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IFAC-PapersOnLine 49-6 (2016) 110–114

Anatomy of Discrete Kalman Filter and Its Implementation for Sensorless Velocity Estimation of Organic Actuator

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Abstract: This paper presents a general derivation of the discrete Kalman filter (DKF) in context of education, with the objective to integrate teaching and research by promoting control and signal processing as a field that embraces science, technology, engineering and mathematics (STEM). In more detail, this contribution showcases a possible lecture structure of the discrete Kalman filter together with an innovative laboratory experiment, suitable for an audience that does not require a strong mathematical background. The importance of finding appropriate didactic methods in the context of KF is due to the intrinsic difficulty which characterises this algorithm to be understood by the students. In general the Kalman filter is one of the most used algorithms in all fields of control systems, thanks to its effectiveness and efficiency. In this contribution the filter is used as a state observer to estimate the velocity of a stimulus-responsive polymerfibre actuator in the proposed laboratory experiment. Besides the theoretical aspects of the discrete Kalman filter algorithm, a step-by-step development for its implementation is presented. The proposed structure is general and can be used as a basic frame for research in the context of control and signal processing. In this sense, this contribution proposes a better understanding of the role of integrating teaching and research in education.

Keywords: Education in Signal Processing and Control, Discrete Kalman filter, Virtual sensor

1. INTRODUCTION

The importance of finding appropriate didactic methods in the context of Kalman filter (KF) is due to the intrinsic difficulty which characterises this algorithm to be understood by the students. In this sense, lecturers should really make an effort to realise appropriate didactic material coated by examples together with the related software to be explained in a parallel panel and to be emphasised during the lectures.

One of the key didactic points in KF is to explain the double nature (deterministic and stochastic nature) of the KF. This weak-constraint nature generates some difficulties in students who are used to conceive problems in terms of only deterministic or only stochastic domains. In fact, the concept of the perfect knowledge of the model is unnecessary in KF and this concept should be stressed by the lecturers. As already emphasised at the beginning, KF finds many application possibilities in variegate technical contexts thanks to its weak-constraint nature. All weak-constraint methods are computationally demanding, and often the choice between KF and other weak-constraint methods is a tactical one, depending on some considerations as for instance programming convenience. In this sense, the lecturer should consider the practical nature of the algorithm using examples with the corresponding developed software in which the amount of sums and products between these weak-constraint methods can concretely and easily be seen. The lecturers can indicate supplementary literature in which in practical applications is coated with measurement results.

Today, Kalman filters are at work in every satellite navigation device, every smart phone, and many computer games as in Faragher (2012) presented. For instance in Mercorelli (2015) and Schimmack et al. (2014) an explicit analysis and comparison between KF and other estimation method of the computational load is carried out. To conclude, a crucial importance is represented by the laboratory experimental part of the lecture. In fact, laboratory experiments offer one way to introduce more realism into the education and in this case the application is based on a stimulus-responsive polymerfibre actuator. The mechanism of movement is based on the characteristic of the thermoplastic polymers coated with metal particles.

This paper is organised as follows: Section 2 shows a possible didactical concept of Kalman filter. Section 3 describes step-by-step the anatomy of the DKF. It is an introduction and a possible lecture structure for the implementation in MATLAB®/SIMULINK®. Section 4 presents the w hole source code of the discrete Kalman filter and shows the implementation in a dSPACE DS1103 PPC controller board, which is used to estimate the velocity of the stimulus-responsive polymerfibre actuator and to demonstrate the laboratory experiment. Conclusions close the paper.

10.1016/j.ifacol.2016.07.162

THE MAIN NOMENCLATURE

$\mathbf{A}_k$:	State transition matrix
$\mathbf{B}_k$:	Input control transition matrix
$\mathbf{H}_k$:	Output observer matrix
$\mathbf{K}_k$:	Kalman gain matrix
$\mathbf{P}_k$:	Gain matrix
$\mathbf{Q}_k$:	Process noise covariance matrix
$\mathbf{R}_k$:	Measurement noise covariance matrix
T_s:	Sampling time
$\mathbf{u}_k$:	Control input vector
$\mathbf{v}_k$:	Measurment noise vector
$\mathbf{w}_k$:	Process noise vector
$\mathbf{x}_k$:	System state vector
$\mathbf{x}_1$:	State vector of position
$\mathbf{x}_2$:	State vector of velocity

2. A DIDACTICAL CONCEPT OF A KALMAN FILTER LECTURE

During the Apollo-11 landing, when Neil Armstrong first voiced the statement "Thats one small step for (a) man, one giant leap for mankind!" there were likely only a few people in front of the television on July 21st 1969 at 2:56 AM UTC, that thought of the Extended Kalman filter (EKF) algorithm. Yet it was this very algorithm that helped enable the safe moon landing and also brought home the three astronauts Neil Armstrong, Michael Collins and Edwin Aldrin securely. The mathematic method was implemented in the main program of the Apollo Guidance Computer (ACG). By the way: the ACG is considered as the first "embedded system", having about the same computing power as the Game Boy, a 8-bit handheld video game device developed and manufactured by Nintendo®.

After the publication of Rudolf Emil Kalman in Kalman (1960), the KF is used today as the standard tool in calculation of satellite positions and GPS. In general the Kalman filter is one of the most used algorithms in all fields of control systems, thanks to its effectiveness and efficiency.
It follows that a didactical way is developed with the following structure:

- Introduction into KF and its variations in context of historical background
- Contemplation of deterministic vs. stochastic model in the theoretical part of the lecture
- State-space representation in the theoretical part of the lecture
- Definition and explanation together with the functional sequences of KF in the theoretical part of the lecture
    - · State estimation
    - · State covariance estimation
    - · Riccati equation
- Implementation and verification in graphical programming environment for modelling, simulating and analysing multidomain dynamic systems in the laboratory
- Students implement autonomously the algorithm in the frame of laboratory experiment in a prototyping system

As explained before, one of the key didactic points in KF is to explain the double nature (deterministic and stochastic nature) of the KF. This weak-constraint nature generates some difficulties in students who are used to conceive problems in terms of only deterministic or only stochastic domains. The concept of perfect knowledge of the model is unnecessary in KF which is why this concept should be one main stress of the lecture. In this context, one way to explain this aspect is to emphasis the "two-steps" structure of the algorithm: a *priori* and a *posteriori* estimation inside each sampling time. The lecturers should consider few but extreme important aspects.

- during the a *priori* estimation the deterministic aspect through the mathematical model plays the most important role
- an optimal stochastic correction through the real measurements in the a *posterior* phase of the algorithm

These two points should be clarified by the lecturers using examples including real applications with implemented software in the classes.

3. ANATOMY OF THE LINEAR KALMAN FILTER

Considering the following continuous linear system

$$\mathbf{x}(t) = \mathbf{A}\ \mathbf{x}(t) + \mathbf{B}\ \mathbf{u}(t) + \mathbf{w}_t, \tag{1}$$

which is comparable as in Welch and Bishop (2006), is linear and can be described as follows. Consider that the state-space model, depicted in Fig. 1, is described by the process model with the state equation

$$\mathbf{x}_{k+1} = \mathbf{A}_k\ \mathbf{x}_k + \mathbf{B}_k\ \mathbf{u}_k + \mathbf{w}_k, \tag{2}$$

where $\mathbf{x}_k$ is the n_s dimensional state vector at time k, $\mathbf{u}_k$ is the n_u dimensional known input control vector, $\mathbf{B}_k$ is the input transition matrix and $\mathbf{A}_k$ is the state transition matrix. According to Forward Euler discretisation it follows that

$$\mathbf{A}_k = (\mathbf{I}_{n \times n} + T_s \mathbf{A}), \tag{3}$$

and

$$\mathbf{B}_k = T_s \mathbf{B}, \tag{4}$$

in which matrix $\mathbf{I}_{n_s \times n_s}$ represents the $n \times n$ identity matrix and T_s the sampling time. The zero mean white process noise is denoted by $\mathbf{w}_k$ with the covariance

$$E[\mathbf{w}_k\ \mathbf{w}_k'] = \mathbf{Q}_k. \tag{5}$$

The state vector is defined as

$$\mathbf{x}_k = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{6}$$

where x_1 represents the position and x_2 specified the velocity at time k of the stimulus-responsive polymerfibre actuator. For the measurement $z \in \mathbb{R}^m$ follows

$$z_k = \mathbf{H}_k\ \mathbf{x}_k + \mathbf{v}_k, \tag{7}$$

where $\mathbf{H}_k$ delineates as an output observer matrix and $\mathbf{v}_k$ represents the white measurement noise with the covariance

$$E[\mathbf{v}_k\ \mathbf{v}_k'] = R_k. \tag{8}$$

In practice, the process noise covariance $\mathbf{Q}_k$ and the measurement noise covariance R_k matrices change at each time step k with the sampling time T_s.