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Brief paper

Distributed consensus tracking of a class of asynchronously switched nonlinear multi-agent systems ☆

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ABSTRACT

This paper addresses the distributed consensus tracking problem for a class of switched nonlinear multi-agent systems in pure-feedback form under directed communication networks. For a practical environment, switched non-affine nonlinearities for followers are unknown and their switching signals are assumed to be arbitrary and asynchronous. Compared with existing results in the literature, the main contribution of this paper is to provide a universal control strategy to deal with asynchronously switched nonlinear multi-agent systems in the consensus tracking field. By using the function approximation technique, an approximation-based novel recursive design approach for asynchronously switched followers is presented to design local common adaptive controllers without requiring the knowledge on the signs of control gain functions and on the leader's velocity. From the common Lyapunov function method, it is shown that the consensus tracking errors are asymptotically bounded and converge to a neighborhood of the origin. A simulation example is included to demonstrate the effectiveness of the proposed methodology.

1. Introduction

Distributed consensus problems have been widely investigated for the cooperative control in the presence of the limitation of communication flow among multi-agent systems. These problems have been recently extended to nonlinear multi-agent systems (see Liu & Chen, 2011; Su, Chen, Wang, & Lin, 2011; Zhao, Duan, Wen, & Chen, 2016 and the references therein). Among them, multi-agent systems with unmatched nonlinearities are particularly appealing from the dynamics of many practical applications. In Yoo (2013a,b), consensus tracking approaches were proposed for nonlinear strict-feedback multi-agent systems with unity controlling efforts. To consider controlling efforts dependent on the state variables, a consensus problem was studied in Shen and Shi (2015). Zhang, Liu, and Feng (2015) presented a consensus tracking approach for time-varying nonlinear multi-agent systems in strict-feedback form. These results were recently applied to the distributed consensus problem of non-affine nonlinear multi-agent systems (Shahvali & Shojaei, 2016; Yang & Yue, 2016). Despite these research efforts, nonlinearities of multi-agent systems considered in Liu and Chen (2011), Shahvali and Shojaei (2016), Shen and Shi (2015), Su et al. (2011), Yang and Yue (2016), Yoo (2013a,b), Zhang et al. (2015), and Zhao et al. (2016) are non-switched. That is, the consensus problem of multi-agent systems with switched nonlinearities has not yet been fully treated unlike that of non-switched multi-agent systems. In addition, it is necessary for practical applications to deal with arbitrary and asynchronous switching signals for this problem.

On the other hand, recent years have witnessed intensive research activities in the design and analysis of control schemes for switched systems. Among them, the common Lyapunov function method is particularly popular because switching signals independence of average dwell-time conditions can be considered (see Liberzon, 2003; Xiang & Xiao, 2014 and the references therein). By using the backstepping technique (Krstic, Kanellakopoulos, & Kokotovic, 1995), control problems of switched systems with known unmatched nonlinearities have been recently addressed in Long and Zhao (2012), Ma and Zhao (2010), Niu and Zhao (2013), and Wu (2009). These results have stimulated the development of function-approximation-based adaptive control schemes in the presence of unknown switched nonlinearities (Jiang, Shen, & Shi, 2015; Zhao, Zheng, Niu, & Liu, 2015). However, to the best of our knowledge, the distributed consensus control problem of switched nonlinear multi-agent systems is still open, which is important and

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challenging in both theory and real world applications. This point is a motivation of this paper.

The purpose of this study is to provide a distributed adaptive consensus tracking scheme for a class of multi-agent systems with arbitrary and asynchronous switched non-affine nonlinearities. The switched nonlinearities for followers are assumed to be unknown and unmatched in the control input. Compared with the related works in the literature, the main contributions of this paper are as follows: (i) multi-agent systems with unknown asynchronously switched nonlinearities are firstly considered in the consensus control field and additionally the switched nonlinearities are in completely non-affine form; and (ii) the distributed consensus tracking scheme and its stability analysis for asynchronously switched nonlinear multi-agent systems are developed without requiring the information on the signs of control gain functions and on the leader's velocity. To this end, the common Lyapunov function method is extended to the consensus tracking problem in order to construct a universal design formula of approximation-based local common controllers using only neighbors' information. It is shown that consensus tracking errors are asymptotically bounded and converge to a neighborhood of the origin in the sense of Lyapunov stability criterion. Finally, a simulation example is given for testifying to the validity of the proposed theoretical result.

2. Preliminaries and problem formulation

2.1. Nussbaum-type gain

Some continuous function $\mathcal{N}(s):\mathbb{R}\mapsto\mathbb{R}$ is called a Nussbaum-type function if the following equalities are satisfied (Nussbaum, 1983): $\lim_{s\to+\infty}\sup\frac{1}{s}\int_0^s\mathcal{N}(\vartheta)d\vartheta=\infty$ and $\lim_{s\to+\infty}\inf\frac{1}{s}\int_0^s\mathcal{N}(\vartheta)d\vartheta=-\infty$. In this paper, an even function $e^{\vartheta^2}\cos((\pi/2)\vartheta)$ satisfying the above conditions will be used to deal with unknown signs of control gain functions of followers.

Lemma 1 (Ge, Hong, & Lee, 2004). *Let $V(t)$ and $\xi(t)$ be smooth functions defined on $[0,\bar{t})$ with $V(t)\geq 0,\forall t\in[0,\bar{t})$, and $\mathcal{N}(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds $V(t)\leq c_0+e^{-c_1t}\int_0^t[h(\vartheta)\mathcal{N}(\xi)+1]\dot{\xi}e^{c_1\vartheta}d\vartheta$ where c_0 and c_1 denote positive constants and $h(\cdot)$ is a time-varying parameter which takes values in the unknown closed interval $I=[\underline{l},\bar{l}]$ with $0\notin I$, then $V(t)$, $\xi(t)$, and $\int_0^t h(\vartheta)\mathcal{N}(\xi)\dot{\xi}d\vartheta$ are bounded on $[0,\bar{t})$.*

2.2. Problem statement

Suppose that there exist N followers, labeled as agents 1 to N, and a leader, labeled as an agent 0, under a directed network. The dynamics of N followers is described by the following asynchronously switched uncertain pure-feedback systems:

$$\begin{aligned}\dot{x}_{f,k}&=g_{f,k}^{\sigma_f(t)}(\bar{x}_{f,k},x_{f,k+1})+\varpi_{f,k},\\ \dot{x}_{f,n_f}&=g_{f,n_f}^{\sigma_f(t)}(x_f,u_f^{\sigma_f(t)})+\varpi_{f,n_f},\\ y_f&=x_{f,1},\end{aligned}\tag{1}$$

where $f=1,\ldots,N$, $k=1,\ldots,n_f-1$, $\bar{x}_{f,k}=[x_{f,1},\ldots,x_{f,k}]^\top\in\mathbb{R}^k$ and $x_f=[x_{f,1},\ldots,x_{f,n_f}]^\top\in\mathbb{R}^{n_f}$ are state variables of the fth follower, $\varpi_{f,k}\in\mathbb{R}$ are external disturbances, and $\sigma_f(t):[0,+\infty)\to M_f=\{1,2,\ldots,m_f\}$ is the switching signal for the fth follower. Here, M_f denotes the switching mode set of the fth follower and m_f is the number of modes to be switched. Thus, $\sigma_f(t)$ describes the switching mode of the fth follower and each follower has a different switching signal $\sigma_f(t)$ which leads to the asynchronous switching among followers. For any $j\in M_f$, $f=1,\ldots,N$, and $k=1,\ldots,n_f$, $u_f^j(t)\in\mathbb{R}$ is the input of the jth switched subsystem on the fth follower, and $g_{f,k}^j(\cdot)\in\mathbb{R}$ are unknown C^1 nonlinear functions. Notice that system (1) includes heterogeneous switched nonlinearities $g_{f,k}^{\sigma_f(t)}$ under asynchronous switching $\sigma_f(t)$ among followers.

From the N followers and a leader, the communication topology is defined as a directed graph $\mathcal{G}\triangleq(\mathcal{V},\mathcal{E})$ with $\mathcal{V}\triangleq\{0,1,2,\ldots,N\}$ and the edge set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. In $\mathcal{G}$, an edge $(l,f)\in\mathcal{E}$ denotes that the agent f can receive information from agent l but not vice versa. The set of neighbors of a node f is defined as $\mathcal{N}_f=\{l|(l,f)\in\mathcal{E}\}$ which is the set of nodes with edges incoming to node f. To describe the communication among the followers, a subgraph is defined as $\bar{\mathcal{G}}\triangleq(\bar{\mathcal{V}},\bar{\mathcal{E}})$ with $\bar{\mathcal{V}}\triangleq\{1,2,\ldots,N\}$. The adjacency matrix $\bar{\mathcal{A}}$ of $\bar{\mathcal{G}}$ is $\bar{\mathcal{A}}=[a_{fl}]\in\mathbb{R}^{N\times N}$ where $f=1,\ldots,N$, $l=1,\ldots,N$, $a_{fl}>0$ if $(l,f)\in\bar{\mathcal{E}}$, $a_{fl}=0$ otherwise, and $a_{ff}=0$. Then, the Laplacian matrix $\mathcal{L}$ related with $\mathcal{G}$ is defined as $\mathcal{L}=\begin{bmatrix}0 & 0_{1\times N}\\ -b & \bar{\mathcal{L}}+\mathcal{B}\end{bmatrix}$ where $b=[b_1,\ldots,b_N]^\top$, with $b_f>0,f=1,\ldots,N$, if the leader $0\in\mathcal{N}_f$ and $b_f=0$ otherwise, denotes the communication weight from the leader to the followers, $\mathcal{B}=\text{diag}[b_1,\ldots,b_N]$, and $\bar{\mathcal{L}}=\bar{\mathcal{D}}-\bar{\mathcal{A}}$ is the Laplacian matrix of $\bar{\mathcal{G}}$ with $\bar{\mathcal{D}}=\text{diag}[d_1,\ldots,d_N]$; $d_f=\sum_{l=1,l\neq f}^N a_{fl}$ is the diagonal element of the degree matrix $\bar{\mathcal{D}}$. See Olfati-Saber, Fax, and Murray (2007) for more details on the graph theory.

Remark 1. The directed graph $\mathcal{G}$ with a spanning tree satisfies $\text{rank}(\mathcal{L})=N$ (Ren, 2008). Then, using $(\bar{\mathcal{L}}+\mathcal{B})1_N=b$, it holds that $\text{rank}(\bar{\mathcal{L}}+\mathcal{B})=N$ where 1_N is an N-vector of all ones.

Assumption 1. The leader output signal $r(t)$ is continuous, bounded, and available for the fth followers satisfying $0\in\mathcal{N}_f$, $f=1,\ldots,N$ and $\dot{r}(t)$ is bounded, but not available for all followers.

Assumption 2. The asynchronously switched nonlinear functions $g_{f,k}^j$ are *unknown* on a directed graph $\mathcal{G}$ where $f=1,\ldots,N$, $k=1,\ldots,n_f$, and $\forall j\in M_f$.

Assumption 3. Let $h_{f,k}^j(\bar{x}_{f,k},x_{f,k+1})\triangleq\partial g_{f,k}^j/\partial x_{f,k+1}$. Then, $h_{f,k}^j\neq 0$, their signs are changeable and *unknown*, and there exist unknown real constants $\underline{h}_{f,k}^j>0$ and $\bar{h}_{f,k}^j>0$ such that $\underline{h}_{f,k}^j\leq|h_{f,k}^j|\leq\bar{h}_{f,k}^j$ where $f=1,\ldots,N$, $k=1,\ldots,n_f$, $x_{f,n_f+1}=u_f^j$ and $\forall j\in M_f$.

Assumption 4. The external disturbances $\varpi_{f,k}$ are bounded as $|\varpi_{f,k}|\leq\bar{\varpi}_{f,k}$ where $f=1,\ldots,N$, $k=1,\ldots,n_f$, and $\bar{\varpi}_{f,k}>0$ are unknown constants.

Definition 1 (*Chen & Lewis, 2011*; *Yoo, 2013a*). The distributed consensus tracking errors for followers (1) are said to be cooperatively semi-globally asymptotically bounded if there exist adjustable constants $c_1>0$, $c_2>0$ and the bounds $\beta_1>0$, $\beta_2>0$, such that for every $\alpha_1\in(0,c_1)$ and $\alpha_2\in(0,c_2)$, $\|y_f(t_0)-r(t_0)\|\leq\alpha_1\Rightarrow\|y_f(t)-r(t)\|\leq\beta_1$ and $\|y_f(t_0)-y_l(t_0)\|\leq\alpha_2\Rightarrow\|y_f(t)-y_l(t)\|\leq\beta_2$ as $t\to\infty$ where t_0 is an initial time, $f=1,\ldots,N$, $l=1,\ldots,N$, and $f\neq l$.

The objective of this paper is to design distributed common adaptive consensus tracking laws $u_f^j\triangleq u_f$ for N followers (1) with unknown switched non-affine nonlinearities so that the follower outputs y_f synchronize to the dynamic leader output r while all signals in the total closed-loop systems are bounded.