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Errors-in-variables system identification using structural equation modeling☆

David Kreiberg[a,b], Torsten Söderström[c], Fan Yang-Wallentin[a]

[a] Department of Statistics, Uppsala University, PO Box 513, SE-75120, Uppsala, Sweden

[b] Department of Economics, Norwegian Business School, 0442 Oslo, Norway

[c] Division of Systems and Control, Department of Information Technology, Uppsala University, PO Box 337, SE-75105, Uppsala, Sweden

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ABSTRACT

Errors-in-variables (EIV) identification refers to the problem of consistently estimating linear dynamic systems whose output and input variables are affected by additive noise. Various solutions have been presented for identifying such systems. In this study, EIV identification using Structural Equation Modeling (SEM) is considered. Two schemes for how EIV Single-Input Single-Output (SISO) systems can be formulated as SEMs are presented. The proposed formulations allow for quick implementation using standard SEM software. By simulation examples, it is shown that compared to existing procedures, here represented by the covariance matching (CM) approach, SEM-based estimation provide parameter estimates of similar quality.

1. Introduction

Several estimation methods have been proposed for identifying linear dynamic systems from noise-corrupted output measurements, see for instance Ljung (1999) and Söderström and Stoica (1989). On the other hand, estimation of the parameters of systems in which the input signal is also affected by noise, here referred to as "errors-in-variables" (EIV) models, is recognized as a more delicate problem. Studying such systems is of interest due to their potential usage in the engineering sciences and elsewhere.

Established techniques for handling the EIV problem include the bias-eliminating least squares (Zheng, 1998, 2002), the Frisch estimator (Beghelli, Castaldi, Guidorzi, & Soverini, 1993; Beghelli, Guidorzi, & Soverini, 1990; Diversi & Guidorzi, 2012; Diversi, Guidorzi, & Soverini, 2003, 2004, 2006; Guidorzi, Diversi, & Soverini, 2008; Söderström, 2008) and various forms of bias-compensated least squares (Ekman, 2005; Ekman, Hong, & Söderström, 2006; Mahata, 2007). An overview of EIV system identification, containing various solutions from the literature, can be found in Söderström (1981, 2007, 2012). The topic is also treated from different points of view in the books (Cheng & Van Ness, 1999; Fuller, 2006). A more recent development is represented by the covariance matching (CM) approach introduced in Mossberg and Söderström (2011), Söderström and Mossberg (2011) and Söderström, Mossberg, and Hong (2009). Mossberg and Söderström (2012) and Söderström, Kreiberg, and Mossberg (2014). This approach has been shown to be related to structural equation modeling (SEM) techniques. In Kreiberg, Söderström, and Yang-Wallentin (2013), it is demonstrated how SEM can be applied to the problem of EIV system identification.

The objective of the present study is to further extend and refine the SEM approach. As compared to Kreiberg et al. (2013), we provide a more thorough analysis of how SEM can be applied to the EIV problem. Two different and quite general formulations of the EIV system as SEMs are presented, and their relation is analyzed. To facilitate the SEM implementation of such systems, several extensions of the standard SEM framework are proposed. The suggested formulations are evaluated in terms of statistical and numerical performance using simulated data. Aspects concerning the implementation, which were only briefly considered in Kreiberg et al. (2013), are studied in more detail. In the simulation examples, standard software developed for SEM-based estimation is used.

The study is organized as follows. First, in Section 2, we outline the background of the EIV problem. In Section 3, the standard

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E-mail addresses: David.Kreiberg@statistics.uu.se, David.Kreiberg@bi.no (D. Kreiberg), Torsten.Soderstrom@it.uu.se (T. Söderström), Fan.Yang@statistics.uu.se (F. Yang-Wallentin).

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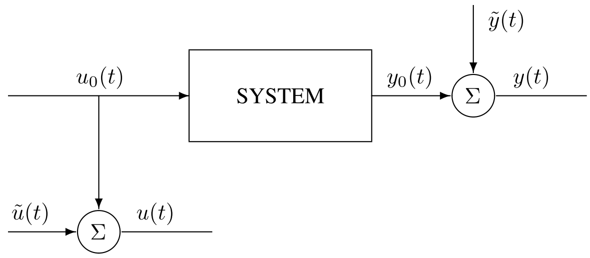

Fig. 1. Basic setup for the dynamic EIV problem.

SEM framework for static systems is reviewed. In Section 4, it is shown how EIV systems can be formulated as SEMs, and in Section 5, simulation examples of the two formulations are presented. Finally, in Section 6, concluding remarks are given.

2. EIV system formulation

First, we define the signals entering the system and then describe the general EIV problem for linear dynamic systems. The usual setup of the EIV problem is illustrated in Fig. 1.

Our interest lies in the linear Single-Input Single-Output (SISO) system described by

$$A(q^{-1})y_0(t) = B(q^{-1})u_0(t), \tag{1}$$

where $y_0(t)$ and $u_0(t)$ are the noise-free output and input signals, respectively, and $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1} of the form

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}, \tag{2}$$

$$B(q^{-1}) = b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}. \tag{3}$$

We allow the noise-free signals to be corrupted by additive measurement noises $\tilde{y}(t)$ and $\tilde{u}(t)$. The available signals are in discrete time and are given by

$$y(t) = y_0(t) + \tilde{y}(t), \tag{4}$$

$$u(t) = u_0(t) + \tilde{u}(t). \tag{5}$$

Since $y_0(t)$ and $u_0(t)$ are not directly observable, the signals are considered *latent*.

The assumptions related to the system and its components are as follows:

$\mathcal{A}1$. All signals and disturbances are zero mean stationary processes.
$\mathcal{A}2$. The polynomials $A(q^{-1})$ and $B(q^{-1})$ are coprime and their respective degrees n_a and n_b are known.
$\mathcal{A}3$. Data records of the noisy output and input signals $\{y(t), u(t)\}_{t=1}^{N}$ are known.
$\mathcal{A}4$. The noise-free input $u_0(t)$ is unknown as well as its second order properties such as its spectrum $\phi_{u_0}(\omega)$.
$\mathcal{A}5$. The measurement noises $\tilde{y}(t)$ and $\tilde{u}(t)$ are white and mutually uncorrelated. Moreover, $\tilde{y}(t)$ and $\tilde{u}(t)$ are both uncorrelated with $u_0(t-\tau)$ for all τ. Their unknown variances are denoted $\psi_{\tilde{y}}$ and $\psi_{\tilde{u}}$.

Our concern is to determine the system transfer function described by

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}}{1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}}. \tag{6}$$

It follows that the parameter vector to be estimated from the noisy data is

$$\boldsymbol{\theta}_0 = \begin{pmatrix} a_1 & \cdots & a_{n_a} & b_1 & \cdots & b_{n_b} \end{pmatrix}^T, \tag{7}$$

where the superscript T denotes the transpose. It may also be of interest to determine other system characteristics such as the measurement noise variances $\boldsymbol{\psi}_0 = (\psi_{\tilde{y}}\ \psi_{\tilde{u}})^T$.

3. Structural equation modeling

In multivariate statistics, SEM is a well established statistical technique which has become popular within many disciplines of social science research. The popularity of SEM stems from its versatility, in which estimation problems involving latent variables and measurement errors can be handled. The versatility is also seen from the fact that numerous types of statistical problems can be formulated within the SEM framework. In what follows, we only briefly summarize the basics of SEM. For a more thorough introduction, see Bartholomew, Knott, and Moustaki (2011) and Bollen (1989).

3.1. Model formulation

The basic framework of SEM is described by the following three equations

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\delta}, \tag{8}$$

$$\mathbf{x}_1 = \boldsymbol{\Lambda}_1\boldsymbol{\eta} + \boldsymbol{\epsilon}_1, \tag{9}$$

$$\mathbf{x}_2 = \boldsymbol{\Lambda}_2\boldsymbol{\xi} + \boldsymbol{\epsilon}_2. \tag{10}$$

The first equation is referred to as the *structural equation*, while the latter two equations are known as the *measurement equations*. The random vectors $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ consist of unobserved (or latent) quantities, whereas the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ consist of observed quantities.

The structural equation describes the relationship among the latent quantities, wherein $\boldsymbol{\eta}$ is endogenous and $\boldsymbol{\xi}$ is exogenous. The parameter matrices $\mathbf{B}$ and $\boldsymbol{\Gamma}$ consist of elements that represent the effect of $\boldsymbol{\eta}$ on $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ on $\boldsymbol{\eta}$, respectively. It is assumed that $\mathbf{I} - \mathbf{B}$ is nonsingular such that $\boldsymbol{\eta}$ can be uniquely determined by $\boldsymbol{\xi}$ and the noise vector $\boldsymbol{\delta}$. It is further assumed that $\boldsymbol{\delta}$ has expectation zero and is mutually uncorrelated with $\boldsymbol{\xi}$.

The measurement equations describe how the observed quantities depend on the latent quantities. The parameter matrices $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$ are so-called *loading* matrices whose elements represent the effect of $\boldsymbol{\eta}$ on $\mathbf{x}_1$ and $\boldsymbol{\xi}$ on $\mathbf{x}_2$, respectively. The measurement noises $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ may or may not be correlated, but are assumed to be mutually uncorrelated with $\boldsymbol{\eta}$, $\boldsymbol{\xi}$ and $\boldsymbol{\delta}$. Note that the measurement equations are modeling devices in their own right. When a measurement equation is implemented without considering the remaining equations, the model is referred to as a *Confirmatory Factor Analysis* (CFA) model. Additional details are given in Bartholomew et al. (2011) and Bollen (1989).

The dimensions of the parameter matrices in (8)–(10) follow from the dimensions of the random vectors. Let n_η, n_ξ, n_{x_1} and n_{x_2} denote the number of elements in $\boldsymbol{\eta}$, $\boldsymbol{\xi}$, $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. The dimensions are then given by

$$\mathbf{B}\left(n_\eta \times n_\eta\right), \qquad \boldsymbol{\Gamma}\left(n_\eta \times n_\xi\right), \tag{11}$$

$$\boldsymbol{\Lambda}_1\left(n_{x_1} \times n_\eta\right), \qquad \boldsymbol{\Lambda}_2\left(n_{x_2} \times n_\xi\right). \tag{12}$$

The model framework additionally include the following covariance matrices

$$\mathsf{E}\left\{\boldsymbol{\xi}\boldsymbol{\xi}^T\right\} = \boldsymbol{\Phi}, \qquad \mathsf{E}\left\{\boldsymbol{\delta}\boldsymbol{\delta}^T\right\} = \boldsymbol{\Psi}_\delta, \tag{13}$$

$$\mathsf{E}\left\{\boldsymbol{\epsilon}_1\boldsymbol{\epsilon}_1^T\right\} = \boldsymbol{\Psi}_{\epsilon_1}, \qquad \mathsf{E}\left\{\boldsymbol{\epsilon}_2\boldsymbol{\epsilon}_2^T\right\} = \boldsymbol{\Psi}_{\epsilon_2}, \tag{14}$$

where E is the expectation operator. The dimensions of the matrices in (13) and (14) follow immediately from the dimensions of the involved vectors. Depending on the noise structure, $\boldsymbol{\Psi}_\delta$, $\boldsymbol{\Psi}_{\epsilon_1}$ and $\boldsymbol{\Psi}_{\epsilon_2}$ may or may not be diagonal.

The elements of $\mathbf{B}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Lambda}_1$, $\boldsymbol{\Lambda}_2$, $\boldsymbol{\Phi}$, $\boldsymbol{\Psi}_\delta$, $\boldsymbol{\Psi}_{\epsilon_1}$ and $\boldsymbol{\Psi}_{\epsilon_2}$ are either free or constrained. An element is said to be constrained if it is assigned a specific value, or if it is a function (linear or non-linear) of other elements. In SEM, it is common to constrain a large number of elements to zero. An example is when any or all of the