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Constrained distributed optimization: A population dynamics approach ☆

Julian Barreiro-Gomez [a,b], Nicanor Quijano [a], Carlos Ocampo-Martinez [b]

[a] *Departamento de Ingeniería Eléctrica y Electrónica, Universidad de los Andes, Carrera 1 No 18A-10, Bogotá, Colombia*

[b] *Automatic Control Department, Universitat Politècnica de Catalunya, Institut de Robòtica i Informàtica Industrial (CSIC–UPC), Llorens i Artigas, 4-6, 08028 Barcelona, Spain*

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ABSTRACT

Large-scale network systems involve a large number of states, which makes the design of real-time controllers a challenging task. A distributed controller design allows to reduce computational requirements since tasks are divided into different systems, allowing real-time processing. This paper proposes a novel methodology for solving constrained optimization problems in a distributed way inspired by population dynamics. This methodology consists of an extension of a population dynamics equation and the introduction of a mass dynamics equation. The proposed methodology divides the problem into smaller sub-problems, whose feasible regions vary over time achieving an agreement to solve the global problem. The methodology also guarantees attraction to the feasible region and allows to have few changes in the decision-making design when a network suffers the addition/removal of nodes/edges. Then, distributed controllers are designed with the proposed methodology and applied to the large-scale Barcelona Drinking Water Network (BDWN). Some simulations are presented and discussed in order to illustrate the control performance.

1. Introduction

An approach to design control systems is to express the desired performance of the plant as an optimization problem with multiple constraints, e.g., minimization of the error, minimization of the norm of states, minimization of the energy associated to control actions, all of those objectives subject to physical and/or operational constraints. When the system involves a large number of states, the design of optimization-based controllers becomes challenging, because of the lack of centralized information or because of other implications associated to information (e.g., communication issues, costs, reliability). The limitation regarding information availability demands the development of distributed optimization techniques that achieve an optimal point of a performance cost function for the total system by using only local and partial information. There are many distributed optimization applications in engineering, and most of them using a network systems approach (Bertsekas, 2012; Gao & Cheng, 2005; Simonetto, Keviczky, & Babuska, 2010, 2011). These problems have been solved by using distributed optimization algorithms based on the Newton method (Jadbabaie, Ozdaglar, & Zargham, 2009; Wei, Ozdaglar, & Jadbabaie, 2013), the sub-gradient method (Johansson, Keviczky, Johansson, & Johansson, 2008; Zhu & Martinez, 2012), and the consensus protocol (Johansson et al., 2008; Notarstefano & Bullo, 2011; Zhang & Liu, 2014), among other techniques. On the other hand, game theory studies the interaction of decision makers and the interconnection of decision making elements based on local information. From this perspective, game-theoretical tools become very useful to describe the behavior of distributed engineered systems (Marden & Shamma, 2015). One important characteristic of this theoretical approach is the Nash equilibrium concept, which describes how a global objective is reached based only on local decisions. The task to reach a global objective with partial information is one of the main aspects in distributed optimization problems. This problem may be seen as a multi-agent case in which there are local interactions

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E-mail addresses: j.barreiro135@uniandes.edu.co, jbarreiro@iri.upc.edu (J. Barreiro-Gomez), nquijano@uniandes.edu.co (N. Quijano), cocampo@iri.upc.edu (C. Ocampo-Martinez).

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among them. Furthermore, evolutionary game theory describes the previously mentioned model of agents interacting but also considering a determined population structure, i.e., constraints in the interaction among agents, (Nowak, 2006). From this point of view, this theory is suitable to design intelligent systems and controllers for systems where there are local decision makers (local controllers) and achieving a global performance and/or global goal under a specific structure, which is given by the topology of the system (e.g., energy networks, water networks, transportation networks, etc.). Also, game theory has become an important and powerful tool for solving optimization problems since the Nash equilibrium corresponds to the extreme of a potential function satisfying the Karush–Kuhn–Tucker (KKT) first order condition (Sandholm, 2010). This property is commonly used in a class of games known as potential games, which have gotten special importance in the solution of engineering problems. For instance, in Marden (2012) potential games are widely studied from the perspective of state-based games. Furthermore, some kind of optimization problems can be solved by finding a Nash equilibrium for an appropriate designed game, and the consideration of only local information allows to solve distributed optimization problems (Arslan & Shamma, 2004). For instance, in Gharesifard and Cortes (2013) a distributed convergence to Nash equilibria in two networks is discussed for zero-sum games. In Pantoja and Quijano (2011), distributed optimization has been applied using replicator dynamics (one of the six fundamental population dynamics), based on local information. In Li and Marden (2014), the design of utility functions for each agent in order to decouple constraints is presented, and the usage of penalty functions and barrier functions is discussed. The design of local control laws for individual agents to achieve a global objective is proposed in Li and Marden (2013), which has been extended in Zhang, Qi, and Zhao (2013) by using matrix theory. The consideration of dynamics in the system-equivalent graph that describes information sharing among decision variables is paramount since some network systems in engineering might grow (e.g., drainage network systems, drinking water networks, distributed generation systems). These dynamics represent an addition or removal of elements to/from the network. Moreover, the connectivity of the network elements could change over time (e.g., re-configuration systems), which could affect availability of information. In Li and Marden (2012), variations on the graph that determines the system information sharing are studied, where the set of communication links varies with a certain probability.

The main contribution of this paper is to introduce a novel methodology to solve constrained optimization problems in a distributed way, inspired by the population dynamics studied in Sandholm (2010). Different from the already published population dynamics approaches, this method adds dynamics to the population masses, making the population simplex vary properly over time making the method robust (Barreiro-Gomez, Quijano, & Ocampo-Martinez, 2014a). The method consists in considering the global problem as a society, where there is limitation of information sharing. The society is divided into several populations, where there is full available information. Then, a local optimization problem is solved at each population whose feasible region varies dynamically, i.e., there is an interchange of masses among populations. The feasible regions vary until all populations agree to solve the global optimization problem. In addition to this, applications in the control field may involve disturbances that could lead the trajectories to leave the feasible region (given by constraints that impose a desire performance) (Barreiro-Gomez, Quijano, & Ocampo-Martinez, 2014b). Another relevant difference with respect to already published distributed population dynamics approaches is that the proposed method guarantees that the feasible region is attractive. The last mentioned feature potentially improve the control performance rejecting disturbances. Finally, the design of the decision-making distributed system allows to have a reduced number of modifications when the graph topology changes, i.e., there are new nodes/edges in the graph or there are nodes/edges that disappear. Also, some redundant links can be identified, i.e., links in the graph that are not necessary in the connection among cliques.

The remainder of the paper is organized as follows. Section 2 shows preliminaries of graphs, population dynamics, and introduces the mathematical formalism that is used throughout the paper. Section 3 presents the population dynamics and the mass dynamics, including relevant characteristics. Then, the stability analysis of the dynamics is presented in Section 4. Section 5 shows the different possible changes that the social graph might suffer, and explains the implication over the design. Section 6 presents the optimization problem forms that could be solved with the population dynamics and the mass dynamics, presenting also some illustrative examples and results. Afterwards, the robustness of the method is shown by applying disturbances in Section 7. Then, Section 8 presents a large-scale system and the design of optimal controllers by using the proposed methodology. Controllers consider both a model-based approach, and a model-free approach. Section 9 shows the results and discussion about the performance of controllers designed with the proposed methodology. In Section 10 the main conclusions are drawn.

Notation. The sub-index is associated to a node of a graph, or to a strategy in a game. On the other hand, the super-index refers to a population. For instance, the sub-index i in x_i, $\mathcal{P}_i$, x_i^p or F_i refers either to a node in a graph or to a strategy, and the super-index p in m^p, $\mathbf{x}^p$, x_i^p or N^p indicates a population. Also it should be clear that the super-index is not an operational number, i.e., N^3 refers to population three but $N^3 \neq NNN$. We use bold font for column vectors and matrices, e.g., $\mathbf{x}$, and $\mathbf{H}$; and non-bold style is used for scalar numbers, e.g., N^p. Calligraphy style is used for sets, e.g., $\mathcal{S}$. The column vector with N unitary entries is denoted by $\mathbb{1}_N$, and the column vector with null entries and suitable dimension is denoted by $\mathbf{0}$. The identity matrix with dimension $N \times N$ is denoted by $\mathbb{I}_N$. The cardinality of a set $\mathcal{S}$ is denoted by $|\mathcal{S}|$. The continuous time is denoted by t, and it is mostly omitted throughout the manuscript in order to simplify the notation. Finally, $\mathbb{R}_+$ represents the set of all non-negative real numbers, and $\mathbb{Z}_+$ represents the set of positive integer numbers.

2. Preliminaries

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected non-complete connected graph that exhibits the topology of a society, where $\mathcal{V}$ is the set of vertices of $\mathcal{G}$ that represents the set of N available strategies in a social game denoted by $\mathcal{S} = \{1, \dots, N\}$; and $\mathcal{E} \subset \{(i, j) : i, j \in \mathcal{V}\}$ is the set of edges of $\mathcal{G}$ that determines the possible interactions among society strategies. The graph $\mathcal{G}$ is divided into M sub-complete graphs known as cliques (a complete sub-graph), where each clique represents a population within the society. The set of populations is denoted by $\mathcal{P} = \{1, \dots, M\}$, and the set of cliques is denoted by $\mathcal{C} = \{\mathcal{C}^p : p \in \mathcal{P}\}$. The clique of the population $p \in \mathcal{P}$ is a graph given by $\mathcal{C}^p = (\mathcal{V}^p, \mathcal{E}^p)$, where the set $\mathcal{V}^p$ represents the set of N^p available strategies in a population game denoted by $\mathcal{S}^p = \{i : i \in \mathcal{V}^p\}$, and $\mathcal{E}^p = \{(i, j) : i, j \in \mathcal{V}^p\}$ is the set of all the possible links in $\mathcal{C}^p$ determining full interaction among the population strategies.

It is assumed that cliques are already known, i.e., the number of cliques M, the set of vertices $\mathcal{V}^p$, and the set of edges $\mathcal{E}^p$ for all $p \in \mathcal{P}$ are known. Although, if it is desired to obtain the optimal set of cliques (i.e., the minimum amount of cliques M such that