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Formation control with mismatched compasses☆

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ABSTRACT

This article addresses the formation control problem with mismatched compasses. Depending on the sensing and communication technology, compass mismatches may arise due to biases in measurement, drift in inertial sensing despite initial alignment, and even spatial variations in the earth's magnetic field. To illustrate the key concepts underlying what happens, we first consider the two agent case and show that the agents converge to a fixed, but distorted formation exponentially fast. In contrast to the matched compass case, the formation is not asymptotically stationary. The distance error and the angular error between the actual final formation and the desired formation are explicitly given, as is the steady state velocity of the formation. The case of time-varying mismatched compasses is also studied. Based on the results, we then propose estimators to obtain the mismatched angle, which allow a compensation algorithm to be proposed such that the desired formation shape is achieved. Finally, the extensions to the n agent case are also considered and similar phenomena are encountered. Simulations are provided to validate the theoretical results.

1. Introduction

The formation control problem for multi-agent systems has received increasing attention during the last decade due to its broad applications in spacecraft formation flying, search and rescue, and formation control of mobile robots (Cortes, Martinez, & Bullo, 2006; He, Qian, Lam, Chen, Han & Kurths, 2015; Meng, Anderson, & Hirche, 2015; Sieber, Deroo, & Hirche, 2013; Yang, Roy, Wan, & Saberi, 2011; Zavlanos & Pappas, 2008). There are many variations on the formation control problem, including problems with a leader or without a leader (Ren, 2007; Shi & Hong, 2009), problems with underlying graph structure which is directed or undirected (Hatano & Mesbahi, 2005; Moreau, 2005), problems in which the formation is achieved with velocity consensus leading to a moving final formation or solutions where the final formation is stationary (Jiang, Deghat, & Anderson, 2013; Lin, Broucke, & Francis, 2004). One particular distinction is between securing a formation with both a prescribed shape and a prescribed orientation, as opposed to simply aiming for a prescribed shape. Seeking a prescribed shape with prescribed orientation is in fact one of the easier problems. It can be solved using a linear consensus-based algorithm, where the control input is a combination of the neighbor-based relative position term and a nonzero bias term representing the formation objective (Fax & Murray, 2004; Olfati-Saber, Fax, & Murray, 2007). This is contrasted with an approach for shape control without orientation which uses gradient-based control, grounded in the theory of graph rigidity and often derived from system structural potentials (Cao, Morse, Yu, Anderson, & Dasgupta, 2011; Krick, Broucke, & Francis, 2009). In the gradient-based approach, agents do need again to measure relative positions, but only in a local coordinate basis associated with the measuring agent, which does not have to be directionally aligned with the coordinate bases of other agents. In contrast, the consensus-based approach requires all agents to have knowledge of where the common/global north is. Equivalently, coordinate bases of the different agents have to be directionally aligned. In

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this article, the consensus-based approach is considered. For this approach, common knowledge of where north is may sometimes be expressed by saying that all agents require a compass. In practice, they may acquire knowledge of north from inertial navigation properly initialized, or landmark data, the use of which likely requires inter-agent transmissions.

It is evident that it will often be physically unrealistic to claim that all agents have common error-free knowledge of where north is: biases can exist in instruments; drift can occur in inertial navigation systems; spatial variation can occur in the earth's magnetic field. This article explores the consequence of postulating the existence of errors in the direction of north, i.e. agents have differing views of where north is. To exhibit the key ideas, which apply to formations of any size, it is convenient to consider first a very simple case. Hence we start from the matched compass formation control for two agents in a two-dimensional plane,

$$\dot{\mathbf{A}}_1 = (\mathbf{A}_2 - \mathbf{A}_1) - \mathbf{D}, \tag{1a}$$

$$\dot{\mathbf{A}}_2 = (\mathbf{A}_1 - \mathbf{A}_2) + \mathbf{D}, \tag{1b}$$

where $\mathbf{A}_1 = [x_1, y_1]^{\mathrm{T}} \in \mathbb{R}^2$, $\mathbf{A}_2 = [x_2, y_2]^{\mathrm{T}} \in \mathbb{R}^2$ are the positions of agents 1 and 2, and $\mathbf{D} = [d_x, d_y]^{\mathrm{T}} \in \mathbb{R}^2$ is a given desired relative position and known for each agent. The objective is to drive agents 1 and 2 to form a stationary formation in the plane such that $\mathbf{A}_2 = \mathbf{A}_1 + \mathbf{D}$. It is straightforward to show that $\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{A}_2 - \mathbf{A}_1 - \mathbf{D}) = -2(\mathbf{A}_2 - \mathbf{A}_1 - \mathbf{D})$. This implies that $\lim_{t\to\infty}(\mathbf{A}_2(t) - \mathbf{A}_1(t)) = \mathbf{D}$, $\lim_{t\to\infty}\dot{\mathbf{A}}_1(t) = 0$, and $\lim_{t\to\infty}\dot{\mathbf{A}}_2(t) = 0$ exponentially fast. Therefore, agents converge to the desired formation and the velocities converge to zero exponentially fast.

In considering the above simple algorithm one should notice that the algorithm is constructed based on the assumption that the relative position measurement $\mathbf{A}_2 - \mathbf{A}_1$ for agent 1 and the relative position measurement $\mathbf{A}_1 - \mathbf{A}_2$ for agent 2 are identical (up to the sign). However, in real systems this assumption is unlikely to be satisfied for reasons as noted above. As already indicated, it is the directional error, i.e., a compass mismatch, that will concern us. For convenience but without any loss of generality, suppose that the global coordinates coincide with the coordinate basis of agent 1. We next seek to express the equation of motion of agent 2 in global coordinates. Suppose A_i denotes the position of agent i, $i = 1, 2$ in global coordinates and ${}^2A_i$ denotes its position in agent 2's coordinates. Suppose agent 2's view of north is that it is ϕ radians in a clockwise direction from agent 1's view, where $\phi \in (-\pi, \pi]$. An illustration is given in Fig. 1. We then know that a vector defining a line segment in global coordinates, denoted by ${}^1\mathbf{W} = [x, y]^{\mathrm{T}}$ is described in agent 2's coordinate basis as ${}^2\mathbf{W} = R(-\phi)\,{}^1\mathbf{W}$, where $R^{-1}(\phi) = R(-\phi) = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$ is the rotation matrix.

Then, in each agent's own coordinate basis, the actual kinematics of each agent with mismatched compasses are given by

$${}^1\dot{\mathbf{A}}_1 = \mathbf{A}_2 - \mathbf{A}_1 - \mathbf{D}, \tag{2a}$$

$${}^2\dot{\mathbf{A}}_2 = R(-\phi)(\mathbf{A}_1 - \mathbf{A}_2) + \mathbf{D}, \tag{2b}$$

where $\mathbf{A}_1 - \mathbf{A}_2$ is expressed in global coordinates, ${}^1\dot{\mathbf{A}}_1$ and ${}^2\dot{\mathbf{A}}_2$ are the velocity vectors of agents 1 and 2 expressed in each agent's own coordinate basis.

A relevant work for this problem is (Oh & Ahn, 2014), where the authors considered that there exists the orientation mismatch of local reference frames of the agents for the formation shape control problem. A combination algorithm aimed at coordinate frame orientation alignment and formation control was proposed and the assumption was imposed that the orientation of each agent's coordinate basis can be exchanged between neighbors. Distance errors have been considered in the context of formation shape control without orientation in Belabbas, Mou, Morse, and Anderson (2012) and Sun, Mou, Anderson, and Morse (2013). It was shown in Belabbas et al. (2012) that if the agents have different understandings of either the desired distance for each pair of agents or of the actual distance between them (perhaps due to measurement bias), the resulting formation shape in the limit is fixed but distorted relative to the desired shape, and generically the actual motions converge to circular closed orbits in the two-dimensional plane. A nongeneric situation can also arise in which the radius of the circular orbit goes to infinity, and then the formation simply translates at a constant velocity. This actually always happens for a two-agent formation. The extension to the case of the 3D tetrahedron formation shape control problem (and indeed more general 3D shapes) was considered in Sun et al. (2013) and it was shown that the motion behavior is a typical helix. It is in fact not hard to vary the conclusions of those papers and establish that a distance error for the two agent formation above leads the formation to take up a steady state spacing close to the desired distance and to move with a velocity parallel to the relative position vector at a speed proportional to the distance mismatch. One could in fact postulate directional and distance errors simultaneously. It would appear that the overall effect is just the superposition of the two individual effects.

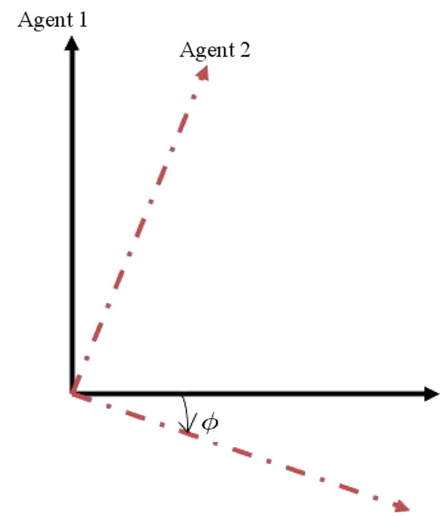

Fig. 1. Coordinates of agents 1 and 2.

In this article, we first focus on the compass mismatch problem for the two agent case (2) and then study the n agent case. In particular, we show that the agents converge to a fixed, but (relative to the desired formation) distorted formation exponentially fast for all the cases. The shape error between the actual final formation and the desired formation is explicitly given. The case of time-varying mismatched compasses and the estimation algorithms for the mismatched angle are also studied. Based on the design of the estimators, the compensation algorithm is proposed such that the desired formation shape is achieved. We finally include discussions on the n agent case where $n \geq 3$.

The organization of this article is as follows. In Section 2, we study the two agent case. Both the cases of constant error and time-varying error are considered. The estimation algorithms and compensation algorithms are proposed in Section 3. We also extend the results to the n agent case where $n \geq 3$ in Sections 4 and 5. Concluding remarks are given in Section 6.

2. Two agent case

Let us go back to (2) and assume that ϕ is constant. By noting the fact that ${}^1\dot{\mathbf{A}}_2 = R(\phi)\,{}^2\dot{\mathbf{A}}_2$, it is not hard to show that (2) can be written as

$$\dot{\mathbf{A}}_1 = \mathbf{A}_2 - \mathbf{A}_1 - \mathbf{D}, \tag{3a}$$