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Brief paper

Bounded extremum seeking with discontinuous dithers[☆]

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ABSTRACT

The analysis of discontinuous extremum seeking (ES) controllers, e.g. those applicable to digital systems, has historically been more complicated than that of continuous controllers. We establish a simple and general extension of a recently developed bounded form of ES to a general class of oscillatory functions, including functions discontinuous with respect to time, such as triangle or square waves with dead time. We establish our main results by combining a novel idea for oscillatory control with an extension of functional analytic techniques originally utilized by Kurzweil, Jarnik, Sussmann, and Liu in the late 80s and early 90s and recently studied by Dürr et al. We demonstrate the value of the result with an application to inverter switching control.

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1. Introduction

Classical Extremum Seeking Optimization (ESO) began in the 1920s, using a slowly oscillating perturbation to guide a system's output to an extremum value (Leblanc, 1922). A series of papers in the 50s and 60s (Meerkov, 1967; Morosanov, 1957; Obabkovm, 1967; Ostrovskii, 1957; Volosov, 1962), were the first analytic studies of the stability of ESO. These results were refined and expanded to a wider class of systems in Krstić and Wang (2000), Tan, Nešić, and Mareels (2006), Tan, Nešić, Mareels, and Astolfi (2009), Guay and Zhang (2003), Srinivasan (2007), Guay (2014).

This form of ESO has been used in many applications (Moase, Manzie, Nesic, & Mareels, 2010) with unknown/uncertain systems (Rotea, 2000) and with discontinuous sliding-mode type ES perturbations (Pan, Özgünder, & Acarman, 2003). Furthermore, ESO has been used for applications such as active flow control (Henning et al., 2007; Kim, Kasnakoglu, Serrani, & Samimy, 2009), aeropropulsion (Wiederhold et al., 2009), cooling systems (Li, Rotea, Chiu, Mongeau, & Paek, 2005), wind energy (Creaby, Li, & Seem, 2009), photovoltaics (Lei, Li, Chen, & Seem, 2011), electromagnetic valve actuation (Peterson & Stefanopoulou, 2004), human exercise machines (Zhang, Dawson, Dixon, & Xian, 2006), enhancing mixing in magnetohydrodynamic channel flows (Ou et al., 2008), controlling Tokamak plasmas (Luo & Schuster, 2009), and recently, utilizing a multivariable Newton-based ESO scheme, for the power optimization of photovoltaic microconverters (Ghaffari, Krstic, & Seshagiri, 2014) as well as Newton-based stochastic extremum seeking (Liu & Krstic, 2014).

In Scheinker and Krstic (2013) a new form of Extremum Seeking Control and Optimization (ESCO) was introduced which, unlike ESO, is applicable to unstable and time-varying systems and utilizes the extremum seeker as the feedback controller itself. ESCO is closely related to the field of vibrational control and highly oscillatory systems (Dürr, Stanković, Ebenbauer, & Johansson, 2013; Kapitza, 1951; Kurzweil & Jarnik, 1987; Martinez, Cortes, & Bullo, 2003; Meerkov, 1980; Moreau & Aeyels, 2000; Sussmann, 1992; Sussmann & Liu, 1991). Whereas ESO has been used to optimize the output of a-priori stable, controlled systems, ESCO can be used to control and optimize uncertain, time-varying, open-loop unstable systems.

Since its inception ESCO has been used to optimize a high voltage converter modulator (Scheinker, Bland, Krstic, & Audia, 2013); to stabilize and expand the region of attraction of a pendulum's vertical equilibrium point (Michalowsky & Ebenbauer, 2013); it has been studied on manifolds (Dürr, Stanković, Ebenbauer, & Johansson, 2014); a non-differentiable form of ESCO was developed in which the system's control efforts settle to zero as equilibrium is approached (Scheinker & Krstic, 2014a); and a bounded form of ESCO was developed for unknown and possibly open-loop unstable systems in which the control efforts and parameter update rates have analytically known

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bounds (Scheinker & Krstic, 2014b), and implemented in hardware to tune a particle accelerator based only on an extremely noisy measurement signal (Scheinker, Baily, Young, Kolski, & Prokop, 2014).

Our main result, Theorem 1, extends the study of bounded ESCO from smoothly varying sinusoidal functions, to a much larger useful class of not necessarily continuous functions, e.g., a perturbing signal common in digital systems, a square wave with dead time between pulses. Theorem 1 has four useful properties:

(1) It establishes feedback control that is, on average, immune to additive, state-independent measurement noise.
(2) It is applicable to time-varying nonlinear systems.
(3) It establishes the on-average equivalence of variety of control choices that may be used with a range of different types of hardware.
(4) The proof is simpler and the result more general than the related work in Dürr et al. (2013), Kurzweil and Jarnik (1987), Sussmann and Liu (1991), Sussmann (1992), Moreau and Aeyels (2000).

2. Main theoretical result

We use the notation $u(y,t) = u(\hat{\psi}(x,t),t)$ to emphasize that the controller need not have direct access to x, i.e., that u is a function of t and of a noise corrupted measurement of a potentially unknown function $\hat{\psi}(x,t) = \psi(x,t) + n(t)$. We recall that a sequence of functions $\{f_k\} \subset L^2[0,1]$ is said to converge weakly to f in $L^2[0,1]$, denoted $f_k \rightharpoonup f$, if

$$\lim_{k\to\infty} \langle f_k, g\rangle = \lim_{k\to\infty} \int_0^1 f_k(\tau)g(\tau)d\tau = \int_0^1 f(\tau)g(\tau)d\tau = \langle f, g\rangle, \quad \forall g \in L^2[0,1].$$

Our primary result is the following.

Theorem 1. *Consider the vector-valued system*

$$\dot{x} = f(x,t) + g(x,t)u(y,t), \tag{1}$$

$$y = \psi(x,t) + n(t) = \hat{\psi}(x,t), \tag{2}$$

where $x \in \mathbb{R}^n$, and the functions $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$, $g : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times n}$, $\psi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$, and $n(t) : \mathbb{R} \to \mathbb{R}$ are unknown and twice continuously differentiable with respect to x. Also, ψ and $\partial\psi/\partial t$ are bounded with respect to t for x in a compact set, and $n(t)$, $\dot{n}(t)$ are bounded. Consider a controller $u : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^n$, given by

$$u(y,t) = \sum_{i=1}^{m} k_i(y,t)h_{i,\omega}(t), \quad k_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^n, \tag{3}$$

where the functions $k_i(y,t)$ are continuously differentiable and the functions $h_{i,\omega}(t)$ are piece-wise continuous. System (1)–(3) has the following equivalent closed-loop form

$$\dot{x}(t) = f(x,t) + \sum_{i=1}^{m} b_i(x,t)h_{i,\omega}(t), \tag{4}$$

$$b_i(x,t) = g(x,t)k_i\left(\hat{\psi}(x,t),t\right). \tag{5}$$

Suppose that the integrals of the functions $h_{i,\omega}(t)$ satisfy the uniform limits

$$\lim_{\omega\to\infty} H_{i,\omega}(t) = \lim_{\omega\to\infty} \int_{t_0}^{t} h_{i,\omega}(\tau)d\tau = 0, \tag{6}$$

and the weak limits

$$h_{i,\omega}(t)H_{j,\omega}(t) \rightharpoonup \lambda_{i,j}(t). \tag{7}$$

Consider also the average system related to (1)–(3) as follows

$$\dot{\bar{x}} = f(\bar{x},t) - \sum_{i,j=1}^{n} \lambda_{i,j}(t)\frac{\partial b_i(\bar{x},t)}{\partial \bar{x}} b_j(\bar{x},t), \quad \bar{x}(0) = x(0). \tag{8}$$

For any compact set $K \subset \mathbb{R}^n$, any $t_0, T \in \mathbb{R}_{\geq 0}$, and any $\delta > 0$, there exists $\omega^\star$ such that for each $\omega > \omega^\star$, the trajectories $x(t)$ and $\bar{x}(t)$ of (4) and (8), satisfy

$$\max_{t\in[t_0,t_0+T]} \|x(t) - \bar{x}(t)\| < \delta. \tag{9}$$

Furthermore,

$$\lim_{t\to\infty} \|\bar{x}(t)\| = 0 \Longrightarrow \lim_{t\to\infty} \|x(t)\| < \delta.$$

In other words, uniform asymptotic stability of (8) over K implies the semiglobal practical uniform asymptotic stability of (1)–(3).

The proof and definitions are given in the Appendix.

A simple example of a system of form (1)–(3) will illustrate the consequences of the theorem using the controller that motivated this work.

Example 1. Consider the system

$$\dot{x} = ax + bu, \quad u = \sqrt{\alpha\omega}\cos\left(\omega t + kx^2\right) \tag{10}$$

noting that when the sign of b is unknown, one cannot design a classical PID type stabilizing controller. Theorem 1 implies that the closed loop average system related to (10) is given by

$$\dot{\bar{x}} = \left(a - k\alpha b^2\right)\bar{x}, \tag{11}$$

which is stabilized by a sufficiently large choice of $k\alpha > \frac{a}{b^2}$, regardless of the sign of b. We now provide the details of how Theorem 1 is applied, carrying out weak limit calculations which we will routinely omit in the remainder of the paper. In the notation used in Theorem 1, system (10) may be written as

$$\dot{x} = \underbrace{ax}_{f(x)} + \underbrace{b}_{g(x)}u, \qquad y = \underbrace{x}_{\psi(x)} \tag{12}$$

$$u = \sqrt{\alpha\omega}\cos\left(\omega t + kx^2\right) = \underbrace{\sqrt{\alpha\omega}\cos(\omega t)}_{h_{1,\omega}(t)}\underbrace{\cos\left(kx^2\right)}_{k_1(x)}\underbrace{-\sqrt{\alpha\omega}\sin(\omega t)}_{h_{2,\omega}(t)}\underbrace{\sin\left(kx^2\right)}_{k_2(x)}, \tag{13}$$

which has closed loop form

$$\dot{x} = ax + \underbrace{\sqrt{\alpha\omega}\cos(\omega t)}_{h_{1,\omega}(t)}\underbrace{b\cos\left(kx^2\right)}_{b_1(x)} \underbrace{-\sqrt{\alpha\omega}\sin(\omega t)}_{h_{2,\omega}(t)}\underbrace{b\sin\left(kx^2\right)}_{b_2(x)}. \tag{14}$$

Consider the sequence of functions $\{h_{1,\omega}(t)\} = \{\sqrt{\alpha\omega}\cos(\omega t)\}$ and $\{h_{2,\omega}(t)\} = \{-\sqrt{\alpha\omega}\sin(\omega t)\}$ where $\omega \to \infty$. Consider corresponding sequences $\{H_{i,\omega}(t) = \int_0^t h_{i,\omega}(\tau)d\tau\}$ where

$$H_{1,\omega}(t) = \sqrt{\frac{\alpha}{\omega}}\sin(\omega(t)), \qquad H_{2,\omega}(t) = \sqrt{\frac{\alpha}{\omega}}\cos(\omega(t))$$

and notice that for each i, the functions $H_{i,\omega}(t)$ converge uniformly to 0 as $\omega \to \infty$. In the present example, according the Riemann–Lebesgue Lemma (Conway, 1990),

$$h_{1,\omega}(t)H_{1,\omega}(t) = \alpha\cos(\omega t)\sin(\omega t) \rightharpoonup \lambda_{1,1} = 0$$

$$h_{1,\omega}(t)H_{2,\omega}(t) = \alpha\cos^2(\omega t) \rightharpoonup \lambda_{1,2} = \frac{\alpha}{2}$$

$$h_{2,\omega}(t)H_{1,\omega}(t) = -\alpha\sin^2(\omega t) \rightharpoonup \lambda_{2,1} = -\frac{\alpha}{2}$$

$$h_{2,\omega}(t)H_{1,\omega}(t) = -\alpha\sin(\omega t)\cos(\omega t) \rightharpoonup \lambda_{2,2} = 0.$$