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Brief paper Passivity-based output-feedback control of turbulent channel flow^{*}

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a r t i c l e i n f o

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A B S T R A C T

This paper describes a robust linear time-invariant output-feedback control strategy to reduce turbulent fluctuations, and therefore skin-friction drag, in wall-bounded turbulent fluid flows, that nonetheless gives performance guarantees in the nonlinear turbulent regime. The novel strategy is effective in reducing the supply of available energy to feed the turbulent fluctuations, expressed as reducing a bound on the supply rate to a quadratic storage function. The nonlinearity present in the equations that govern the dynamics of the flow is known to be passive and can be considered as a feedback forcing to the linearised dynamics (a Lur'e decomposition). Therefore, one is only required to control the linear dynamics in order to make the system close to passive. The ten most energy-producing spatial modes of a turbulent channel flow were identified. Passivity-based controllers were then generated to control these modes. The controllers require measurements of streamwise and spanwise wall-shear stress, and they actuate via wall transpiration. Nonlinear direct numerical simulations demonstrated that these controllers were capable of significantly reducing the turbulent energy and skin-friction drag of the flow.

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1. Introduction

Turbulent channel flows are characterised by their selfsustaining chaotic motions and are known to induce high skinfriction drag. Conversely, laminar channel flow has the lowest sustainable skin-friction drag [\(Bewley](#page--1-3) [&](#page--1-3) [Aamo,](#page--1-3) [2004\)](#page--1-3) and is stable to infinitesimal perturbations (turbulent fluctuations) for *Re* < 5772 [Trefethen,](#page--1-4) Trefethen, [Reddy,](#page--1-4) [and](#page--1-4) [Driscoll](#page--1-4) [\(1993\)](#page--1-4).^{[2](#page-0-4)} However, experiments and simulations show that channel flow can sustain turbulence for *Re* as low as 1000 [\(Schmid](#page--1-5) [&](#page--1-5) [Henningson,](#page--1-5) [2001\)](#page--1-5) and transition to turbulence can occur at these low Reynolds numbers to perturbations of finite amplitude. This is because the nonlinearity in the Navier–Stokes equations plays a significant role, so a full consideration of stability must take it into account. When

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attempting to control turbulent wall-bounded flows, difficulties arise in several areas. The first of these is modelling. The equations that model the dynamics of all incompressible Newtonian fluids, the incompressible Navier–Stokes equations, are a set of nonlinear partial differential algebraic equations (PDAEs). The linear dynamics of turbulent fluids are known to be responsible for all of the energy production [\(Schmid,](#page--1-6) [2007\)](#page--1-6). Therefore, it is justifiable to only control these dynamics as long as the effect of the nonlinearity is modelled appropriately as a source of uncertainty. In order to form a finite-dimensional linear control model, these equations must first be linearised around a known equilibrium solution and then discretised resulting in a set of differential algebraic equations. The scales of the flow that need to be controlled are not always known. Therefore, there is a balancing act of ensuring that the state dimension of the model is large enough to resolve the smaller scales but making it small enough so that controller synthesis is feasible. Adequate estimation and actuation are other issues. Practically, it is likely that sensors and actuators will be restricted to the walls. This limits the accuracy of flow estimations and efficacy of control actuation away from the walls. These issues combine to limit the drag reduction that can be achieved by output-feedback control.

There has been a significant amount of research into the use of modern control theory to reduce skin-friction drag in turbulent channel flow. A significant portion of this research has investigated [t](#page--1-7)he performance of state-feedback linear quadratic regulator [\(Bew](#page--1-7)[ley](#page--1-7) [&](#page--1-7) [Liu,](#page--1-7) [1998;](#page--1-7) [Högberg,](#page--1-8) [Bewley,](#page--1-8) [&](#page--1-8) [Henningson,](#page--1-8) [2003;](#page--1-8) [Lim,](#page--1-9) [2003\)](#page--1-9)

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 2 *Re* $:=$ $\frac{U_{cl}h}{v}$, centreline Reynolds number, where U_{cl} is the maximum laminar centreline velocity, *h* is the channel half-height and ν is the kinematic viscosity.

and dynamic output-feedback linear quadratic Gaussian controllers [\(Lee,](#page--1-10) [Cortelezzi,](#page--1-10) [Kim,](#page--1-10) [&](#page--1-10) [Speyer,](#page--1-10) [2001\)](#page--1-10). Model predictive control has also been used for drag reduction, both state-feedback [\(Bewley,](#page--1-11) [Moin,](#page--1-11) [&](#page--1-11) [Temam,](#page--1-11) [2001\)](#page--1-11) and output-feedback controllers [\(Lee,](#page--1-12) [Kim,](#page--1-12) [&](#page--1-12) [Choi,](#page--1-12) [1998\)](#page--1-12) have been investigated. Furthermore, static output-feedback control laws have been derived capable of globally stabilising low-Reynolds number two-dimensional channel flows [\(Balogh,](#page--1-13) [Liu,](#page--1-13) [&](#page--1-13) [Krstic,](#page--1-13) [2001\)](#page--1-13). For an overview of the field of flow control in general, the reader is referred to the books by [Aamo](#page--1-14) [and](#page--1-14) [Krstić](#page--1-14) [\(2003\)](#page--1-14), [Barbu](#page--1-15) [\(2011\)](#page--1-15) and [Gad-el-Hak](#page--1-16) [\(2000\)](#page--1-16).

There are many sources of uncertainty when controlling fluidic systems; examples include exogenous disturbances to flow variables, parametric uncertainty and modelling uncertainty. Passivity-based control has been proven to be both effective and robust to disturbance uncertainty. [Sharma,](#page--1-17) [Morrison,](#page--1-17) [McKeon,](#page--1-17) [Limebeer,](#page--1-17) [and](#page--1-17) [Koberg](#page--1-17) [\(2011\)](#page--1-17) designed globally stabilising linear time-invariant (LTI) passivity-based controllers capable of relaminarising $Re_\tau = 100$ channel flow.^{[3](#page-1-0)} Note that laminar incompressible channel flow has the lowest sustainable drag [\(Bewley](#page--1-3) [&](#page--1-3) [Aamo,](#page--1-3) [2004\)](#page--1-3). The controllers required full flow field information of the wall-normal velocity whilst actuation was via body-forcing on the wall-normal velocity throughout the channel. In their approach, they recognised that the nonlinearity in the Navier–Stokes equations acts a passive feedback operator. The passivity theorem states that two passive systems in feedback leads to the global system being passive. Therefore, it was required only to enforce passivity on the linear system to guarantee global stability. With the choice of sensing and actuation used, the controllers were capable of making the linear dynamics passive, but only just. It was found that only the four lowest spatial Fourier modes of the system needed to be controlled in order for flow relaminarisation to occur, suggesting that it is these modes that are most important for energy production. In an earlier work, [Sharma](#page--1-18) [\(2009\)](#page--1-18) found that the passivity framework could also be applied to the linearised Navier–Stokes equations for the purposes of robust model reduction.

The aim of this paper is to extend the work of [Sharma](#page--1-17) [et al.](#page--1-17) [\(2011\)](#page--1-17) towards passivity-based control of turbulent channel flow with actuation *and* sensing restricted to the walls; in this particular case, sensing of streamwise and spanwise wall-shear stress and actuation via wall transpiration. This will show the drag reduction achievable by a passivity-based controller with more realistic sensing and actuation than that used previously. This is the first time an output-feedback passivity-based control method with wall sensing and actuation has been applied to turbulent flow. As will be demonstrated, with the sensing/actuation arrangement employed, it will not be possible to enforce passivity. Instead, passivity-based control shall be used to minimise the upper bound on energy production of the closed-loop system. This is achieved by restricting the supply of energy to the flow's spatial modes. This paper also aims to analyse the linear dynamics of channel flow using the framework of passivity, revealing which modes are responsible for the majority of energy production and therefore which modes need to be controlled. Identifying these spatial modes is a novel contribution and it is hoped that this will aid in future controller design. The work presented in this paper builds on that by [Heins,](#page--1-19) [Jones,](#page--1-19) [and](#page--1-19) [Sharma](#page--1-19) [\(2014\)](#page--1-19) which outlined a linear analysis of this control method. This paper goes substantially further by applying passivity-based controllers to high-fidelity nonlinear simulations in order to gain insight into the performance of this control method on realistic turbulent flows. Finally, the synthesis methodology has been significantly simplified compared

Fig. 1. A schematic displaying the system energy bound ε for LTI system $v(t)$ = *Gf*(*t*).

to [Sharma](#page--1-17) [et al.](#page--1-17) [\(2011\)](#page--1-17). We hope that the new synthesis method will be easier to generalise to complex geometries using matrixfree methods that are currently being developed [\(Theofilis,](#page--1-20) [2011\)](#page--1-20).

In the following, a brief review of passivity and passivity-based control is outlined in Section [2,](#page-1-1) details of linear analysis and nonlinear numerical testing and a discussion of the new results from this testing is presented in Section [3,](#page--1-21) conclusions are given in Section [4.](#page--1-22)

2. Passivity-based control

2.1. Preliminaries

Denote with *G* a LTI representation of a spatially discrete, linearised flow system with the input–output relation $v(t)$ = *Gf*(*t*), where $v(t) \in \mathbb{R}^n$ is a vector of velocities and $f(t) \in \mathbb{R}^n$ is a vector of input forces. The system *G* is passive if it is only capable of storing and dissipating energy and not producing any of its own. There are several types of passivity and the terminology for each has varied over the years. The conventions of [Kottenstette,](#page--1-23) [McCourt,](#page--1-23) [Xia,](#page--1-23) [Gupta,](#page--1-23) [and](#page--1-23) [Antsaklis](#page--1-23) [\(2014\)](#page--1-23) shall be used in the current work. The system *G* is *strictly input passive* (SIP) iff there exists $\varepsilon > 0$ such that:

$$
\langle v(t), f(t) \rangle_{[0,T]} \ge \varepsilon \langle f(t), f(t) \rangle_{[0,T]} - \Gamma_0,\tag{1}
$$

for all *T* > 0, where: $\langle X, Y \rangle_{[t_1,t_2]} := \int_{t_1}^{t_2} X^\top Y dt$, denotes an inner product and $\Gamma_0 \in \mathbb{R}$ is the initial stored energy. The system constant $\varepsilon \in \mathbb{R}$ acts as an energy bound; its importance to the current work will be demonstrated throughout this paper. A system is said to be *passive* if $\varepsilon = 0$. For a SIP system, ε bounds energy dissipation from below. However, if $\varepsilon < 0$, the system is not passive and ε bounds energy production from above. This is demonstrated in the schematic in [Fig. 1,](#page-1-2) for the case where $\Gamma_0 = 0$. The graph for *G* can only lie in the shaded region of the figure. When $\varepsilon < 0$, the system is able to dissipate, store *and* produce energy, energy production occurring in the dashed area below the abscissa. The aim of passivity-based control is to minimise $|\varepsilon|$; if possible, forcing $\varepsilon > 0$. Note, that minimising the energy bound $|\varepsilon|$ will give no guarantees of either local or global stability. However, it will guarantee a restriction on system energy production from disturbance $inputsf(t)$.

Passivity is a time-domain concept. However, it has a frequencydomain counterpart named *positive realness*. Taking Laplace transforms of system inputs $f(t)$ and outputs $v(t)$, we can form a transfer function matrix $G(s)$ such that $V(s) = G(s)F(s)$ where $s = \sigma + j\omega$ and $j = \sqrt{-1}$. A system with transfer function matrix *G*(*s*) is *strictly positive real* (SPR) [\(Sun,](#page--1-24) [Khargonekar,](#page--1-24) [&](#page--1-24) [Shim,](#page--1-24) [1994\)](#page--1-24) iff there exists $\varepsilon > 0$ such that $\forall \omega \in [0, \infty)$:

$$
\frac{1}{2}\left[G(j\omega) + G^{\top}(-j\omega)\right] \ge \varepsilon I,\tag{2}
$$

 3 $Re_\tau = \frac{u_\tau h}{v}$, skin-fiction velocity $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$, τ_w is wall-shear stress and ρ is fluid density.

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