Automatica 66 (2016) 155–162

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

The synchronization of instantaneously coupled harmonic oscillators using sampled data with measurement noise[☆]

Jingyi Wang[a,b], Jianwen Feng[c,1], Chen Xu[c,1], Michael Z.Q. Chen[d], Yi Zhao[c], Jiqiang Feng[c]

[a] Key Laboratory of Optoelectronic Devices and Systems of Ministry of Education and Guangdong Province, PR China
[b] College of Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, PR China
[c] College of Mathematics and Computational Science, Shenzhen University, Shenzhen 518060, PR China
[d] Department of Mechanical Engineering, The University of Hong Kong, Hong Kong

ARTICLE INFO

Article history:
Received 26 December 2014
Received in revised form
3 August 2015
Accepted 7 December 2015

Keywords:
Synchronization
Coupled harmonic oscillators
Instantaneous interactions
Sampled data
Measurement noise

ABSTRACT

In this brief, we propose an effective algorithm for synchronization of instantaneously coupled harmonic oscillators by using sampled data which may contain measurement noise. We discuss the convergence of this algorithm for both fixed and switching directed network topologies in the presence or absence of leaders. We also establish sufficient conditions under which the coupled harmonic oscillators could attain synchronized oscillatory motions in the presence of noise. And moreover, it is shown that synchronization can be attained even when the velocity information is exchanged only at discrete-time instants. Finally, we give some numerical examples to illustrate the effectiveness of the theoretical results.

1. Introduction

Synchronization phenomena are common in nature and hold particular promise for applications to many fields (for example, population dynamics, power systems and automatic control, see Boccaletti, Kurths, Osipov, Valladares, & Zhou, 2002; Dörfler & Bullo, 2014; Tanner, Jadbabaie, & Pappas, 2007; Wang & Chen, 2002 and Wang, Chen, & Wang, 2015b). Generally speaking, synchronization is the process in which two or more (coupled) dynamical systems seek to adjust a certain prescribed property of their motions to a common behavior in the limit as time tends to infinity (Boccaletti et al., 2002). Many studies have therefore been conducted on these problems (see Boccaletti et al., 2002; Dörfler & Bullo, 2014; Olfati-Saber, Fax, & Murray, 2007; Wang et al., 2015a and the references therein) because of their great relevance to real world applications.

Coupled harmonic oscillators are interesting because both their frequencies and amplitudes can be made to converge to the same values over time, irrespective of their initial displacements (Ballard, Cao, & Ren, 2010). As a result, some authors have devoted to investigating different synchronization algorithms for coupled harmonic oscillators from various perspectives. For instance, Ren (2008) studied the synchronization of continuous-time coupled harmonic oscillators with local interactions over both fixed and switching network topologies, while Ballard et al. (2010) also addressed discrete-time coupled harmonic oscillators and applied it to the synchronized motions of multiple mobile robots. In addition, Zhou, Zhang, Xiang, and Wu (2012) considered the synchronization of coupled harmonic oscillators with local instantaneous interactions and obtained some sufficient conditions for fixed and switching topologies in either the presence or the absence of leaders. Zhang and Zhou (2012) also investigated the synchronization of undirected networks of coupled harmonic oscillators using sampled data measurements with controller failures. Zhou, Zhang, Xiang, and Wu (2013) further addressed sampled data synchronization of coupled harmonic oscillators with controller failure and communication delays. Furthermore, Sun, Lü, Chen, and Yu (2014) investigated the synchronization of instantaneously coupled harmonic oscillators with directed

☆ This work was supported by the National Science Foundation of China under Grant No. 61273220, 61373087, 61374053, 61377017, 61472257 and 61401283. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hyeong Soo Chang under the direction of Editor Richard Middleton.

E-mail addresses: jingyi.jy.wang@gmail.com (J. Wang), fengjw@szu.edu.cn (J. Feng), xuchen_szu@szu.edu.cn (C. Xu), mzqchen@hku.hk (M.Z.Q. Chen), yzhao@szu.edu.cn (Y. Zhao), fengjq@szu.edu.cn (J. Feng).

[1] Tel.: +86 755 26532312; fax: +86 755 26538881.

http://dx.doi.org/10.1016/j.automatica.2016.01.012
0005-1098/

topology. Recently, Sun, Yu, Lü, and Chen (2015) focused on synchronization of coupled harmonic oscillators in the presence of random noise. In particular, the synchronization of coupled harmonic oscillators plays an important role in applications such as motion coordination, consensus seeking and collective tracking, see Casau, Sanfelice, Cunha, Cabecinhas, and Silvestre (2015); Chiorescu et al. (2004); Li and Ding (2015); Ren (2008); Ren and Beard (2005); Zhang, Yang, and Zhao (2013) and Zhou et al. (2012).

With the aid of high-speed computers and modern control systems and signal processing techniques, samples of the control input signals need only be taken at discrete-time instants. Meanwhile, many synchronization studies of dynamical networks have been performed recently using sampled data as the main means of information transfer among the network agents. For example, synchronization of coupled harmonic oscillators was considered via sampled data control by Sun et al. (2014) and Zhou et al. (2013). In addition, sampled data synchronization control of dynamical networks was studied by stochastic sampling by Shen, Wang, and Liu (2012) and aperiodic sampling by Wen, Yu, Chen, Yu, and Chen (2014). No studies, however, have ever been conducted on the synchronization of instantaneously coupled harmonic oscillators using sampled data that contains measurement noise.

In this paper, we consider the dynamic behaviors of a set of identical harmonic oscillators. The main aim of this paper is to identify and analyze the synchronization conditions for instantaneously coupled harmonic oscillators using only sampled velocity data with measurement noise in either the presence or the absence of leaders. The communication topology of the proposed algorithm is directed, because the sampling cost of algorithms with directed information flow is smaller than their undirected counterparts. The convergence analysis uses tools as algebraic graph theory, matrix theory, statistics and impulsive differential equations, and the theoretical results are then applied to collective tracking of some specific multi-agent systems.

The rest of this paper is organized as follows. In Section 2, we introduce the general model for a system of instantaneously coupled harmonic oscillators with measurement noise. In Section 3, we establish some synchronization criteria for such networks when leaders are either present or absent and we illustrate the theoretical results with some numerical examples in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries

First, we write down the notations that will be used throughout this paper. Let $\mathbf{1}_n = [1, 1, \ldots, 1]^\top \in \mathbb{R}^n$ and I_n be the n-dimensional identity matrix and we use the superscript $\top$ to denote the transpose of a matrix and $\rho(\cdot)$ denotes the spectral radius. We write $x \doteq y$ to mean that x is defined to be another name for y. Finally, for square matrices A and B, we write $A \succ B$ (respectively, $\succeq, \prec$ and $\preceq$) to mean that $A - B$ is positive definite (respectively, positive semi-definite, negative definite and negative semi-definite).

To analyze the convergence conditions for a set of coupled harmonic oscillators over any arbitrary network topology, the oscillator interactions shall be modeled by a directed graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph for some set of nodes $\mathcal{V} = \{1, 2, \ldots, n\}$ and set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. And denote $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ as the adjacency matrix associated with $\mathcal{G}$ such that a_{ij} is a positive weight for all $(i, j) \in \mathcal{E}$ (so that node i receives information from node j while j being called the parent of i) and $a_{ij} = 0$ for all $(i, j) \notin \mathcal{E}$. Define the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ by $l_{ii} = \sum_{i=1, j\neq i}^{n} l_{ij}$, where $l_{ij} = -a_{ij} (\leq 0)$ for $i \neq j$. A sequence of edges $(i_1, i_2), (i_2, i_3), \ldots$, for $i_j \in \mathcal{V}$, is called a directed path of $\mathcal{G}$. And a directed graph is strongly connected if there is a directed path from every node to every other node. Moreover, a directed graph is balanced if $\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji}$ for every node i.

The measured values used in the sampled data control methods (Zhang & Shi, 2012), however, are usually contaminated with measurement noise arising from the measuring errors of the physical devices and the various states of the observers (for instance, the observers' relative positions to the instruments and so on). In Fig. 1, let the points $j(x_j, y_j)$, $i(x_i, y_i)$ and $\tilde{i}(x_{\tilde{i}}, y_{\tilde{i}})$ be the state of the observer, the state of the (expected) observed node and the actual measured state of the observed node, respectively. Let $\overrightarrow{ji}$, $\overrightarrow{j\tilde{i}}$ and $\overrightarrow{i\tilde{i}}$ be the real difference, the measured difference and the measurement noise, respectively, and $\omega = [\omega_x, \omega_y]^\top \in \mathbb{R}^2$ be a random noise vector. Clearly, $\overrightarrow{j\tilde{i}} = \overrightarrow{ji} + \overrightarrow{i\tilde{i}}$. More precisely, we have

$$\begin{bmatrix} x_{\tilde{i}} - x_j \\ y_{\tilde{i}} - y_j \end{bmatrix} = \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} + \begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix} \circ \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix},$$

where $\circ$ is the Hadamard product.

Motivated by the aforementioned comments and taking sampled data noise into account, we propose a highly effective algorithm for synchronizing n instantaneously coupled harmonic oscillators that are described by the impulsive differential equations

$$\begin{cases} \dot{r}_i(t) = v_i(t), \quad \dot{v}_i(t) = -\alpha r_i(t), & t \neq t_k, \\ \Delta r_i(t_k) = 0, & t = t_k, \\ \Delta v_i(t_k) = \sum_{j=1}^{n} a_{ij}[\mu(v_j(t_k) - v_i(t_k)) & \\ \quad + \omega_k(v_j(t_k) - v_i(t_k))], & t = t_k, \end{cases} \tag{1}$$

$i = 1, 2, \ldots, n$, for some positive constant α, where $r(t) \in \mathbb{R}$ is the position of the oscillator and $v(t) \in \mathbb{R}$ is its velocity, the coupling strength $\mu > 0$, the coefficients $a_{ij} \geq 0$ if and only if oscillator i can attain the velocity of oscillator j, the time sequence $\{t_k\}_{k=1}^{+\infty}$ satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = +\infty$, $\Delta r_i(t_k) = r_i(t_k^+) - r_i(t_k)$, $\Delta v_i(t_k) = v_i(t_k^+) - v_i(t_k)$ with $r_i(t_k^+) = \lim_{t\to t_k^+} r_i(t)$, $v_i(t_k^+) = \lim_{t\to t_k^+} v_i(t)$, $\lim_{t\to t_k^-} r_i(t) = r_i(t_k)$ and $\lim_{t\to t_k^-} v_i(t) = v_i(t_k)$ (so that $r_i(t)$ and $v_i(t)$ are left continuous), the sequence of random variables $\{\omega_k\}$ is independent and identically distributed with zero-mean and σ^2 variance and the part $\omega_k(v_i(t_k) - v_j(t_k))$ is the measurement noise and depends on the states of oscillators i and j for $k \in \mathbb{N}$.

3. Synchronization analysis

3.1. Synchronization without a leader

Theorem 1. *If the directed graph $\mathcal{G}$ is strongly connected and*

(1) $0 < \lambda_n < \frac{(1-1/\alpha)\sin^2(\sqrt{\alpha}T_k)\zeta_n + 1 - \zeta_n}{(\alpha-1)\sin^2(\sqrt{\alpha}T_k) + 1 - \zeta_n}$;

(2) $\alpha > 1 + \frac{\zeta_n - 1}{1 - \zeta_n + \zeta_n \sin^2(\sqrt{\alpha}T_k)} > 0$;

(3) $T_k \neq \frac{j\pi}{\sqrt{\alpha}}$, *for all $k \in \mathbb{N}$ and $j \in \mathbb{N}$,*

where $\zeta_n > 0$ and $\lambda_n > 0$ are the greatest eigenvalues of $(I_n - \Xi)^\top (I_n - \Xi)$ and $(I_n - \Xi - \mu L)^\top (I_n - \Xi - \mu L) + \sigma^2 L^\top L$, respectively, then the solutions $[r_i(t), v_i(t)]^\top$ of system (1) converge in mean square to the synchronized state

$$\begin{bmatrix} \gamma(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} \xi^\top r(0)\cos(\sqrt{\alpha}t) + \frac{1}{\sqrt{\alpha}}\xi^\top v(0)\sin(\sqrt{\alpha}t) \\ -\sqrt{\alpha}\xi^\top r(0)\sin(\sqrt{\alpha}t) + \xi^\top v(0)\cos(\sqrt{\alpha}t) \end{bmatrix}, \tag{2}$$

where $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^\top$ ($\sum_{i=1}^{n} \xi_i = 1$) is a left eigenvector of L corresponding to the eigenvalue 0, $\Xi = \mathbf{1}_n \xi^\top$ and $r(0)$ and $v(0)$ are the initial values of system (1).