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Multi-agent zero-sum differential graphical games for disturbance rejection in distributed control[☆]

Qiang Jiao [a,1], Hamidreza Modares [b], Shengyuan Xu [a], Frank L. Lewis [b], Kyriakos G. Vamvoudakis [c]

[a] School of Automation, Nanjing University of Science and Technology, Nanjing, 210094, Jiangsu, PR China
[b] University of Texas at Arlington Research Institute, 7300 Jack Newell Blvd. S., Ft. Worth, TX 76118, USA
[c] Center for Control, Dynamical-systems and Computation (CCDC), University of California, Santa Barbara, CA 93106-9560, USA

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ABSTRACT

This paper addresses distributed optimal tracking control of multi-agent linear systems subject to external disturbances. The concept of differential game theory is utilized to formulate this distributed control problem into a multi-player zero-sum differential graphical game, which provides a new perspective on distributed tracking of multiple agents influenced by disturbances. In the presented differential graphical game, the dynamics and performance indices for each node depend on local neighbor information and disturbances. It is shown that the solution to the multi-agent differential graphical games in the presence of disturbances requires the solution to coupled Hamilton–Jacobi–Isaacs (HJI) equations. Multi-agent learning policy iteration (PI) algorithm is provided to find the solution to these coupled HJI equations and its convergence is proven. It is also shown that L_2-bounded synchronization errors can be guaranteed using this technique. An online PI algorithm is given to solve the zero-sum game in real time. A simulation example is provided to show the effectiveness of the online approach.

1. Introduction

The distributed control of multi-agent systems has received much attention in the past years, due to its potential applications in a variety of engineering systems. A rich body of literature has been developed on distributed control methods for consensus and synchronization (Fax & Murray, 2004; Jadbabaie, Lin, & Morse, 2003; Olfati-Saber & Murray, 2004; Qu, 2009; Ren & Beard, 2005, 2008; Ren, Beard, & Atkins, 2005; Tsitsiklis, 1984), and some good surveys can be found in Lewis, Zhang, Hengster-Movric, and Das (2014), Qu (2009) and Ren et al. (2005).

The objective in distributed control is to design for each agent a control protocol, depending only on local neighbor information, to guarantee synchronized behavior of all the agents by reaching agreement on certain quantities of interests. Although agent dynamics may be subject to external disturbances, most of the existing distributed control methods ignore these disturbances. However, to avoid performance degradation, it is necessary to consider the effect of disturbances in the distributed control problem formulation a priori.

Recently, disturbance attenuation has been taken into account in distributed control of multi-agent systems. Lin, Jia, and Li (2008) studied the H_∞ consensus problem for first-order dynamical systems. Li, Duan, and Huang (2009) transformed the problem of the disturbance rejection for multi-agent linear systems to the design of a set of H_∞ controllers for a set of independent systems. Liu and Jia (2010, 2011) derived conditions in terms of linear matrix inequalities (LMIs) to ensure consensus of the multi-agent systems with a prescribed H_∞ level. Li, Duan, and Chen (2011) considered the distributed H_2 and H_∞ control problems for linear multi-agent systems. Wen, Duan, Li, and Chen (2012) addressed a finite L_2-gain performance index for nonlinear multi-agent systems. Yang and Wang (2013) presented notions of L_p-gain and L_2-gain consensus for multi-agent systems in the presence of disturbances.

Most of the existing methods for multi-agent control in the presence of disturbances considered the leaderless consensus (or

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E-mail addresses: qjiao0312@gmail.com (Q. Jiao), modares@uta.edu (H. Modares), syxu@njust.edu.cn (S. Xu), lewis@uta.edu (F.L. Lewis), kyriakos@ece.ucsb.edu (K.G. Vamvoudakis).

[1] Tel.: +86 13813859536.

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distributed regulation) problem, in which all nodes converge to a common value that cannot generally be controlled. On the other hand, the problem of distributed tracking (or leader–follower consensus problem), which is the problem of interest of this paper, requires that all nodes synchronize to a leader (Hong, Hu, & Gao, 2006; Li, Wang, & Chen, 2004; Ren, Moore, & Chen, 2007; Wang & Chen, 2002). Moreover, most of these methods derived conditions based on linear matrix inequalities (LMIs) to solve the problem (except for Wen et al., 2012; Yang & Wang, 2013). A new framework based on differential game theory was developed to achieve consensus of multi-agent systems (Vamvoudakis, Carrillo, & Hespanha, 2013) and synchronization of multi-wheeled mobile robots (Luy, Thanh, & Tri, 2013). Specifically, Vamvoudakis et al. (2013) presented a distributed algorithm, but it is limited to double integrator systems. Moreover, the convergence proof of the presented policy iteration algorithm and the L_2-bounded synchronization error were not shown.

Over the last decade, multi-agent learning systems have been developed to create agents that learn from experience how to best interact with other agents (Busoniu, Babuska, & De Schutter, 2008; Chang, 2009; Lakshmanan & Farias, 2006; Littman, 2001; Vrancx, Verbeeck, & Nowe, 2008; Wheeler & Narendra, 1986). A significant part of the research on multi-agent learning concerns reinforcement learning techniques for finite state Markov decision processes. Convergence of each agent to the optimal response using Q-learning was shown on condition that all other agents converge to their optimal response (Littman, 2001). To the best of our knowledge, the multi-agent reinforcement learning for general continuous-time and continuous-state systems in the presence of disturbance has not yet been considered. Moreover, a rigorous proof of the convergence of the multi-agent learning methods to the optimal Nash equilibrium has not yet been provided in the general case.

The main contributions of the paper are as follows. First, a "Bounded L_2-Gain Synchronization Problem" is formulated for multi-agent systems in the presence of disturbances in the agent dynamics. Second, the concept of differential game theory is utilized to formulate the distributed L_2-Gain control problem into a multi-player zero-sum differential graphical game. Next, it is shown that the solution to this differential graphical game requires the solution to coupled Hamilton–Jacobi–Isaacs (HJI) equations. The Nash solution of the graphical game is investigated and the global synchronization error is shown to be L_2-bounded if the graph has a spanning tree. Then, a multi-agent policy iteration algorithm is presented to find the solution to these HJI equations. A rigorous proof of the convergence of the proposed learning algorithm to the optimal Nash equilibrium is presented. This work extends the work of Vamvoudakis, Lewis, and Hudas (2012) to the cases in which the external disturbances cannot be ignored, on one hand, and the work of Vamvoudakis et al. (2013) to the distributed tracking problem with general linear systems, on the other hand.

This paper is organized as follows. The next section provides mathematical background and the problem formulation for distributed L_2-gain control of multi-agent systems. This problem is then formulated into a multi-player zero-sum differential graphical game in Section 3. The Nash solution to this graphical game is presented in Section 4. Sections 5 and 6 present policy iteration algorithms and their implementation, respectively, for finding the Nash solution to the presented game. Sections 7 and 8 present simulation results and conclusion, respectively.

2. Preliminaries and problem formulation

In this section, a review of communication graphs is given and the problem of synchronization of multi-agent systems subject to external disturbances is formulated.

2.1. Mathematical background

A *directed graph* G consists of a pair (V, E), where $V = \{\alpha_1, \dots, \alpha_N\}$ is a finite nonempty set of nodes and $E \subseteq V \times V$ is a set of ordered pairs of nodes, called edges. $E = [e_{ij}]$ is called the adjacency matrix with $e_{ij} > 0$if $(\alpha_j, \alpha_i) \in E$ and $e_{ij} = 0$otherwise. Note that diagonal elements $e_{ii} = 0$. The set of nodes α_j with edges incoming to node α_i is called the neighbors of node i, namely $N_i = \{\alpha_j : (\alpha_j, \alpha_i) \in E\}$. The graph Laplacian matrix is defined as $L = D - E$, which has all row sums equal to zero. $D = \mathrm{diag}(d_i)$ is called the in-degree matrix, where $d_i = \sum_{j \in N_i} e_{ij}$ is the weighted in-degree of node i.

Definition 1 (*Lewis et al., 2014*)**.** A (directed) tree is a connected digraph where every node except one, called the root, has in-degree equal to one. A graph is said to have a spanning tree if a subset of the edges forms a directed tree.

Throughout the paper, $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$ are denoted as the maximum and minimum singular values of the matrix A, respectively.

2.2. Problem formulation

Consider the communication graph $G = (V, E)$ having N agents, each with dynamics given by

$$\dot{x}_i = Ax_i + B_i u_i + D_i v_i \tag{1}$$

where $x_i(t) \in R^n$, $u_i(t) \in R^{m_i}$ and $v_i(t) \in R^{q_i}$ are the state, control input and external disturbance of node i, respectively. The state of the control or leader node is $x_0(t) \in R^n$ and it is assumed to satisfy the dynamics

$$\dot{x}_0 = Ax_0. \tag{2}$$

Standard Synchronization Problem. Design $u_i(t)$ in (1) so that $\|x_i(t) - x_0(t)\| \to 0$, $\forall i$ when $v_i(t) = 0$.

For each node i, the local neighborhood tracking error $\delta_i \in R^n$ is defined as Khoo, Xie, and Man (2009)

$$\delta_i = \sum_{j \in N_i} e_{ij}(x_i - x_j) + g_i(x_i - x_0) \tag{3}$$

where $g_i \geq 0$ is called the pinning gain and $g_i > 0$ for at least one root node i (Li et al., 2004).

The overall tracking error vector for all nodes is given by

$$\delta = ((L + G) \otimes I_n)(x - \underline{x}_0) = ((L + G) \otimes I_n)\zeta \tag{4}$$

where $x = [x_1^T \quad x_2^T \quad \cdots \quad x_N^T]^T$, $\delta = [\delta_1^T \quad \delta_2^T \quad \cdots \quad \delta_N^T]^T$ are global node state vector and the global tracking error vector, and $\underline{x}_0 = \underline{I}x_0 \in R^{nN}$, with $\underline{I} = \mathbf{1}_N \otimes I_n \in R^{nN \times n}$,$I_n$ the $n \times n$ identity matrix and $\mathbf{1}_N$ the N-vector of ones. The Kronecker product is denoted by $\otimes$. The pinning gain matrix $G \in R^{N \times N}$ is a diagonal matrix with diagonal entries equal to the pinning gains g_i. The (global) synchronization error is

$$\zeta = \left(x - \underline{x}_0\right) \in R^{nN}. \tag{5}$$

The following lemma shows that the small local neighborhood synchronization error implies small global synchronization error.

Lemma 1 (*Khoo et al., 2009*)**.** *Let $(L + G)$ be non-singular. Then the synchronization error is bounded by*

$$\|\zeta\| \leq \|\delta\| / \underline{\sigma}(L + G). \tag{6}$$

Remark 1. The matrix $(L + G)$ is non-singular if the graph has a spanning tree and $g_i \neq 0$ for a root node i (Khoo et al., 2009).