If we consider reaching the segmented target set $\mathcal{T}(t) = \cup_{i=1}^{N} \mathcal{T}_i(t)$, then the number of state variables is $3N$. For example, in Section 4, we present the multiple vehicle problem with 5 vehicles, which requires dealing with a 15-dimensional state space. This large number of dimensions quickly exhausts the computational power for computing the HJB equation solution and the optimal control.