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Brief paper

Leader–follower containment control over directed random graphs☆

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ABSTRACT

The leader–follower consensus problem for multi-agent systems over directed random graphs is investigated. Motivated by the fact that inter-agent communication can be subject to random failure when agents perform tasks in a complex environment, a directed random graph is used to model the random loss of communication between agents, where the connection of the directed edge in the graph is assumed to be probabilistic and evolves according to a two-state Markov Model. In the leader–follower network, the leaders maintain a constant desired state and the followers update their states by communicating with local neighbors over the random communication network. Based on convex properties and a stochastic version of LaSalle's Invariance Principle, almost sure convergence of the followers' states to the convex hull spanned by the leaders' states is established for the leader–follower random network. A numerical simulation is provided to demonstrate the developed result.

1. Introduction

Consensus problems that seek to agree upon certain quantities of interest have attracted significant research attention. A comprehensive review of consensus problems is provided in Olfati-Saber, Fax, and Murray (2007) and Ren, Beard, and Atkins (2007). To achieve consensus, agents are generally required to exchange information over a communication network as a means to coordinate their behaviors, such as achieving a common heading direction in flocking problems (Jadbabaie, Lin, & Morse, 2003; Tanner, Jadbabaie, & Pappas, 2007), agreeing on the group average in distributed sensing (Zhu & Martínez, 2010), or achieving consensus in rendezvous and formation control problems (Dimarogonas & Kyriakopoulos, 2007; Kan, Navaravong, Shea, Pasiliao, & Dixon, 2015), to name a few. In most of these applications, consistent information exchange between agents in either an undirected or directed manner is a common assumption to ensure full cooperation among team members. However, when agents operate in a complex environment, the inter-agent communication could be subject to random failure due to either interference or unpredictable environmental disturbance. Since task completion relies on communication and interaction among agents, achieving consensus over such a stochastic communication network can be challenging.

Leader–follower containment control is a particular class of consensus problems, in which the networked multi-agent system consists of leader agents and follower agents. Generally, the leaders are a small subset of the agents, which are informed of the global task objectives, while the followers act under the influence of both neighboring agents and the leaders through local interactions. A main objective in leader–follower containment control is to drive all followers' states to a desired destination determined by the leaders' states. Hybrid control schemes are developed in Ji, Ferrari-Trecate, Egerstedt and Buffa (2008) to drive the dynamic follower agents into a convex polytope spanned by the stationary leader agents, where the local interaction among agents is modeled as an undirected graph. The work of Ji et al. (2008) is then extended to multiple stationary and dynamic leaders under a directed interaction graph in Cao, Ren, and Egerstedt (2012), Li, Ren, and Xu (2012) and Meng, Ren, and You (2010). Containment control for a leader–follower network under a switching graph is studied in Lou and Hong (2012) and Notarstefano, Egerstedt, and Haque (2011). In Kan, Klotz, and Dixon (2015), containment control is applied to a social network to regulate the emotional states of individuals to a desired end. For networked Lagrangian systems with parametric uncertainties, distributed containment control is developed in Mei, Ren, and Ma (2012). In the aforementioned works, a deterministic dynamic system is considered, where

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dynamic agents communicate and coordinate with other agents over an undirected or directed deterministic communication network. Mean-square containment control of a multi-agent system with communication noise is considered in Wang, Cheng, Hou, Tan, and Wang (2014). Since the results developed in Cao et al. (2012), Ji et al. (2008), Kan et al. (2015), Li et al. (2012), Lou and Hong (2012), Mei et al. (2012), Meng et al. (2010), Notarstefano et al. (2011), Wang et al. (2014) may not be applicable to stochastic communication networks where the existing communication links experience random loss, an extension of the classical containment control from the deterministic network to the stochastic network is desirable.

Building on graph theory and probability theory, several consensus results have been developed for random graphs. One of the earliest consensus results over an undirected random network is reported in Hatano and Mesbahi (2005), which proves that agreement can be achieved almost surely if the communication links between any pair of agents are activated independently with a common probability. The undirected random graph in Hatano and Mesbahi (2005) is extended to a general class of directed random graphs in Porfiri and Stilwell (2007) and Wu (2006). Necessary and sufficient conditions for consensus are developed in Tahbaz-Salehi and Jadbabaie (2008) for graphs that are generated by an ergodic and stationary random process. Mean-square-robust consensus over a network with communication noise and random packet loss is considered in the work of Zhang and Tian (2010) and Zhang and Tian (2012). Stochastic consensus for a multi-agent system with communication noise and Markovian switching topologies is investigated in Wang, Cheng, Ren, Hou, and Tan (2015). However, the convergence results reported in Hatano and Mesbahi (2005), Porfiri and Stilwell (2007), Tahbaz-Salehi and Jadbabaie (2008), Wang et al. (2015), Wu (2006), Zhang and Tian (2010), Zhang and Tian (2012) are only developed for leaderless networks without considering how the leaders can influence the followers to a desired end.

In this paper, the classical leader–follower containment control problem for deterministic systems is extended to a stochastic scenario. The leader–follower network is tasked to drive all followers into a prespecified destination area (i.e., the convex hull spanned by the leaders' states) under the influence of the leaders. Only the leaders are assumed to have the knowledge of the destination. To move toward the specified destination, the followers communicate and update their states with neighboring agents over a communication network. Since wireless communication is subject to random failure due to factors such as fading and packet loss, the inter-agent communication is modeled as a random graph, where each link evolves according to a two-state Markov Model to model the random loss of the existing communication link. In addition, the random communication network is assumed to be directed. Rather than assuming that all edges share a common edge probability and evolve independently with their previous edge connection states as in Hatano and Mesbahi (2005), different edges are allowed to have different transition probabilities in the current work that evolve according to a Markov Model, which can be used to model a large class of real-world networks to reflect the dependence of the current system states on their previous states. Moreover, compared to the works of Hatano and Mesbahi (2005), Porfiri and Stilwell (2007), Tahbaz-Salehi and Jadbabaie (2008), Wu (2006), a hierarchical network structure (i.e., leader–follower network) is considered where one-sided influence of leaders is used to affect the desired behaviors of the followers. Almost sure convergence of the followers' states to the convex hull spanned by the leaders' states over a random communication graph is then established via the convex properties in Boyd and Vandenberghe (2004) and a stochastic version of LaSalle's Invariance Theorem (Kushner, 1971).

2. Problem formulation

A multi-agent system consisting of n agents that communicate over wireless channels is considered. The wireless channels have intermittent connectivity, which cause the connections among the agents to vary with time. The communication graph is modeled as a temporal network, or time-varying graph, $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$. The vertices $\mathcal{V}$ represent the agents, which do not vary with time. The edges $\mathcal{E}(t)$ represent the connections among the agents and do vary with time. The flow of information is assumed to be asymmetric, so the edges in $\mathcal{E}(t) \subset \mathcal{V} \times \mathcal{V}$ are directed. Specifically, the directed edge $\left(v_j, v_i\right) \in \mathcal{E}$ indicates that node v_i can receive information from node v_j, but v_j may not necessarily receive information from v_i. In the directed edge $\left(v_j, v_i\right)$, v_i and v_j are referred to as the child node and the parent node, respectively.

2.1. Directed random graph

Consider first the graph at one particular time, say $t = t_0$. Then, suppressing the time dependence, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a directed graph. The graph $\mathcal{G}$ is called a directed random graph if the set of edges $\mathcal{E}$ is randomly determined. Let $\bar{\mathcal{E}} \subset \mathcal{V} \times \mathcal{V}$ be a set of potential directed edges connecting the nodes in $\mathcal{V}$. Each potential edge $\left(v_j, v_i\right)$ is associated with a weight $w_{ij} \in \mathbb{R}^+$, which indicates how node v_i evaluates the information collected from v_j. We assume that the weight w_{ij} for each $\left(v_j, v_i\right)$ is known initially and there are no self loops, so $(v_i, v_i) \notin \bar{\mathcal{E}}$, $i = 1, 2, \ldots, n$. Associated with each potential edge $\left(v_j, v_i\right) \in \bar{\mathcal{E}}$, let there be a Bernoulli random variable δ_{ij}. An edge $\left(v_j, v_i\right) \in \bar{\mathcal{E}}$ will exist in $\mathcal{E}$ if $\delta_{ij} = 1$ and will not exist in $\mathcal{E}$ if $\delta_{ij} = 0$. It is assumed that, for different edges, the $\left\{\delta_{ij}\right\}$ are statistically independent.

Now, consider the temporal network, $\mathcal{G}(t)$, which consists of a time sequence of directed random graphs in which the edge set varies with t. In particular, each edge (i, j) evolves according to a two-state homogeneous Markov process $\delta_{ij}(t)$ for $i, j \in \{1, 2, \ldots, n\}$ with stationary state transition probability $p_{ij} \in (0, 1]$, which indicates that, at the next time instant t', the edge (i, j) will change its state to $\delta_{ij}\left(t'\right) = 1 - \delta_{ij}(t)$ with probability p_{ij} and will remain the previous state $\delta_{ij}\left(t'\right) = \delta_{ij}(t)$ with probability $1 - p_{ij}$.

Assumption 1. The random processes $\left\{\delta_{ij}(t)\right\}$ do not change infinitely fast, and thus we can choose a sampling time Δ_t such that with arbitrarily high probability, $\delta_{ij}(t) = \delta_{ij}(t + t_0)$ if $0 \leq t_0 < \Delta_t$ for all $i, j \in \{1, 2, \ldots, n\}$.

Note that Assumption 1 will be true for any real system. For example, let T_0 and T_1 denote the expected dwell times in states 0 and 1 for the Markov process $\delta_{ij}(t)$, respectively. Then, the probability of staying in the same state during an observation period can be made arbitrarily large by selecting an appropriate Δ_t. For example, the probability of remaining in state 0 during an interval of length Δ_t is $e^{-\Delta_t/T_0}$.

We assume that the sequence of random graphs can be discretized in the following way. Let $t_k = k\Delta_t$, $k \in \mathbb{Z}^+$ be a time sequence, where $\Delta_t \in \mathbb{R}^+$ is a sufficiently small sampling period during which we may assume the edge set is constant over each time interval $[t_k, t_{k+1})$. Let $\mathcal{G}(k)$ denote the random graph $\mathcal{G}(t)$ at $t = t_k$. Note that $\mathcal{G}(k)$ is drawn from a finite sample space, which we denote by $\bar{\mathcal{G}} = \{\mathcal{G}_1, \ldots, \mathcal{G}_M\}$, and $\left|\bar{\mathcal{G}}\right| \leq 2^{|\bar{\mathcal{E}}|}$, which is determined by the power set of $\bar{\mathcal{E}}$. In a directed graph, a *directed path* from node v_1 to node v_k is a sequence of edges (v_1, v_2), $(v_2, v_3), \ldots, (v_i, v_k)$. If a directed graph contains a *directed spanning tree*, every node has exactly one parent node except for one node, called the root, and the root has directed paths