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Evaluating Performance Limits in FOTD Plant Control

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Abstract: The paper deals with first order time delayed plant control. It starts with introducing and comparing several alternative approaches comprising PI control and some model based approaches. This comparison focuses on speed of transients and contemplates a possibility of approaching its absolute limits by keeping additional constraints of transient shapes at the plant input and output. When estimating achievable performance limits in nominal situations, we can see an equivalence of several formally originally and possible areas of their application. Then the motivational and educational aspects of managing diversity of considered solutions are discussed. The aim of the experimental phase of this contribution is to demonstrate common points and differences of these approaches by their application to laboratory plant models exhibiting several features typically observed in industrial plants control.

Keywords: Optimal control, performance analysis, PID control, dead time compensator, control education.

1. INTRODUCTION

PI and PID control represent the most frequently used control technology in practice (Åström and Hägglund, 2006). Their technological basis has been established in the period before the WWII, but it still represents the field of active research. Later, several methods srived to improve its performance. When wishing to discuss the challenges arising from this area to control education and research, we should be able to evaluate the achievable limits, these existing alternative methods can reach. Furthermore, since alternatives resulting from properties of the Diophantine equation solutions formulated for a linear plant model should be nominally equivalent, the question arises, if they are fully equivalent also within a real time control design context and how this diversity may be used in practice.

2. FOTD PLANT'S CONTROL PERFORMANCE

In control design the primary attention is paid to the first order time delayed (FOTD) plant models

$$S(s) = \frac{Y(s)}{U(s)} = \frac{K_s e^{-T_d s}}{s+a} \tag{1}$$

Once wishing to discuss the success of particular solutions which may be interesting for both research, and control education, we should start with identifying the ideal performance limits and defining the performance measures to evaluate their performance.

2.1 Time related performance measures

When considering the time aspects of a FOTD plant control, usually the IAE (Integral of Absolute Error) is used (Shinskey, 1990) defined as

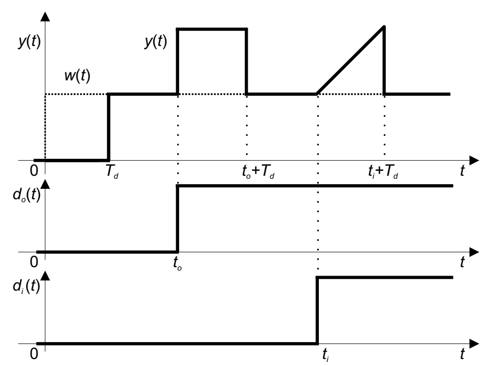

Fig. 1. Ideal (limit) responses of an IPDT plant ($a = 0$) with a setpoint $w(t)$, output disturbance d_o and an input disturbance step d_i

$$IAE = \int_0^\infty |e(t)|\, dt\ ;\ \ e = w - y \tag{2}$$

Since the PI control is known by its Pareto character (Arrieta and Vilanova, 2011; Grimholt and Skogestad, 2012; Huba, 2013a), it is recommended to formulate a controller optimization problem by considering both values of the setpoint (IAE_s) and the (input or output) disturbance (IAE_d) responses with a cost function formulated, for Example, as

$$IAE_\Sigma = IAE_s + IAE_d \tag{3}$$

All FOTD plants may be modeled by an integral plus dead time (IPDT) model extended by an internal feedback. Thus it is always useful to check the results related to FOTD control for an IPDT plant representing their simplest version (Ťapák and Huba, 2016). Thanks to the dead time delaying response of a control, an ideal setpoint step response (Fig. 1) corresponding to a unit step of $w(t)$ may be characterized by a figure not lower than

$$IAE_{sopt} = T_d \tag{4}$$

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For an output disturbance unit step d_o added to the plant output $y(t)$, the IAE value may not be lower than

$$IAE_{oopt} = T_d \quad (5)$$

For an input disturbance unit step d_i added to the plant input $u(t)$, the output starts firstly to linearly increase and the corresponding IAE_i value may not be lower than

$$IAE_{iopt} = 0.5 K_s T_d^2 \quad (6)$$

These figures give performance limits related to the control speed. Apart from this, however, we also need to think about the transient shapes at the input and output.

2.2 Shape related performance measures

An important property of all step responses is their smoothness requiring a piecewise monotonicity. Output deviations from the monotonicity (MO) may be evaluated by using the TV_0 measure (relative total variance modified from TV introduced by Skogestad (2003))

$$TV_{0y} = \sum_i |y_{i+1} - y_i| - |y_\infty - y_0| \quad (7)$$

As proven in Huba (2013a), for integral and unstable first order plants a MO output step response is always related to a one-pulse (1P) input, i.e. to an input consisting of two MO intervals separated by an extreme point, or an extreme interval. For stable plants a MO output may also be achieved by MO input, but such transients are usually much slower. [1] The deviations of the plant input $u(t)$ from the 1P behavior may be evaluated in terms of

$$TV_{1u} = \sum_i |u_{i+1} - u_i| - |2u_m - u_\infty - u_0| \quad (8)$$

Thereby, u_0 and u_∞ represent the initial and final input values and $u_m \notin (u_0, u_\infty)$ is the extreme control value.

Similarly, deviations of the output disturbance step responses from ideal 1P shapes may be evaluated by TV_{1yd} measure. In control of FOTD systems a MO output step responses may also be combined with a higher number of input pulses, but the performance is still dominated by the above mentioned situations.

3. PI CONTROL

3.1 Analytical "optimal" (TP) PI controller tuning

When wishing to modify dynamics of the setpoint and disturbance responses separately, the PI controller

$$R(s) = K_c \frac{1 + T_i s}{T_i s} \quad (9)$$

may be extended to a two degree of freedom (2DOF) control by a setpoint weighting, or by a prefilter

$$F_p(s) = \frac{b T_i s + 1}{T_i s + 1} \quad (10)$$

Large number of methods for an "optimal" PI controller tuning signalizing already an inflation may be found in O'Dwyer (2009). In case of occurrence more than one, we should try (to help our students) to understand, why it is so. Having being confronted with this question for several decades and having being inspired by success of the first known controller tuning method by Ziegler and Nichols (1942), we have developed a new performance portrait method (Huba, 2013a,b). Similarly as in the former approach, it is based on checking the closed loop performance for all possible tunings and choosing the best one which would guarantee the chosen performance criteria. This allows to find both the optimal nominal tuning for precisely known systems and the optimal robust PI tuning for systems with uncertainties. Evaluation of the PI controller tunings for the integral plus first order (IPDT) plant in Huba (2013a) has shown that in the nominal case one may analytically derive nearly optimal results by the triple real dominant pole (TP) method (Vítečková and Víteček, 2008, 2010; Huba, 2016). Thereby, a pole s_o of a characteristic quasi-polynomial $P(s)$ has to fulfill conditions $P(s_o) = 0$, $\dot{P}(s_o) = 0$ a $\ddot{P}(s_o) = 0$. It yields

$$\begin{aligned} & s_o = -\frac{A_d + 4 - S_d}{2T_d}\ , \quad A_d = aT_d\ , \quad S_d = \sqrt{A_d^2 + 8} \\ & K_o = K_c K_s T_d = (S_d - 2) e^{(S_d - A_d - 4)/2} \\ & \tau_o = \frac{T_i}{T_d} = \frac{2(2 - S_d)}{A_d^2 + 2A_d + 28 - (A_d + 10) S_d} \\ & b = \frac{1}{\tau_o T_d s_o} = \frac{A_d^2 + 2A_d + 28 - (A_d + 10) S_d}{(S_d - 2)(S_d - A_d - 4)} \end{aligned} \quad (11)$$

For an IPDT plant with $a = 0$ simplified formulas yield

$$\begin{aligned} & s_o = -(2 - \sqrt{2})/T_d \approx -0.586/T_d \\ & K_o = K_c K_s T_d = 2(\sqrt{2} - 1) e^{\sqrt{2} - 2} \approx 0.461 \\ & \tau_o = T_i/T_d = (2\sqrt{2} + 3) \approx 5.828 \\ & b = T_p/T_i = (2 - \sqrt{2})/2 \approx 0.293 \end{aligned} \quad (12)$$

The resulting performance characterized by the IAE values may in a nominal case be calculated by the Laplace transform as IE (integral of error) which yields

$$\begin{aligned} IAE_{s,PI} &= (1 - b) T_i \approx 4.121\, T_d \\ IAE_{i,PI} &= T_i / K_P \approx 12.639\, K_s T_d^2 \end{aligned} \quad (13)$$

For the output disturbance steps one gets $IAE_o = 0$, which signalizes sign changes of the control error. The PI control is unable to get a MO decayed attenuation of output disturbance steps approaching the ideal shape in Fig. 1. The IAE_o values corresponding to (12) have to be calculated by a simulation. Obviously, all obtained IAE figures (Tab. 1) are much higher than the performance limits (4)-(6). This fact may be neglected for $T_d \approx 0$, whereas for longer T_d it provides an incentive for looking for improvements.

3.2 2DOF PI tuning by PP method

Some improvements may be achieved by the performance portrait (PP) method (Huba, 2013a). It is based on evaluating the closed loop performance over a grid of all the meaningful closed loop parameters. Then the aim may be formulated, for example, to find tuning yielding $\min(IAE_\Sigma)$ under four shape related constraints in form of tolerable deviations from ideal responses at the output and input

$$\begin{aligned} TV_0(y_s) \le \epsilon_{ys}\ ;\ TV_1(y_d) \le \epsilon_{yd} \\ TV_1(u_s) \le \epsilon_{us}\ ;\ TV_1(u_d) \le \epsilon_{ud} \end{aligned} \quad (14)$$

Optimization results in Tab.1 considering

$$IAE_\Sigma = IAE_s + IAE_i \quad (15)$$

with

$$\epsilon = \epsilon_{ys} = \epsilon_{yd} = \epsilon_{us} = \epsilon_{ud} = 0.001 \quad (16)$$

[1] For first order systems with a long dead time the input associated with a MO output may also consists of several MO intervals