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Brief paper

Distributed containment tracking of multiple stochastic nonlinear systems ☆

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ABSTRACT

This paper investigates distributed containment tracking for multiple stochastic nonlinear systems with multiple dynamic leaders under directed network topology. The control input of each agent can only use its local state and the states of its neighbors. With the backstepping design method, distributed tracking controllers are designed. By using stochastic analysis and graph theory, it is shown that the followers' outputs will exponentially converge to the convex hull spanned by the dynamic leaders's outputs with tunable tracking errors while all the states of the closed-loop system remain bounded in probability. A numerical example is provided to illustrate the effectiveness of the theoretical results.

1. Introduction

Since stochastic stabilization theory was introduced in 1960s by Kushner in Kushner (1967), much progress has been made on stabilization of stochastic nonlinear systems described by Itô stochastic differential equations. The existing literature on controller design for stochastic nonlinear systems can be mainly divided into two types: using Lyapunov functions in the form of quadratic functions multiplied by different weighting functions (Pan & Basar, 1999) and adopting Krstić and Deng's quartic Lyapunov functions method (Deng & Krstić, 1997, 2000; Krstić & Deng, 1998). Subsequently, these design techniques are further developed in Barbu (2012), Barbu, Da Prato, and Rockner (2009), Li and Wu (2013) and Shi, Xia, Liu, and Rees (2006).

Recently, the study on leader-following multi-agent systems has attracted a great deal of attention in the control community due to their wide practical applications in areas such as large scale robotic systems (Belta & Kumar, 2002) and biological systems (Olfati-Saber, 2006). The authors in Hu and Hong (2007) and Zhu and Cheng (2010) consider the leader-following consensus problem for a group of second-order autonomous agents with time-varying delays. When there exist noises in communication channels, the authors in Djaidja and Wu (2015), Djaidja, Wu, and Fang (2015) and Huang and Manton (2009) focus on the mean square consensus tracking problem of multi-agent systems by employing stochastic analysis and algebraic graph theory. A common feature of the above works is that only a single leader is considered.

The consensus-like problem with multiple leaders called the containment control, where the followers are driven into the convex hull spanned by the multiple leaders, is studied in Cao, Ren, and Egerstedt (2012), Li Ren, and Xu (2012) and Liu, Xie, and Wang (2012). The authors in Cao et al. (2012) study the distributed containment control of a group of mobile autonomous agents with multiple stationary or dynamic leaders under both fixed and switching directed network topologies. The authors in Liu et al. (2012) establish continuous-time and sampled-data based protocols for networked multi-agent systems. The authors in Li et al. (2012) investigate the containment control problem for a group of autonomous vehicles modeled by double-integrator dynamics.

It is noted that most of the physical systems are nonlinear (Khalil, 2002) and stochastic (Crandall & Mark, 1963) in nature, such as stochastic mobile autonomous vehicles (Schadschneider, Chowdhury, & Nishinari, 2010) and stochastic Lagrangian systems (Cui, Wu, Xie, & Shi, 2013). To our best knowledge, there is few results on containment control of multiple stochastic nonlinear

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systems in the open literature which motivates this study. The main contributions of this paper include:

(1) Only the information of the leaders' outputs is required to be available to a subgroup of followers, and the leaders' dynamics can be completely unknown. Therefore, the leaders' model can be more general than those in existing containment results for the leaders with known dynamics (Li, Xie, & Zhang, 2015; Lou & Hong, 2012; Notarstefano, Egerstedt, & Haque, 2011).
(2) This work considers the followers with inherently nonlinear diffusion terms and arbitrary number of integrators, and also considers coupling terms in agents dynamics and limited information exchange among neighboring agents. Containment control of such multiple stochastic nonlinear systems is very challenging, and no result has been reported. A distributed containment control scheme has been proposed in this work.
(3) It can be proved that the followers' outputs will eventually converge to the convex hull spanned by the dynamic leaders's outputs with tunable tracking errors. In other words, not only can the convergence of the followers' outputs be proved, but also can the convergent points of the followers' outputs be specified.

The remainder of the paper is organized as follows. Section 2 describes the problem to be investigated. Section 3 addresses containment controller design and performance analysis of the closed-loop control system. Section 4 gives a simulation example, which is followed by a conclusion in Section 5. Appendices A and B collect some useful lemmas and the proof of a proposition.

2. Problem formulation and some preliminaries

In this paper, we consider a network composed of N stochastic nonlinear systems as followers and K leaders. The followers' dynamics are described as follows:

$$\begin{aligned} dx_{ij} &= x_{i,j+1}dt + \Omega_{ij}(\bar{x}_{ij})d\omega, \quad j = 1, \dots, n_i - 1, \\ dx_{i,n_i} &= u_i dt + \Omega_{i,n_i}(\bar{x}_{i,n_i})d\omega, \\ y_i &= x_{i1}, \end{aligned} \tag{1}$$

where $\bar{x}_{ij} = (x_{i1}, \dots, x_{ij})^T \in R^j, u_i \in R, y_i \in R$ are the state, input, output of the ith follower, respectively, $i = 1, \dots, N$. ω is an m-dimensional standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions, that is, it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets. $\Omega_{ij}(\bar{x}_{ij}) : R^j \to R^{1\times r}, i = 1, \dots, N, j = 1, \dots, n_i$, are $\mathcal{C}^1$ functions.

The leaders' outputs are defined as $r_s(t) \in R, s = 1, \dots, K$.

Remark 1. From (1), the diffusion terms $\Omega_{ij}(\bar{x}_{ij}), i = 1, \dots, N, j = 1, \dots, n_i$, are inherently nonlinear, which is completely different from some semilinear conditions such as global Lipschitz condition (Li, Ren, Liu, & Fu, 2013).

We use a digraph $\mathcal{G} = (\mathcal{V}_f, \mathcal{E}, A)$ to describe the relationship among N followers (1) with the set of nodes $\mathcal{V}_f = \{1, 2, \dots, N\}$, set of arcs $\mathcal{E} \subset \mathcal{V}_f \times \mathcal{V}_f$, and a weighted adjacency matrix $A = (a_{ij})_{n\times n}$ with nonnegative elements. $(j, i) \in \mathcal{E}$ means that agent j can directly send information to agent i. The set of neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, i \neq j\}$. $a_{ij} > 0$ if node j is a neighbor of node i and $a_{ij} = 0$ otherwise. A sequence $(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)$ of edges is called a directed path from node i_1 to node i_k. The diagonal matrix $D = diag(\kappa_1, \kappa_2, \dots, \kappa_n)$ is the degree matrix, whose diagonal elements $\kappa_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. The Laplacian of a weighted digraph $\mathcal{G}$ is defined as $L = D - A$.

To describe the relationship between the followers (1) and the leaders, define a directed topology $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \mathcal{V}_f \cup \mathcal{V}_l, \mathcal{V}_f = \{1, 2, \cdots, N\}, \mathcal{V}_l = \{1, 2, \dots, K\}$, and its set of arcs $\bar{\mathcal{E}} \subset (\mathcal{V}_f \times \mathcal{V}_f \cup \mathcal{V}_f \times \mathcal{V}_l)$. If for every node i in $\mathcal{V}_f$, one can find a node j in $\mathcal{V}_l$, such that there is a path in $\bar{\mathcal{G}}$ from node j to i, we say that the set $\mathcal{V}_l$ is globally reachable in $\bar{\mathcal{G}}$. A diagonal matrix $B = diag(\sum_{s=1}^K b_{1s}, \dots, \sum_{s=1}^K b_{Ns})$ is the leader adjacency matrix associated with $\bar{\mathcal{G}}$, where $b_{is} > 0$ if node s in $\mathcal{V}_l$ is a neighbor of node i in $\mathcal{V}_f$ and $b_{is} = 0$ otherwise. Denote $H = L + B$.

To proceed further, we need the following assumptions.

Assumption 1. The leaders' outputs $r_i(t)$ and their derivatives $\dot{r}_i(t), i = 1, \dots, K$, are bounded. The jth leader's output $r_j(t) \in R$ and $\dot{r}_j(t)$ are only available for the ith follower satisfying $j \in \mathcal{N}_i, i = 1, \dots, N$.

Assumption 2. The leaders set $\mathcal{V}_l$ is globally reachable in the directed graph $\bar{\mathcal{G}}$.

Now, we are ready to give the definition for distributed containment output tracking.

Definition 1. The distributed containment output tracking problem for system (1) is solvable if one can find nonnegative constants $\kappa_{ij} (j = 1, \dots, K)$ satisfying $\sum_{j=1}^K \kappa_{ij} = 1$ and for any given $\varepsilon > 0$, there exists a set of distributed control laws such that:

(a) all the states of the closed-loop system are bounded in probability;
(b) for any initial value $x(t_0)$, there is a finite-time $T(x(t_0), \varepsilon)$ such that

$$E\left|y_i(t) - \sum_{j=1}^K \kappa_{ij} r_j(t)\right| < \varepsilon,$$
$$\forall t > T(x(t_0), \varepsilon), i = 1, \dots, N.$$

Remark 2. From Definition 1, one knows that $\sum_{j=1}^K \kappa_{ij} r_j(t)$ belongs to the convex hull spanned by the dynamic leaders' outputs. In other words, if the distributed containment output tracking problem defined in Definition 1 is solved, the followers' outputs will eventually converge to the convex hull spanned by the dynamic leaders's outputs with tunable tracking errors. Definition 1 not only describes the convergence, but also points out the specific limit point the followers will converge to, which is different from the available results (such as Lou & Hong, 2012) which only addresses the convergence.

Remark 3. Assumption 2 is necessary for the solvability of the containment output tracking problem of the system (1). If the leaders set $\mathcal{V}_l$ is not globally reachable in the digraph $\bar{\mathcal{G}}$, as demonstrated by Lou and Hong (2012), one can find some followers separated from all leaders and form several sub-systems without any interconnections between them. Therefore, it is impossible to achieve output tracking for these followers.

The following lemma is crucial for the distributed controller design.

Lemma 1. *All the eigenvalues of the matrix $H = L + B$ have positive real parts if and only if Assumption 2 holds.*

Proof. From the definition of H, it is easy to find that this lemma is a special case of Lemma 1 in Li et al. (2015).

The objective of this paper is to design distributed controllers to solve the containment output tracking problem for system (1).