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Brief paper

Design of adaptive finite-time controllers for nonlinear uncertain systems based on given transient specifications[☆]

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ABSTRACT

In this paper, we consider designing adaptive finite-time controllers for a class of SISO strict feedback nonlinear plants with parametric uncertainties based on given specifications. In addition to system stability and for the system output and virtual control errors to converge to zero, the specifications also include requirement on transient response in terms of convergence time and convergence rate. If the bound of the compact set in which unknown parameter lies is incorporated, the designed finite-time controller can ensure convergence within two stages. In the first stage the squared norm of the system states including the virtual control errors converges to a given invariant set faster than a specified linear rate. In the second stage the states converge to the origin with a convergence rate faster than a given exponential speed. If the bound of system initial states is also incorporated into the controller, then the states converge to the origin within the given time at a rate faster than the pre-specified exponential rate.

1. Introduction

In controller design, a set of specifications is usually given in advance to guide the designing of controller. The essential requirement is the system stability. The others include steady state accuracy and transient performances. However for nonlinear systems including adaptive control systems, controller design based on transient specifications is very challenging, as it is not an easy task to establish and quantify the transient responses of closed-loop systems.

Adaptive control strategy has been proved to be an efficient and effective way to control systems with parametric uncertainties. In the absence of disturbance and unmodeled dynamics, most adaptive control systems ensure tracking/stabilization errors to converge to the origin. In other words, the given specifications are only system stability and steady state performance of tracking errors, without any consideration of transient performance in controller design. In fact, the resulting transient responses of adaptive control systems may be unacceptable due to non-guaranteed convergence time/rate which may be very long/slow and this clearly limits practical applications of adaptive control schemes. Thus many researchers have put lots of efforts to study and improve the transient performance for adaptive control systems. In Krstic, Kokotovic, and Kannelakopoulos (1993), $\mathcal{L}_2$ and $\mathcal{L}_\infty$ performance bounds are derived for a class of adaptive control schemes and improvement of such bounds can be achieved by tuning controller parameters. In Datta and Ioannou (1994), the mean square tracking error and the $\mathcal{L}_\infty$ tracking error bound are used as criteria to assess the performance of a standard model reference adaptive control. A modified control scheme based on these two criteria is proposed to achieve an arbitrary improved nominal performance. In Zhang, Wen, and Soh (1999) a scheme for designing a totally decentralized adaptive stabilizers for a class of large-scale systems is presented. The transient performance of the adaptive system is also evaluated by both $\mathcal{L}_2$ and $\mathcal{L}_\infty$ bounds of the tracking errors. Some other results related to $\mathcal{L}_2$ and $\mathcal{L}_\infty$ performance bounds are also reported in Zhou, Wen, and Zhang (2004, 2006). These bounds can be made arbitrarily small by properly choosing control design parameters. In Bechlioulis and Rovithakis (2009) two robust adaptive control schemes for strict feedback nonlinear uncertain systems

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are proposed using a prescribed performance bound (PPB) technique. These schemes are capable of guaranteeing prescribed performance bounds semi-globally. In Wang and Wen (2010) a new adaptive controller design scheme is proposed for nonlinear systems with the consideration of uncertain actuator failure based on a prescribed performance bound which characterizes the convergence rate and maximum overshoot of the tracking error. In this way, it ensures the transient performance even in the presence of actuator failure. While only guaranteeing the convergence rate of the tracking error with these schemes, the transient responses of other system states such as virtual control errors cannot be established. Moreover, the prescribed signal depends on the initial value of the tracking error, thus the results are only local in the sense that the designed control schemes can only be applicable to tracking errors satisfying the given initial conditions. On the other hand, there is still no result available taking convergence time together with convergence rates into account in the design of adaptive controllers.

It is noted that continuous finite-time control can make the controlled systems converge to the origin within finite time. Finite-time stabilization controllers have been investigated for different classes of systems. In Bhat and Bernstein (1998) a class of bounded continuous time-invariant finite-time stabilizing feedback laws is given for the double integrator. Lyapunov theory is used to prove finite-time convergence. In Huang, Lin, and Yang (2005) finite-time stabilization problem for a class of nonlinear systems that are dominated by a lower-triangular system is investigated. In Qian and Lin (2001b) continuous global finite-time stabilizer is designed by using a recursive *add a power integrator* technique originally proposed in Coron and Praly (1991). In Hong, Wang, and Cheng (2006) global finite time stabilization is investigated for a class of nonlinear systems in p-normal form with parametric uncertainties based on the backstepping approach.

In this paper, an adaptive control scheme involving an online parameter estimator is designed to guarantee finite time stability of the closed loop system. If the system states including virtual control errors are required to meet additional specifications such as convergence time and convergence rate, the proposed scheme is still able to achieve them. The required convergence rate can be ensured within two stages, by incorporating the bound of a compact set in which the unknown parameter lies into the controller design. In the first stage the squared norm of the system states converges faster than a pre-specified linear jumping-down rate, while in the second stage the states approach to the origin with a convergence rate faster than the given exponential speed. To meet the requirements of pre-specified convergence time and exponential rate, the bound of system initial states is also required to be incorporated into the controller.

The remainder of the paper is organized as follows. The problem is formulated and some useful preliminaries are presented in Section 2. In Sections 3 and 4, adaptive finite-time control schemes are proposed and the transient performances are analyzed, respectively. Simulation results are given in Section 5 to validate the theoretical results. Finally, we conclude the paper in Section 6.

2. Preliminaries and problem formulation

2.1. Preliminaries

Consider $y = x^{\frac{p}{q}}$ where p is a positive integer and q is a positive odd integer. If we ignore all the complex roots, then obviously $y = \operatorname{sign}(x)|x|^{\frac{p}{q}}$ if p is an odd integer; otherwise $y = |x|^{\frac{p}{q}}$.

To establish our results, some preliminary lemmas and definitions are firstly introduced.

Lemma 1 (Bhat & Bernstein, 1998). *Suppose there is a C^1 positive definite Lyapunov function $V(x,t)$ defined on $U \times \Re^+$ where $U \subset R^n$ is the neighborhood of the origin, and there are positive real constants $c > 0$ and $0 < \alpha < 1$, such that $\dot{V}(x,t) + cV^{\alpha}(x,t)$ is negative semidefinite on U. Then $V(x,t)$ is locally in finite-time convergent with a settling time*

$$T \le \frac{V^{1-\alpha}(x_0,t_0)}{c(1-\alpha)}$$

for any given initial condition $x(t_0)$ in the neighborhood of the origin in U.

Lemma 2 (Qian & Lin, 2001a). *If $0 < p = p_1/p_2 \le 1$, where $p_1 > 0$, $p_2 > 0$ are positive odd integers, then $|x^p - y^p| \le 2^{1-p}|x-y|^p$.*

Lemma 3 (Hardy, Littlewood, & Polya, 1952). *For $x_i \in \Re, i = 1,\dots,n$, $0 < p \le 1$, then*

$$\left(\sum_{i=1}^{n}|x_i|\right)^p \le \sum_{i=1}^{n}|x_i|^p \le n^{1-p}\left(\sum_{i=1}^{n}|x_i|\right)^p.$$

Lemma 4 (Qian & Lin, 2001a). *Let d and e be positive constants and $\gamma(x,y) > 0$ is a real value function. Then we have*

$$|x|^d|y|^e \le \frac{d\gamma(x,y)|x|^{d+e}}{d+e} + \frac{e\gamma^{-d/e}(x,y)|y|^{d+e}}{d+e}.$$

Next lemma is the well-known Young's inequality.

Lemma 5. *For nonnegative numbers a, b, and $\kappa > 0$, we have*

$$ab < \frac{a^{1+\kappa}}{1+\kappa} + \frac{\kappa b^{1+\frac{1}{\kappa}}}{1+\kappa} \le a^{1+\kappa} + b^{1+\frac{1}{\kappa}}. \tag{1}$$

Definition 1 (Bhat & Bernstein, 1998). Consider a dynamic system

$$\dot{x} = f(x,t), \quad f(0,t) = 0 \tag{2}$$

where $f : U_0 \times \Re^+ \to \Re^n$ is continuous on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is (locally) finite-time stable if it is Lyapunov stable and for any initial condition $x_{t_0} \in U$ where $U \subset U_0$, if there is a settling time $T > t_0$, such that every solution $x(t;t_0,x_0)$ of system (2) satisfies $x(t;t_0,x_0) \in U\backslash\{0\}$ for $t \in [t_0,T)$, and

$$\lim_{t\to T} x(t;t_0,x_0) = 0, \quad x(t;t_0,x_0) = 0, \ \forall t \ge T.$$

If $U = \Re^n$, then the origin $x = 0$ is a globally finite-time stable equilibrium.

2.2. Problem formulation

We consider the following class of nonlinear systems

$$\begin{aligned}
\dot{x}_1 &= x_2 + f_1(t,x_1,\sigma)\\
\dot{x}_2 &= x_3 + f_2(t,x_1,x_2,\sigma)\\
&\vdots\\
\dot{x}_n &= u + f_n(t,x_1,\dots,x_n,\sigma)
\end{aligned} \tag{3}$$

where x_i, $i = 1,\dots,n$ and $u \in \Re$ are the system state and input respectively, and $f_i : \Re^i \times \Re^+ \to \Re$, $i = 1,\dots,m$, are C^1 functions with $f_i(t,0,\dots,0,\sigma) = 0$, $\forall t$. σ is an uncertain constant. The following assumption is made.