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New developments for matrix fraction descriptions: A fully-parametrised approach☆

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ABSTRACT

This article aims at giving a new answer for the challenging problem of the parametrisation of multi-input multi-output matrix fraction descriptions. In order to reach this goal, new parametrisations of matrix fraction descriptions, called fully-parametrised left matrix fraction descriptions (F-LMFD), are first introduced. Their structural properties as well as their suitability for multi-input multi-output model description are more precisely analysed. As any over-parametrised model description, the F-LMFD cannot describe a transfer function uniquely. The structure of the space of equivalent F-LMFD is then investigated through the determination of its basis. The study carried out in this article is the prelude to a computational improvement of the identification of matrix fraction descriptions with gradient-based optimisation methods.

1. Introduction

In this article, the problem of the parametrisation of Multi-Input Multi-Output (MIMO) Linear Time Invariant (LTI) systems of a given order n_x is addressed. More particularly, Matrix Fractions Descriptions (MFD) of MIMO transfers are considered. As underlined in Gevers (2006), methodological research on MIMO system parametrisation has been neglected since the 1990s in aid of subspace-based system identification methods. This can mainly be explained for two reasons. First, the tricky problem of finding appropriate parametrisations of MIMO systems is bypassed when working with subspace-based methods (Gevers, 2006). Second, optimisation algorithms often showed poor convergence properties with the minimal parametrisations used to ensure the model identifiability.

Mostly in the 1980s, several parametrisations of MIMO systems were derived. First, a lot of studies focused on canonical parametrisations (Clark, 1976; Hazewinkel & Kalman, 1976). These studies were motivated by the search for a unique representation of the true rational MIMO system, whose structure depends on a finite set of *Kronecker indices*. In parallel, in order to bypass numerical problems encountered, *e.g.*, in system identification (Gevers & Wertz, 1987; Glover & Willems, 1974), with canonical parametrisations, *pseudo-canonical* parametrisations (Gevers & Wertz, 1987), also referred to as *overlapping models* (Glover & Willems, 1974; Van Overbeek & Ljung, 1982) or *multistructural models* (Guidorzi & Beghelli, 1982), have been developed. These studies mainly concerned state-space representations (Clark, 1976; Correa & Glover, 1984; Gevers & Wertz, 1987; Glover & Willems, 1974; Van Overbeek & Ljung, 1982). An equivalent work was done about the parametrisation of MIMO transfer functions (Deistler & Hannan, 1981; Guidorzi & Beghelli, 1982) providing a set of parametrisations for both state-space and transfer function representations that have in common three main features: (i) these parametrisations are based on the selection of structural indices, (ii) the set of all rational systems with a fixed order n_x can be covered by a finite number of *pseudo-canonical* parametrisations (Gevers & Wertz, 1987), (iii) they are minimal parametrisations, *i.e.*, they are made up of the minimal number of descriptive parameters. This latest point comes from the pursued objective of canonical parametrisations in describing every system uniquely. Before the 1990s had indeed predominated

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the idea that a parametrisation which uniquely represents a system should be chosen so as to get a well-conditioned parameter estimation problem (Gevers, 2006). Another reason can certainly be found in the optimisation algorithm formulation since minimal parametrisations lead to Jacobian matrices of full rank (Wills & Ninness, 2008).

In this article, we introduce new fully-parametrised matrix fraction descriptions of MIMO systems of a given order n_x. These parametrisations depend upon a set of structural indices. However, for a given set of indices, these descriptions encompass several canonical descriptions and are therefore less specific. Moreover, we will show that, for a specific and unique choice of indices, the resulting fully-parametrised MFD is related to pseudo-canonical parametrisations. Indeed, for a given order, this full parametrisation is unique and encompasses all the pseudo-canonical descriptions defined in Guidorzi and Beghelli (1982) for MIMO matrix fraction descriptions of order n_x. This feature is of main importance when system identification is concerned because the set of model candidates used for model selection must be large enough and well-parametrised to ensure that the algorithm used to optimise the involved cost function gives access to a global minimum.

This article is organised as follows. Section 2 is devoted to the study of the fully-parametrised MFD (F-MFD). In Section 3, the case of F-MFD specified by quasi-constant indices is detailed. Preliminary results about the structure of the equivalence class of MFD are given in this section. In Section 4, the structure of the equivalence class of MFD is further investigated and specified for any choice of structure indices. After comments gathered in Section 5, Section 6 concludes this study and discusses future research perspectives. Before going further, it can eventually be mentioned here that the results and development presented in this article are illustrated with a numerical example which is first introduced in Section 2. The same numerical example is used all along the article with the aim of helping the reader understanding.

2. Fully parametrised matrix fraction descriptions

In the sequel, the integers n_x, n_u and n_y designate the order of the system and its number of inputs and outputs, respectively. The objective of this section is to define a new parametrisation of MFD that represents MIMO transfer functions of a given order n_x. We shall first recall the desirable requirements for this description.

2.1. Requirements for MIMO system description

In this paper, we focus on MIMO LTI black-box transfer functions. This kind of representations is indeed widespread in modern control theory (Goodwin, Graebe, & Salgado, 2001; Ljung, 1999). We more precisely consider irreductible MFD (Kailath, 1980) of linear transfer functions $\mathsf{H}(s)$ (Kailath, 1980). Because the results for right MFD (RMFD) are obtained by transposing those holding for left MFD (LMFD), only the case of LMFD is developed in the sequel. Thus, $\mathsf{H}(s)$ is written as

$$\mathsf{H}(s) = \mathsf{D}^{-1}(s)\,\mathsf{N}(s), \tag{1}$$

where $\mathsf{N}(s)$ and $\mathsf{D}(s)$ are polynomial matrices of dimensions $n_y \times n_u$ and $n_y \times n_y$, respectively while s is the Laplace transform variable. The coefficients of these polynomials are the n_θ unknown parameters of the transfer function, gathered herein into a vector $\theta \in \mathbb{R}^{n_\theta}$, also referred to as *vector of parameters* hereafter. We need now to devise a degree structure on $\mathsf{D}(s)$ and $\mathsf{N}(s)$ (Kailath, 1980), or in other words, a parametrisation, of these matrices so as to describe MIMO transfer functions consistently. For a fixed-order transfer function satisfying $\lim_{s\to\infty}|\mathsf{H}(s)| < \infty$ (Kailath, 1980), three essential conditions can be listed. First, the order of $\mathsf{H}(s)$ should be equal to n_x whatever the values of θ. Second, the description (1) should be proper (Kailath, 1980, p. 382) for all values of θ. Third, every linear system of order n_x should have, at least, one representation $\mathsf{H}(s)$ in the parameter space.

As highlighted in McKelvey (1998) as well as in the introduction of the current paper, the main difficulty with MIMO MFD is to fix, *a priori*, the model parametrisation, *i.e.*, the way the unknown parameters θ enter the matrices $N(s)$ and $D(s)$, respectively. From a practical viewpoint, especially if system identification is concerned, considering a minimal parametrisation, *i.e.*, an injective mapping from the space of parameters to the model space is tempting. Unfortunately, as shown in Kailath (1980), no injective mapping covering all the linear models of a given McMillan degree n_x exists. In order to circumvent this difficulty, fully-parametrised surjective MFD are considered hereafter. Said differently, the aim of this section is to define a maximal parametrisation of LMFD. Herein, *maximal parametrisation* means a parametrisation with the largest number of parameters as possible that fulfils the previous requirements.

2.2. Preliminary definitions

Before going further, we need to introduce notations and definitions. The set of proper transfer functions of dimension $n_y \times n_u$ and of order n_x is denoted by $\mathbb{M}(n_x)$. To avoid any confusion, the LMFD are distinguished from the transfer functions they represent via the notation $(\mathsf{D}(s), \mathsf{N}(s))$. Moreover, we note $\mathbb{S}(n_x)$ the set of LMFD $(\mathsf{D}(s), \mathsf{N}(s))$ of order n_x. With these notations, we can introduce the function

$$\begin{aligned} \pi:\quad \mathbb{S}(n_x) &\longrightarrow \mathbb{M}(n_x) \\ (\mathsf{D}(s), \mathsf{N}(s)) &\longmapsto \mathsf{D}^{-1}(s)\,\mathsf{N}(s). \end{aligned} \tag{2}$$

From Hannan and Deistler (1988), we have $\pi(\mathbb{S}(n_x)) = \mathbb{M}(n_x)$. In the following sections, only irreducible LMFD will be considered (Hannan & Deistler, 1988; Kailath, 1980). Hence, let us finally introduce the set of irreducible LMFD as

$$\mathbb{S}_i(n_x) = \{(\mathsf{D}(s), \mathsf{N}(s)) \in \mathbb{S}(n_x) \mid (\mathsf{D}(s), \mathsf{N}(s)) \text{ is irreducible}\}, \tag{3}$$

with $\mathbb{S}_i(n_x) \subset \mathbb{S}(n_x)$.

2.3. Definition of the set of Fully-parametrised LMFD (F-LMFD)

Definition 1 (*F-LMFD*)**.** Let $\mathsf{H}(s)$ be a transfer function of order n_x and of dimension $n_y \times n_u$. We name *Fully-parametrised LMFD (F-LMFD)* a LMFD $(\mathsf{D}(s), \mathsf{N}(s))$ defined by the following degree structure (Kailath, 1980)

$$\deg(\mathsf{D}(s)) = \begin{bmatrix} -\rho_1- \\ \vdots \\ -\rho_{n_y}- \end{bmatrix}, \quad \deg(\mathsf{N}(s)) = \begin{bmatrix} -\rho_1- \\ \vdots \\ -\rho_{n_y}- \end{bmatrix}, \tag{4}$$

where $\mathsf{D}(s)$ is assumed to be row reduced, *i.e.*, the row degrees ρ_i of $\mathsf{D}(s)$ and $\mathsf{N}(s)$ satisfy the condition (Kailath, 1980)

$$\sum_{i=1}^{n_y} \rho_i = n_x \quad \text{with } \rho_i \geq 0. \tag{5}$$

Regarding the numerator, the degree structure of the F-LMFD is identical to the well known Echelon canonical form (Kailath, 1980) and to the pseudo-canonical form introduced in Guidorzi and Beghelli (1982). The denominator structure is however much simpler. The F-LMFD denominator is indeed only structured by row, every columns degrees being identical while the denominator