Automatica 66 (2016) 25–33

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed disturbance estimator and application to stabilization for multi-dimensional wave equation with corrupted boundary observation☆

Hongyinping Feng[a], Bao-Zhu Guo[b,a,c]

[a] School of Mathematical Sciences, Shanxi University, Taiyuan, Shanxi, 030006, PR China
[b] Key Laboratory of System and Control, Academy of Mathematics and Systems Science, Academia Sinica, Beijing 100190, PR China
[c] School of Computer Science and Applied Mathematics, University of the Witwatersrand, Johannesburg, South Africa

ARTICLE INFO

Article history:
Received 3 November 2014
Received in revised form
11 November 2015
Accepted 14 November 2015

Keywords:
Active disturbance rejection control
Infinite-dimensional system
Disturbance estimation
Stabilization

ABSTRACT

In this paper, we design, in a systematic way, an infinite-dimensional disturbance estimator by the active disturbance rejection control approach. The proposed disturbance estimator can be used to extract real signal from corrupted velocity signal. Its variant form can also be served as a tracking differentiator. The result is applied to stabilization for a multi-dimensional wave equation with position and corrupted velocity measurements.

1. Introduction

Many control approaches initially for lumped parameter systems have been developed to cope with systems with uncertainty coming from un-modeled dynamics and external disturbance. These include the internal model principle for output regulation to deal with some class of external disturbances; the robust control for systems with uncertainties; the sliding mode control for system with internal and/or external disturbance; and the adaptive control for systems with unknown parameters, to name just a few. These methods have also been generalized to the distributed parameter systems. Examples can be found in Cheng, Radisavljevic, and Su (2011), Guo and Jin (2013), Orlov (2000) and Pisano and Orlov (2012) by the sliding mode control; Immonen and Pohjolainen (2006) and Rebarber and Weiss (2003) by the output regulation; and Krstic (2010) and Krstic and Smyshlyaev (2008) by the adaptive control, where the backstepping approach plays an important role. Recently, the adaptive control is proposed within the backstepping approach to deal with the anti-stable one-dimensional wave equation in Bresch-Pietri and Krstic (2014) and Krstic (2010) where the unknown parameter is estimated by employing parameter projection, and a stabilizer is designed based on the passivity principle. Actually, there are many works based on passivity principle in the study of stabilization for PDEs, see Kugi, Schlacher, and Irschik (1999), Lasiecka and Triggiani (2000), Meurer and Kugi (2011), Ortega, van der Schaft, Maschke, and Escobar (2002) and Tucsnak and Weiss (2009) and the references therein.

The active disturbance rejection control (ADRC), as an unconventional design strategy similar to the external model principle (Medvedev & Hillerström, 1995), was first proposed by Han in 1998, for which a nice survey can be found in Han (2009). One of the remarkable features of ADRC is that the disturbance is estimated in real time through an extended state observer (Guo & Zhao, 2011) and is compensated (canceled) in feedback loop which makes the control energy significantly reduced (Zheng & Gao, 2012). The generalization of ADRC to the systems described by PDEs is first for one-dimensional systems in our previous works (Feng & Li, 2013; Guo & Jin, 2013) and then for multi-dimensional system in Guo and Zhou (2014) where the disturbance is dealt with by ODEs reduced from the associated PDE through some special test functions. Very

☆ This work is supported by the NNSFC (61403239, 61374089, 11401351, 61473180), the National Basic Research Program of China (2011CB808002), the STIP in Shanxi (2014101), Science Council of Shanxi Province (2015021010) and the National Research Foundation of South Africa. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Xiaobo Tan under the direction of Editor Miroslav Krstic.

E-mail addresses: fhyp@sxu.edu.cn (H. Feng), bzguo@iss.ac.cn (B.-Z. Guo).

http://dx.doi.org/10.1016/j.automatica.2015.12.008
0005-1098/

recently, the result is developed to deal with one-dimensional PDE with corrupted output in our work (Feng & Guo, 2014).

In this paper, we generalize the result of Feng and Guo (2014) from a one-dimensional PDE to a class of multi-dimensional PDEs. Our design is systematic. We first design, by the active disturbance rejection control approach, a relative independent infinite-dimensional disturbance estimator to estimate the distributed disturbance. This estimator is shown to be served as a tracking differentiator in infinite-dimensional context. The disturbance is then compensated in the feedback loop to stabilization for a class of second order infinite-dimensional systems. The result is applied to a multi-dimensional wave equation as a demonstration.

We proceed as follows. In Section 2, a parameterized infinite dimensional system in Hilbert space is discussed. The convergence with respect to parameter is developed. Section 3 is devoted to a systematic design for disturbance estimator and distributed tracking differentiator. In Section 4, a time-varying gain is introduced to cope with the peaking phenomenon caused by constant high gain. The application to stabilization for a multi-dimensional wave equation with corrupted velocity output is investigated in Section 5. Some concluding remarks are presented in Section 6.

2. Preliminary

In this section, we present some preliminary results. Let U be a Hilbert space and let $G_i, i = 1, 2$ be self-adjoint, strictly positive operators in U. We first consider the following system:

$$\begin{cases} \dot{w}_R(t) = -RG_1 w_R(t) + Rv_R(t), \\ \dot{v}_R(t) = -RG_2 w_R(t) + f(t), \end{cases} \tag{1}$$

where R is a positive constant and $f \in L^2_{loc}(0, \infty; U)$. We consider system (1) in the state Hilbert space $\mathcal{H} := D(G_2^{1/2}) \times U$ with the inner product given by

$$\langle (f_1, g_1), (f_2, g_2)\rangle_{\mathcal{H}} = \langle G_2^{1/2} f_1, G_2^{1/2} f_2\rangle_U + \langle g_1, g_2\rangle_U, \quad \forall\, (f_i, g_i) \in \mathcal{H},\ i = 1, 2. \tag{2}$$

System (1) can be written as an evolutionary equation in $\mathcal{H}$:

$$\frac{d}{dt}(w_R(t), v_R(t)) = \mathcal{A}_R(w_R(t), v_R(t)) + (0, f(t)), \tag{3}$$

where $\mathcal{A}_R : D(\mathcal{A}_R)(\subset \mathcal{H}) \to \mathcal{H}$ is defined by

$$\begin{cases} \mathcal{A}_R(f, g) = (-RG_1 f + Rg, -RG_2 f), \forall (f, g) \in D(\mathcal{A}_R), \\ D(\mathcal{A}_R) = \{(f, g) \mid f \in D(G_2), -G_1 f + g \in D(G_2^{1/2})\}. \end{cases} \tag{4}$$

It is seen from (4) that the domain of operator $\mathcal{A}_R$ is independent of the parameter R.

Lemma 2.1. *Suppose that the operators G_1 and G_2 are commutable, self-adjoint, strictly positive in Hilbert space U. Then the operator $\mathcal{A}_R$ defined by (4) generates a C_0-semigroup of contractions on $\mathcal{H}$. Therefore, for any initial state $(w_R(0), v_R(0)) \in \mathcal{H}, f \in L^2_{loc}(0, \infty; U)$, there exists a unique solution $(w_R, v_R) \in C(0, \infty; \mathcal{H})$ to (3), and if $f \in H^1_{loc}(0, \infty; U)$ or $f \in L^2_{loc}(0, \infty; D(G_2^{1/2}))$ and $(w_R(0), v_R(0)) \in D(\mathcal{A}_R)$, then $(w_R, v_R) \in C(0, \infty; D(\mathcal{A}_R))$ is a classical solution.*

Proof. For any $(f, g) \in D(\mathcal{A}_R)$, since G_1 and G_2 are commutable and hence $G_2G_1 > 0$,

$$\begin{aligned} &\mathrm{Re}\langle \mathcal{A}_R(f, g), (f, g)\rangle \\ &\quad = \mathrm{Re}\langle -RG_1 f + Rg, G_2 f\rangle_U + \mathrm{Re}\langle -RG_2 f, g\rangle_U \\ &\quad = -R\,\langle G_1 f, G_2 f\rangle_U = -R\langle G_2G_1 f, f\rangle_U \le 0. \end{aligned} \tag{5}$$

So $\mathcal{A}_R$ is dissipative in $\mathcal{H}$. It is a trivial exercise that

$$\mathcal{A}_R^{-1}(f, g) = \left(-\frac{1}{R}G_2^{-1}g, \frac{f - G_1G_2^{-1}g}{R}\right), \quad \forall\, (f, g) \in \mathcal{H}.$$

By the Lumer–Phillips theorem (Pazy, 1983, Theorem 1.4.3), $\mathcal{A}_R$ generates a C_0-semigroup of contractions on $\mathcal{H}$. We only need to mention the classical solution. Thanks to assumption on $f(t)$, we have $(0, f) \in H^1_{loc}(0, \infty; \mathcal{H})$ or $(0, f) \in L^2_{loc}(0, \infty; D(\mathcal{A}_R))$. By Pazy (1983, Corollary 2.5, p.107 and Corollary 2.6, p.108), system (1) admits a unique classical solution $(w, v) \in C(0, \infty; D(\mathcal{A}_R))$. □

Proposition 2.1. *Let U be a Hilbert space and let $G_i, i = 1, 2$ be self-adjoint, strictly positive operators in U. Suppose that $f \in L^2_{loc}(0, \infty; U)$ is measurable and there exists a positive constant $M_f > 0$ such that*

$$\|f(t)\|_U \le M_f, \quad \forall\, t \ge 0. \tag{6}$$

Assume further that the operators G_1 and G_2 satisfy:

$$\|g\|_U \le c_0\|G_1^{1/2} g\|_U, \quad \forall\, g \in D(G_1) \tag{7}$$

and

$$\alpha G_2 = G_1 + I, \tag{8}$$

where c_0 is a positive constant and the constant α satisfies:

$$\alpha > \max\{c_0 + 1, 2\}. \tag{9}$$

Then, for any initial state $(Rw_0, v_0) \in \mathcal{H}$ and any $a > 0$, where (w_0, v_0) is independent of R, system (1) admits a unique solution $(w_R, v_R) \in C(0, \infty; \mathcal{H})$ such that

$$\lim_{R\to\infty} [\|w_R(t)\|_U + \|v_R(t)\|_U] = 0 \quad \textit{uniformly in } [a, \infty). \tag{10}$$

Proof. Since $\mathcal{A}_R$ is the generator of a C_0-semigroup on $\mathcal{H}$, for any initial state $(Rw_0, v_0) \in \mathcal{H}$ and inhomogeneous term $f \in L^2_{loc}(0, \infty; U)$, system (1) admits a unique mild solution $(w_R, v_R) \in C(0, \infty; \mathcal{H})$ which is given by

$$(w_R(t), v_R(t)) = e^{\mathcal{A}_R t}(Rw_0, v_0) + \int_0^t e^{\mathcal{A}_R(t-s)}(0, f(s))ds. \tag{11}$$

Since $D(\mathcal{A}_R)$ is dense in $\mathcal{H}$, for any $R > 0$ and $n \in Z^+$, we can find $(w^n_{0R}, v^n_{0R}) \in D(\mathcal{A}_R)$ such that

$$\|(w^n_{0R}, v^n_{0R}) - (Rw_0, v_0)\|_{\mathcal{H}} \le \frac{1}{nR}. \tag{12}$$

Owing to assumption (6), by exploiting the technique of mollifier, we can find $f_n \in C^\infty(0, \infty; U)$ such that $|f_n(t)| \le M_f$ and

$$\|f_n(t) - f(t)\|_{L^1_{loc}(0,\infty;U)} \to 0 \quad \text{as } n \to \infty. \tag{13}$$

By (8), G_1 and G_2 are commutable, self-adjoint, strictly positive in U. Owing to Lemma 2.1, $(w^n_R, v^n_R) \in C(0, \infty; D(\mathcal{A}_R))$ is the classical solution:

$$(w^n_R(t), v^n_R(t)) = e^{\mathcal{A}_R t}(w^n_{0R}, v^n_{0R}) + \int_0^t e^{\mathcal{A}_R(t-s)}(0, f_n(s))ds, \quad \forall\, n \in \mathbb{N}. \tag{14}$$

We first claim that for any given $a > 0$,

$$\lim_{R\to\infty} [\|w^n_R(t)\|_U + \|v^n_R(t)\|_U] = 0 \quad \text{uniformly in } n \in \mathbb{N} \text{ and } t \in [a, \infty). \tag{15}$$