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Event-based state estimation of linear dynamic systems with unknown exogenous inputs ☆

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ABSTRACT

In this work, an event-based optimal state estimation problem for linear-time varying systems with unknown inputs is investigated. By treating the unknown input as a process with a non-informative prior, the event-based minimum mean square error (MMSE) estimator is obtained in a recursive form. It is shown that for the general time-varying case, the closed-loop matrix of the optimal event-based estimator is exponentially stable and the estimation error covariance matrix is asymptotically bounded for each sample path of the event-triggering process. The results are also extended to the multiple sensor scenario, where each sensor is allowed to have its own event-triggering condition. The efficiency of the proposed results is illustrated by a numerical example and comparative simulation with the MMSE estimators obtained based on time-triggered measurements. The results are potentially applicable to event-based secure state estimation of cyber-physical systems.

1. Introduction

The increasing demand on safe, secure and high performance operation of civil and industrial engineering systems has given birth to cyber-physical systems (CPSs). Different from the traditional control systems, CPSs are normally composed of networks of interacting components (e.g., sensor/actuator networks). Despite the encouraging features and new opportunities brought on by this type of systems, CPSs have introduced several new challenges to control system design.

One of these challenges is the limitation of communication and power resources. In many CPS applications, a large number of sensors and actuators linked through communication networks are utilized to accomplish certain tasks (e.g., quality control, remote monitoring). When all the components transmit their updates to each other or the computing center, the number of available communication channels acts as a natural limitation; moreover, in many occasions, some of these components are battery powered (e.g., mobile sensor networks), making the amount of available power a restriction on system performance as well. The event-triggered data transmission policies (Åström & Bernhardsson, 2002; Meng & Chen, 2012; Yook, Tilbury, & Soparkar, 2002) provide an efficient remedy to handle these limitations, and event-based state estimation, which is the scope of this work, has received a lot of attention in the control community during the past few years.

Earlier results on this topic focus on optimal event-triggering policy design. The optimal event-based finite-horizon sensor transmission scheduling problems were studied in Imer and Başar (2005) and Rabi, Moustakides, and Baras (2006) for continuous-time and discrete-time scalar linear systems, respectively; the results were extended to vector linear systems in Li, Lemmon, and Wang (2010). In Marck and Sijs (2010), a sampling protocol was proposed based on the Kullback–Leibler divergence of the probability distributions obtained when incorporating or not incorporating a measurement. Adaptive sampling for state estimation was considered in Rabi, Moustakides, and Baras (2012) for continuous-time linear systems. For further results on this line of research, see also Shi, Johansson, and Qiu (2011), Li and Lemmon (2011), Molin and Hirche (2012), Wang and Fu (2014) and references therein.

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Recent developments focus on the optimal estimator design for certain pre-specified event-triggering conditions. A general description of event-based sampling was proposed in Sijs and Lazar (2012), and a low-complexity event-based estimator with a hybrid update was proposed based on the approximation of the uniform distribution with the sum of a finite number of Gaussian distributions. Utilizing a Gaussian assumption on the distribution of the state conditioned on all past available measurement information, the event-based minimum mean square error (MMSE) estimator was derived in Wu, Jia, Johansson, and Shi (2013), and the tradeoff between communication rate and performance was explicitly analyzed; the extension to more general event-triggering conditions and the multiple sensor scenario was considered in Shi, Chen, and Shi (2014a). The Gaussian assumption was shown to be maintained in Han, Mo, Wu, Sinopoli, and Shi (2013), based on a stochastic event-triggering condition that introduced randomization in the triggering sets. In Sijs, Noack, and Hanebeck (2013), an event-based state estimator was obtained by minimizing the maximum possible mean squared error and treating the event-triggering conditions as non-stochastic uncertainty. In Shi, Chen, and Shi (2014b), a constrained optimization approach was utilized to solve an event-based estimation problem for a triggering scheme quantifying the magnitude of the innovation. A variance-triggered state estimation problem was considered in Trimpe and D'Andrea (2014), and the asymptotic periodicity of the triggering pattern was proved for an unstable scalar system. A nonlinear event-based state estimation problem was considered in Lee, Liu, and Hwang (2014), where a Markov chain approximation algorithm was proposed. The event-triggered estimation problem of systems with mixed time delays was considered by Zou, Wang, Gao, and Liu (2015) using sampled-data information for the continuous-time case. The problem of event-based state estimation for discrete-state hidden Markov models was investigated in Shi, Elliott, and Chen (2016).

On the other hand, the complex structure and extensive utilization of communication networks have made CPSs fragile and prone to unknown and unpredictable cyber attacks, which can cause disastrous consequences to infrastructure, national security, and even human life. In this context, a number of interesting attempts on secure detection and estimation have been recently reported, through graph-theoretic methods (Pasqualetti, Dorfler, & Bullo, 2013), by exploring the sparsity of the attack signals (Fawzi, Tabuada, & Diggavi, 2014), and using game-theoretic approaches (Miao, Pajic, & Pappas, 2013; Mo & Sinopoli, 2014). An alternative way to consider the secure estimation problems, however, is to treat the attack signals as unknown exogenous inputs and solve a problem of estimating the states in the existence of the unknown inputs. For the scenario of time-triggered measurements, this type of problems has been extensively investigated using the unbiased minimum variance (UMV) estimation approach in the literature, see, e.g., Kitanidis (1987), Darouach and Zasadzinski (1997), Darouach, Zasadzinski, and Boutayeb (2003), Cheng, Ye, Wang, and Zhou (2009), Fang, Shi, and Yi (2011) and references therein for the related developments; however, for the case of event-triggered measurement information, the estimation problem for systems with unknown inputs has not been investigated. The main difficulty is that when the measurements are assumed to be available at each time instant, the UMV estimators are normally obtained by directly minimizing the estimation error covariance matrices and the effect of the measurement information on the conditional distributions of the states (which is crucial in optimal event-based estimator design) is not explored; and thus the UMV estimation approach developed for the time-triggered measurement case cannot be generalized to consider the event-triggered scenario.

Meanwhile, it is interesting to note that by treating the unknown input as a process with non-informative prior, the Bayesian inference approach was successfully utilized to find the optimal MMSE estimate for systems with partially observed inputs (Li, 2013), and the results were shown to reduce to those obtained by the UMV approach for the unknown input case. For a system with an unknown exogenous input, it is normally not possible to find an appropriate state estimate or prediction, when no information about the current state is available from the sensor.[1] Although the Bayesian approach allows the exploitation of the implicit information contained in the event-triggering conditions (Han et al., 2013; Shi et al., 2014a; Wu et al., 2013), it is not yet clear whether this implicit information is "informative" enough to ensure the existence of an appropriate state estimate that is optimal in certain sense without exactly knowing the value of the current sensor measurement, which is investigated in this paper. To do this, an event-based optimal state estimation problem for linear time-varying systems with unknown exogenous inputs and stochastic event-triggering conditions is considered. The main contributions are summarized as follows:

(1) Under some mild conditions, it is shown that the conditional distribution of the state on the event-triggered measurement information is Gaussian, and the event-based MMSE estimate is developed in a recursive form. The obtained results generalize the time-triggered state estimation results obtained in Kitanidis (1987), Darouach and Zasadzinski (1997), Li (2013) to the case of event-triggered measurements.
(2) For each sample path of the event-triggering process, we show that the event-based MMSE estimator is exponentially stable with bounded estimation error covariance for the linear time-varying case. The results are equally applicable to the UMV estimator for linear time-varying systems as well, as it has the similar filter structure with the proposed event-based MMSE estimator.
(3) For the multiple sensor scenario with separate event-triggering conditions on each sensor, we show that the event-based MMSE estimator can also be developed under a rank condition on the lumped measurement matrices of all sensors. The differences of the estimator equations from the classic time-triggered Kalman filter as well as approximate event-based MMSE estimator developed for deterministic event-triggering conditions in the multiple sensor scenario are discussed.

For the multiple-sensor scenario, the problems of distributed event-based state estimation have been extensively investigated in Trimpe and D'Andrea (2011), Weimer, Araujo, and Johansson (2012) and Trimpe (2014). The main differences of the results developed for the multiple-sensor scenario in this work from Trimpe and D'Andrea (2011); Weimer et al. (2012) and Trimpe (2014) include: (1) the communication among sensors is not considered and the problem of event-based state estimation is considered in a centralized fashion; and (2) the effect of an unknown exogenous input term in the process equation is considered in event-based estimator design.

On the other hand, the problem of the unknown input observer design for continuous-time deterministic systems was presented in Darouach, Zasadzinski, and Xu (1994), where the necessary and sufficient conditions were provided; these conditions were related to the relative degree of the transfer function between the unknown input and the measured output. In Cristofaro and Johansen (2014), the authors utilized the unknown input observers for detection, isolation and control reconfiguration in overactuated systems. Recently, Johansen, Cristofaro, Sorensen, Hansen, and Fossen (2015) used the unknown input observer

[1] The intuition is that without the sensor measurement update, no clue of the current unknown input can be inferred.