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Stability analysis of sampled-data switched systems with quantization[☆]

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ABSTRACT

We propose a stability analysis method for sampled-data switched linear systems with finite-level static quantizers. In the closed-loop system, information on the active mode of the plant is transmitted to the controller only at each sampling time. This limitation of switching information leads to a mode mismatch between the plant and the controller, and the system may become unstable. A mode mismatch also makes it difficult to find an attractor set to which the state trajectory converges. A switching condition for stability is characterized by the total time when the modes of the plant and the controller are different. Under this condition, we derive an ultimate bound on the state trajectories by using a common Lyapunov function computed from a randomized algorithm. The switching condition can be reduced to a dwell-time condition.

1. Introduction

The recent advance of networking technologies makes control systems more flexible. However, the use of networks also raises new challenges such as packet dropouts, variable transmission delays, and real-time task scheduling. Switched system models provide a mathematical framework for such network properties because of their versatility to include both continuous flows and discrete jumps; see Donkers, Heemels, van de Wouw, and Hetel (2011), Lin and Antsaklis (2005), Song, Kim, and Karray (2008), Zhang and Yu (2010) and references therein for the application of switched system models to networked control systems.

On the other hand, many control loops in a practical network contain channels over which only a finite number of bits can be transmitted. We need to quantize data before sending them out through a network. Therefore the effect of data quantization should be taken into consideration to achieve stability and desired performance. In addition to the practical motivation, literature such as Nair and Evans (2003), Okano and Ishii (2014), Tatikonda and Mitter (2004), Wong and Brockett (1999) has answered the theoretical question of how much information is necessary/sufficient for a given control problem.

Switched systems and quantized control have been studied extensively but separately; see, e.g., Liberzon (2003b), Lin and Antsaklis (2009), Shorten, Wirth, Mason, Wulff, and King (2007) for switched systems and Ishii and Tsumura (2012), Matveev and Savkin (2009), Nair, Fagnani, Zampieri, and Evans (2007) for quantized control. However, quantized control of switched systems has received increasing attention in recent years. One particular research line deals with control problems with limited information for discrete-time Markovian jump linear systems (Ling & Lin, 2010; Liu, Daniel, & Lu, 2009; Nair, Dey, & Evans, 2003; Xiao, Xie, & Fu, 2010; Xu, Zhang, & Dullerud, 2013). In most of the above studies, the switching behavior of the plant is available to the controller *at all times*.

In contrast, in *sampled-data* switched systems with quantization, the controller receives the quantized measurement and the active mode of the plant *only at each sampling time*. Since the controller side does not know the active mode of the plant between sampling times, we do not always use the controller mode consistent with the plant mode at the present time. The closed-loop system may therefore become unstable when switching occurs

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between sampling times. Moreover, for the stability of quantized systems, it is important to obtain regions to which the state belongs. However, mode mismatches yield complicated state trajectories, which make it difficult to find such regions.

Stabilization of sampled-data switched system with *dynamic* quantizers has been first addressed in Liberzon (2014), which has proposed an encoding strategy for state feedback stabilization. This encoding method has been extended to the output feedback case (Wakaiki & Yamamoto, 2014a) and to the case with disturbances (Yang & Liberzon, 2015a). A crucial ingredient in the dynamic quantization is a reachable set of the state trajectories through sampling intervals. Propagation of reachable sets is used to set the quantization values at the next sampling time, and the dynamic quantizer achieves increasingly higher precision as the state approaches the origin. On the other hand, we study the stability analysis of sampled-data switched systems with *finite-level static* quantizers. For such a closed-loop system, asymptotic stability cannot be guaranteed. The objective of the present paper is therefore to find an ultimate bound on the system trajectories as in the single-modal case, e.g., Elia and Mitter (2001), Ishii, Başar, and Tempo (2004), Ishii and Francis (2002), Haimovich, Kofman, and Seron (2007). Since frequent mode mismatches make the trajectories diverge, a certain switching condition is required for the existence of ultimate bounds. After stating the problem formally in Section 2, we will show the differences between this work and the previous work (Liberzon, 2014).

As in Ma and Zhao (2015) for switched systems with time delays, we here characterize switching behaviors by the total time when the controller mode is not synchronized with the plant one, which we call the *total mismatch time*. We derive a sufficient condition on the total mismatch time for the system to be stable, by using an upper bound on the error due to sampling and quantization. Moreover, an ultimate bound on the state trajectories is obtained under the switching condition. For the stability analysis, we use a common Lyapunov function that guarantees stability for all individual modes in the non-switched case. We find such Lyapunov functions in a computationally efficient and less conservative way by combining the randomized algorithms in Liberzon and Tempo (2004), Ishii et al. (2004) together.

From the total mismatch time, we can obtain an asynchronous switching time ratio. If the controller mode is synchronized with the plant one, then the closed-loop system is stable. Otherwise, the system may be unstable. Hence the total mismatch time is a characterization similar to the total activation time ratio (Zhai, Hu, Yasuda, & Michel, 2001) between stable modes and unstable ones. The crucial difference is that the unstable modes we consider are caused by switching within sampling intervals. Using this dependence of the instability on the sampling period, we can reduce the switching condition on the total mismatch time to a dwell-time condition, which is widely used for the stability analysis of switched systems. In Section 4, we will discuss in detail the relationship between the total mismatch time and the dwell time of switching behaviors.

This paper is organized as follows. In Section 2, we present the closed-loop system, the information structure, and basic assumptions. In Section 3, we first investigate the growth rate of the common Lyapunov function in the case when switching occurs in a sampling interval. Next we derive an ultimate bound on the state, together with a sufficient condition on switching for stability. Section 4 is devoted to reducing the derived switching condition to a dwell-time condition. We illustrate the results through a two-tank system in Section 5. Finally, concluding remarks are given in Section 6.

This paper is based on a conference paper (Wakaiki & Yamamoto, 2014b). In the conference version, some of the proofs were omitted due to space limitations. The present paper provides complete results on the stability analysis in addition to an illustrative example. We also made structural improvements in this paper.

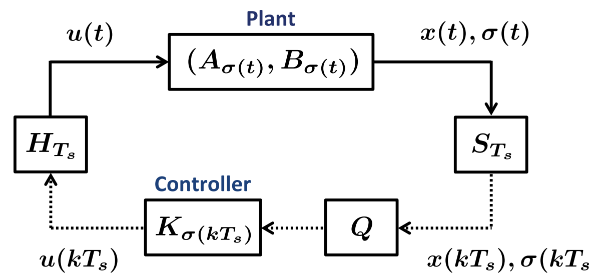

Fig. 1. Sampled-data switched system with quantization, where T_s is the sampling period and S_{T_s}, H_{T_s}, and Q are a sampler, a zero-order hold, and a static quantizer, respectively.

Notation. We denote by $\mathbb{Z}_+$ the set of non-negative integers $\{k \in \mathbb{Z} : k \geq 0\}$. For a set $\Omega \subset \mathbb{R}^n$, Cl (Ω), Int (Ω), and $\partial\Omega$ are its closure, interior, and boundary, respectively. For sets Ω_1, Ω_2, let $\Omega_1 \setminus \Omega_2$ be the relative complement of Ω_2 in Ω_1, i.e., $\Omega_1 \setminus \Omega_2 := \{\omega \in \Omega_1 : \omega \notin \Omega_2\}$. Let us denote by $|\mathcal{S}|$ the number of elements in a finite set $\mathcal{S}$.

Let $M^\top$ denote the transpose of a matrix $M \in \mathbb{R}^{n\times m}$. The Euclidean norm of a vector $v \in \mathbb{R}^n$ is defined by $\|v\| := (v^\top v)^{1/2}$. For a matrix $M \in \mathbb{R}^{m\times n}$, its Euclidean induced norm is defined by $\|M\| := \sup\{\|Mv\| : v \in \mathbb{R}^n, \|v\| = 1\}$. Let $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the largest and the smallest eigenvalue of a square matrix $P \in \mathbb{R}^{n\times n}$. Let $\mathcal{B}(L)$ be the closed ball in $\mathbb{R}^n$ with center at the origin and radius L, that is, $\mathcal{B}(L) := \{x \in \mathbb{R}^n : \|x\| \leq L\}$.

Let T_s be the sampling period. For $t \geq 0$, we define $[t]^-$ by

$$[t]^- := kT_s \quad \text{if } kT_s \leq t < (k+1)T_s \quad (k \in \mathbb{Z}_+).$$

2. Sampled-data switched systems with quantization

2.1. Switched systems

Consider the following continuous-time switched linear system

$$\dot{x} = A_\sigma x + B_\sigma u, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state and $u(t) \in \mathbb{R}^m$ is the control input. For a finite index set $\mathcal{P}$, the mapping $\sigma : [0, \infty) \to \mathcal{P}$ is right-continuous and piecewise constant, which indicates the active mode $\sigma(t) \in \mathcal{P}$ at each time $t \geq 0$. We call σ a *switching signal*, and the discontinuities of σ *switching times* or simply *switches*. The plant sends to the controller the state x and the switching signal σ.

The first assumption is stabilizability of all modes.

Assumption 1. For every mode $p \in \mathcal{P}$, (A_p, B_p) is stabilizable, i.e., there exists a feedback gain $K_p \in \mathbb{R}^{m\times n}$ such that $A_p + B_pK_p$ is Hurwitz.

2.2. Quantized sampled-data system

Consider the closed-loop system in Fig. 1. Let $T_s > 0$ be the sampling period. The sampler S_{T_s} is given by

$$S_{T_s} : (x, \sigma) \mapsto (x(kT_s), \sigma(kT_s)) \qquad (k \in \mathbb{Z}_+),$$

and the zero-order hold H_{T_s} by

$$H_{T_s} : u_d \mapsto u(t) = u_d(k), \quad t \in [kT_s, (k+1)T_s) \quad (k \in \mathbb{Z}_+).$$

The second assumption is that at most one switch happens in each sampling interval.

Assumption 2. Every sampling interval $(kT_s, (k+1)T_s)$ has at most one switch.

See Remark 5 (5) below for the reason why we need this switching assumption.