Automatica 69 (2016) 78–86

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Stability of switched nonlinear systems with delay and disturbance ☆

Yue-E Wang [a], Xi-Ming Sun [b,1], Frédéric Mazenc [c]

[a] School of Mathematics and Information Science, Shaanxi Normal University, Xi'an 710119, PR China

[b] School of Control Science and Engineering, Dalian University of Technology, Dalian 116024, PR China

[c] EPI Inria-Saclay DISCO, Laboratoire des Signaux et Systémes, CNRS-CentraleSupélec, 3 rue Joliot Curie, 91192 Gif-sur-Yvette, France

ARTICLE INFO

Article history:
Received 26 June 2015
Received in revised form
24 November 2015
Accepted 29 January 2016

Keywords:
Switched nonlinear systems
Lyapunov–Krasovskii functionals
Input delay
Input-to-state stability

ABSTRACT

We consider a class of nonlinear time-varying switched control systems for which stabilizing feedbacks are available. We study the effect of the presence of a delay in the input of switched nonlinear systems with an external disturbance. By contrast with most of the contributions available in the literature, we do not assume that all the subsystems of the switched system we consider are stable when the delay is present. Through a Lyapunov approach, we derive sufficient conditions in terms of size of the delay ensuring the global exponential stability of the switched system. Moreover, under appropriate conditions, the input-to-state stability of the system with respect to an external disturbance is established.

1. Introduction

Switched systems in continuous-time are systems with discrete switching events. They rely on a switching signal, which indicates at each instant which subsystem operates. Switched systems have extensive applications in domains which pertain to complex dynamical networks (Zhao, David, & Liu, 2009), mobile robot (Sankaranarayanan & Mahindrakar, 2009), flight control (Yang, Cocquempot, & Jiang, 2009), and many others (see Liberzon, 2003). Control problems for these systems have been studied in many contributions (Caliskan, Ozbay, & Niculescu, 2013; Liberzon, 2014; Sun, 2008; Zhao & Hill, 2008; Zhao, Zhang, Shi, & Liu, 2012). For instance, the work Zhao and Hill (2008) solved the L_2-gain analysis and an H_∞ control problem and gave necessary and sufficient conditions of stability of switched systems derived from multiple generalized Lyapunov-like functions. In Sun (2008), a robust switching signal was designed to render the switched system exponentially stable and robust against switching perturbations, based on the notion of relative distance.

It is well-known that in many applications, delays are present due in particular to measurement and transport phenomena. For instance, switched systems with delay model drilling systems and cellular mobile communication systems. In these systems, each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately switched to the available channel, which generally includes time delays and disturbances (Rappaport, 1996). Delays can degrade the performances of the controllers or even destabilize the systems whose stability in the absence of delay is ensured by an appropriate choice of feedback. In fact, the presence of delays in the input and of additive disturbances may cause the system breakdown or the system crash if the switching signal is designed inappropriately. For these reasons, the last ten years have witnessed significant developments in the domain of switched systems when a delay is present, as illustrated for instance by the contributions (Colaneri, Geromel, & Astolfi, 2008; Ma & Zhao, 2015; Vu & Kristi, 2010; Wang, Shi, Wang, & Wang, 2012). Interestingly, the presence of a delay may destabilize some of the subsystems of a switched system, without destabilizing it, which implies that stability conditions based on the assumption that the delay does not destabilize any subsystem are conservative. Since establishing stability results with the largest possible delay is desirable, the research subject of the stability of switched systems with unstable subsystems is very appealing from an applied point of view. The first contribution devoted to it is Zhai, Hu, Yasuda, and Michel

☆ This work was supported by National Natural Science Foundation of China under Grants 61403241, 61325014, the China Postdoctoral Science Foundation under Grant 2014M560748, the Fundamental Research Funds for the Central Universities under Grant GK201503011 and the PHC Bosphore 2016. The material in this paper was not presented at any conference This paper was recommended for publication in revised form by Associate Editor Hiroshi Ito under the direction of Editor Andrew R. Teel.

E-mail addresses: baihewye@snnu.edu.cn (Y.-E. Wang), sunxm@dlut.edu.cn (X.-M. Sun), frederic.mazenc@l2s.centralesupelec.fr (F. Mazenc).

[1] Tel.: +86 411 84707580; fax: +86 411 84707581.

http://dx.doi.org/10.1016/j.automatica.2016.02.015
0005-1098/

(2001). It presents results for linear systems only. From then on, the stability and the stabilization for switched linear time delay systems with both stable and unstable subsystems have been studied in many works. On the other hand, switched nonlinear time-varying time delay systems received less attention (with Liu, Liu, & Xie, 2011, Müller & Liberzon, 2012, Sun & Wang, 2012, Wang, Wang, & Zhao, 2013 as notable exceptions), probably because of the difficulty of analyzing general families of these systems. But the study of these systems is fundamental because many models are nonlinear and tracking problems lead to time-varying systems. In fact, this subject is strongly motivated by recent applications, notably in the domains of mechanical systems (Moon & Kalmar-Nagy, 2001), wireless communications (Rappaport, 1996), and mobile robots (Malisoff, Mazenc, & Zhang, 2012).

These remarks motivate the present work. Before describing more precisely its purpose, a few preliminary comments are needed. Lyapunov–Krasovskii Functionals (LKFs) are very powerful tools when it comes to establishing stability conditions for time delay systems. For constructions of LKFs for linear time delay systems, the research monographs (Fridman, 2014; Gu, Kharitonov, & Chen, 2003) have provided systematic methods, which have been already used to solve problems for switched linear time delay systems. But, in a general nonlinear context, even for stable systems with delay without switch, only a few constructions of Lyapunov–Krasovskii Functionals (LKFs) are available. Fortunately, recently, in Mazenc, Malisoff, and Dinh (2013) (see also Mazenc, Niculescu, & Krstic, 2012), a new family of Lyapunov–Krasovskii Functionals (LKFs) has been proposed to analyze the stability of systems belonging to a broad family of nonlinear time-varying systems with delays without switch. The delay is not supposed to be known, and it is established that a system in closed-loop with a feedback which globally asymptotically stabilize its origin is not destabilize by the delay provided it is smaller than an upper bound (given by an explicit formula). Our aim is to extend the preceding study (Wang, Sun, Wang, & Zhao, 2014), which considered the case of a switched systems with delay whose subsystems are all stable, to show how the functionals constructed in Mazenc et al. (2013) can be used to establish a similar result for switched systems, when a constant point wise delay in the input destabilize some of the subsystems of the system. Besides, we aim at determining estimates of the norm of solutions when additive disturbances in the input are present. As in Wang et al. (2014), we shall prove the input-to-state stability (ISS) for switched nonlinear time delay systems with respect to the disturbance. It is worth pointing out that the assumptions imposed in Wang et al. (2014) guarantee the stability of the system by implying that all its subsystems are stable, which leads to conservative conditions in terms of the size of the delay and of the minimum allowable dwell time of the switching signal. Our aim is to relax these hypotheses.

The analysis we shall propose decomposes in two steps. First, by constructing a new LKF, we will determine sufficient conditions, involving the size of the delay, the growth properties of the vector fields and minimum dwell time, guaranteeing the stability of the switched nonlinear system and of all its subsystems. These conditions are less restrictive than those imposed in Wang et al. (2014) because, the LKF given in Wang et al. (2014) involves a common Lyapunov function, which results in a conservative constraint on the input delay upper bound. The new LKF we propose allows us to derive less conservative input delay upper bound and average dwell time. Second, we focus on the case where the input delay is larger than the upper bound given in the first step, so that some subsystems may be unstable. By modifying the LKF and considering a special switching signal, the ISS of the system is guaranteed for delays larger than the admissible delays of the first step.

Finally, we wish to observe that, although the main results of the present paper assume the existence of stabilizing control laws for the system without delay, they can be used in the context of the design of control laws because they make it possible to select, amongst the possible control laws, those that lead to good robustness properties with respect to the presence of a delay and of additive disturbances.

The paper is organized as follows. The problem is stated in Section 2. Section 3 gives two preliminary result, which are instrumental in establishing the main results, which are given in Section 4, where are given, results for switched systems whose subsystems are all stable and Section 5 which presents a solution in the case where some of the subsystems of the studied switched system are unstable. Section 6 is devoted to an illustrative example. Concluding remarks in Section 7 end paper.

Notations. The notation will be simplified, e.g., by omitting the arguments of the functions, whenever no confusion can arise from the context. By τ, we denote the delay satisfying $0 \leq \tau \leq \bar{\tau}$, where $\bar{\tau}$ is a finite positive constant. The Banach Space of absolutely continuous functions $\phi : [-\bar{\tau}, 0] \rightarrow \mathbb{R}^{n_x}$ with $\dot{\phi} \in L_2([-\bar{\tau}, 0]; \mathbb{R}^{n_x})$, equipped with the norm $\|\phi\|_W = \max_{m\in[-\bar{\tau},0]} |\phi(m)| + \left(\int_{-\bar{\tau}}^{0} |\dot{\phi}(s)|^2 ds\right)^{\frac{1}{2}}$ is denoted by $W[-\bar{\tau}, 0]$. For $t \in \mathbb{R}^+$, set $x_t(s) := x(t+s), s \in [-\bar{\tau}, 0]$. For a measurable and essentially bounded function $u : \mathbb{R}^+ \rightarrow \mathbb{R}^p$, $\|u\|_\infty = ess \sup_{t\geq 0} |u(t)|$. If $\|u\|_\infty < \infty$, we write $u \in L_\infty^p$. For a continuous-time signal $w(t)$, set $\|w[t_1, t_2]\| = \sup_{t_1\leq s\leq t_2}\{|w(s)|\}$. For any $t_2 > t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the number of switching of $\sigma(t)$ over (t_1, t_2). If $N_\sigma(t_1, t_2) \leq N_0 + \frac{t_2-t_1}{\tau_a}$ holds for two constants $\tau_a > 0, N_0 \geq 0$, then τ_a is called average dwell time. A function $V : \mathbb{R}^{n_x} \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is called uniformly proper and positive definite provided there are $\alpha_s, \alpha_l \in \mathcal{K}_\infty$ such that $\alpha_s(|x|) \leq V(x, t) \leq \alpha_l(|x|)$, for all $x \in \mathbb{R}^{n_x}$ and $t \geq 0$.

2. Problem formulation

Throughout the paper, we consider the switched nonlinear time-varying system:

$$\begin{aligned} \dot{x}(t) &= f_{\sigma(t)}(x,t) + g_{\sigma(t)}(x,t)[u_{\sigma(t)}(x(t-\tau),t) + w(t)], \\ x_{t_0}(m) &= \xi(m), \quad m \in [-\bar{\tau}, 0], \end{aligned} \tag{1}$$

where $t_0 \geq 0$, $x \in \mathbb{R}^{n_x}$ is the state, $\dot{x}(t)$ denotes the right-hand derivative of $x(t)$, the delay τ satisfies $\tau \in [0, \bar{\tau}]$, $w \in L_\infty^p$ represents a disturbance and ξ is a differentiable initial function. The function $\sigma : [0, \infty) \rightarrow N = \{1, 2, \ldots, n\}$ is the switching signal. Associated with σ, we have the switching sequence $\{(i_0, t_0), \ldots, (i_k, t_k), \ldots, |i_k \in N, k \in \mathbb{N}\}$, which is such that the i_kth subsystem is active when $t \in [t_k, t_{k+1})$. For any $i \in N$, $u_i(x, t) \in \mathbb{R}^p$ is C^1 and is the predesigned stabilizing controller for the ith subsystem, f_i, g_i are locally Lipschitz with respect to x, and, for all $t \geq t_0$, $u_i(0, t) = 0$ and $f_i(0, t) = 0$. We assume that, for any finite time interval, there is only a finite number of switches and no jump occurs in the state at a switching instant.

For the sake of clarity, we give the definition of ISS for switched systems with delay, which is analogous to the one introduced in Pepe and Jiang (2006).

Definition 1. System (1) is said to be input-to-state stable (ISS) if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$, such that for any initial condition $\xi \in W[-\bar{\tau}, 0]$ and for any $w \in L_\infty^p$, the solution of (1) exists over $[0, +\infty)$ and satisfies

$$|x(t)| \leq \beta(\|\xi\|_W, t - t_0) + \gamma(\|w[t_0, t]\|), \quad t \geq t_0 \geq 0. \tag{2}$$

We introduce an assumption.

Assumption 1. There are known C^1 uniformly proper and positive definite functions $V_i : \mathbb{R}^{n_x} \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$, $i \in N$ such that