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IFAC-PapersOnLine 49-6 (2016) 099–104

Anatomy of Haar Wavelet Filter and Its Implementation for Signal Processing

Manuel Schimmack, * **Susan Nguyen,** * **Paolo Mercorelli** *

* *Institute of Product and Process Innovation, Leuphana University of Lueneburg, Volgershall 1, D-21339 Lueneburg, Germany (e-mail: schimmack, mercorelli@ uni.leuphana.de)*

Abstract: This paper gives an insight to the workings of discrete wavelet transformation (DWT) in context of education, with the objective to integrate teaching and research by promoting signal processing and control as a field that embraces science, technology, engineering and mathematics (STEM). In more detail, this contribution showcases a possible lecture structure of the basic principle of orthogonal wavelets in general, and the discrete wavelet decomposition method. The architecture of the presented software structure are described step-by-step, to provide an elementary guideline for a possible implementation into an embedded system. Herein, the focus is set on the Haar wavelet specifically, thus as an illustrative example, the code for the use of it is presented. With the wavelet packet transform as a method of discrete wavelet transform, the algorithm is able to decompose and reconstruct an input signal with reduction of noise. The noise of a sequence can be located, so that the wavelet basis can be rearranged. In particular, this allows for the elimination of any incoherent parts that make up the unavoidable measuring noise of the acquired signal, which was tested in GNU Octave and MATLAB®.

Keywords: Education in Signal Processing and Control, Noise reduction, Haar wavelet, Discrete wavelet transformation

1. INTRODUCTION

The Wavelet transformation is an important mathematical tool that has proven its relevance in the use of general signal analysis as for example noise reduction. Also in the application of process control, wavelets have become the defacto tool in context of signal compression, estimation, filtering, and identification Tangirala et al. (2013).

The wavelets operate similarly to the commonly used Fourier transform, often even characterised as an extension to that, since its functions provide results in both time domain and frequency domain. Therefore, the main reason to prefer the wavelet transform over the traditional Fourier transform is that its better resolution gives accurate information about frequencies at certain times. Moreover it is also more suited for approximation of signals with discontinuities and sharp spikes. Trigonometric wavelets are wavelet functions with sinusoidal properties. They are essentially equivalent to the windowed FFT, since they also make use of cosine functions for approximation and a scale-able window function.

The presented software structure is based on an orthogonal wavelet and makes it possible to locate noise. Using this information, adjustments in the signal can be taken to remove those noise components in a possible control or signal processing application. By altering the input signal and removing unwanted or unnecessary parts, the reconstruction process also results in data compression, reducing the needed memory space of an embedded system. Along several different possible wavelet methods, this work gives an introduction to the Haar wavelet as well as its implementation into the frame of a lecture. In the case of this lecture example where a discrete signal is analysed, the Haar wavelet is most suited. Therefore a example which is built based on the free WaveLab850 of Stanford University Buckheit et al. (2005) using this particular Haar wavelet family. Simply put, the motivation herein is to prove the noise reduction ability of the wavelet transform and to clear measured signals from unwanted errors, be it from biological or environmental causes. This way it enables a more precise detection of measured signals, thus also providing an accurate signal analysis method for an application in control or signal processing.

This paper is organised as follows: Section 2 shows the anatomy of the Haar wavelet in context of wavelet transformation, the mother wavelet's structure and Nyquist frequency in a didactical way. It presents a possible lecture structure for a future implementation in an embedded system and devoided background aspects of the Haar wavelet in context of the computation of the discrete wavelet transformation. Section 3 describes the Haar wavelet framework and shows the developed software tool in context of education, that is utilising the wavelet packet transform, with the "MakeWaveletPacket" which is based on the free WaveLab850 library. The implementation of the source code is presented in section 4 and conclusions close the paper.

10.1016/j.ifacol.2016.07.160

The main nomenclature

b:	Frequency-dependent parameter
c_n:	Wavelet coefficients
D:	Wavelet tree depth
d:	Index scale
$e(t)$:	Noise
I:	Interval of time
k:	Time-dependent parameter
$\mathcal{L}(\omega)$:	Trigonometric polynomial
$m_0(\omega)$:	Generating function
N:	Vanishing moment
n:	Samples
$y(t)$:	Input signal
$P_N(\omega)$:	Polynomial
$s_{(d,b,k)}$:	Wavelet coefficient
$\mathbf{V}$:	Vector space
$wp_{(d,b,k)}$:	Wavelet coefficient tree
λ:	Scalar
$\psi^n(t)$:	Wavelet family
z:	Discrete complex variable
ω:	Angular frequency

2. ANATOMY OF THE HAAR WAVELET

The general wavelet transform method of all is the Haar wavelet, which was also the first one to be proposed in 1910 by the mathematician Alfred Haar. The Haar wavelet has a simple rectangular structure, making it memory efficient and preferably used in analysing compact discrete signals in Mercorelli (2007). For wavelet transformation, a

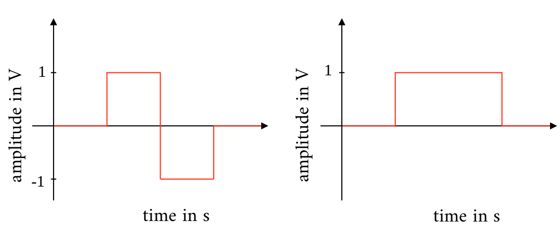

Fig. 1. Haars mother wavelet function (left) and its corresponding scaling function (right)

wavelet function and a scaling function are needed. The Haar wavelet's basis wavelet function, also known as the mother wavelet, can be described as follows

$$\psi_{(d,b,k)}(t) = \psi_b(2^d t - n), \tag{1}$$

with a support of size 2^{-d} of the Nyquist frequency and the indices (d, b, k). Herein, d is set as a scale index describing the tree level, b as a phase parameter for the frequency shift and k as a time related parameter indicating the shift in time. Also the mother wavelet's structure is shown by the dependencies

$$\psi^h(t) = \begin{cases} 1; & \text{if } 0 \le t < \dfrac{1}{2}; \\ -1; & \text{if } \dfrac{1}{2} \le t < 1; \\ 0; & \text{for all other } t. \end{cases} \tag{2}$$

From this, its rectangular form becomes evident. It is on a consistent value positive and negative for an equal amount of time, and zero for the remaining time, resulting in an average value of zero as well. This is the basic wavelet function which serves as the form of the wavelet family that will be used to approximate the signal which is to be analysed. A corresponding scaling function with the integral of 1 is used to adjust the wavelet to the observed signal. Herein, scale is related to frequency, so that by the impact of the frequencies, high parameters have the ability to compress and lower parameters to stretch the wavelet. In addition to the scale variation, the wavelet function can also be adjusted through time shifts, which results in either the hastening or the delaying of the function. Because of this, rather than only acting upon a time-frequency domain, wavelet transform is considered to work with a time-scale domain. Figure 2 shows a couple of different Haar wavelet functions $\psi^h_{(d,b,k)}(t)$ with varying scales and timeshifts and a constant time parameter k.

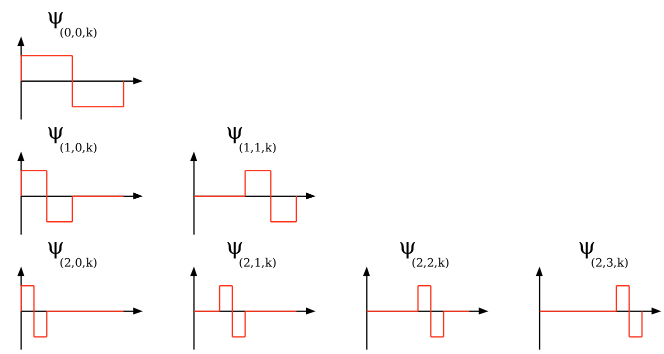

Fig. 2. Schematic of Haar wavelet functions for varying scales and timeshift

2.1 Packets and Nyquist Frequency

With a window of four samples an orthogonal basis with four functions is defined. The following illustration Fig. 3 shows the decomposition process for the first two levels up to $d = 2$ of the Haar wavelet packet tree. According to the Nyquist theorem, a signal needs to be sampled with the Nyquist frequency that is at least twice the signal frequency in order to be reconstructed. This principle here is applied to the Haar frequency.

With four samples, it can be shown that, for the first level $d = 1$, two subspaces are created. Therein the sampling rate of the low frequency one ranges from 0 to 0.5 of the Nyquist frequency and the high frequency one from 0.5 up to 1 of the Nyquist frequency. It is also shown how all of the samples are divided into low frequency sequences on one side and high frequency sequences on the other. Furthermore the schematic makes clear how the two subspaces hold 50 percent of the samples each. The same can be applied to the second level of the tree, where four subspaces are created and hold 25 percent of the samples each. This means, that the subspaces range from 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75 and 0.75 to 1 of the Nyquist Frequency. Again, the subspaces are divided into low and high frequency sections, with the low frequency on the left and high frequency on the right. All of these created subspaces make up the packets with variables (d, b, k) of the wavelet packet tree.