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Brief paper

A robust extremum seeking scheme for dynamic systems with uncertainties and disturbances[☆]

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ABSTRACT

This paper studies a numerical optimization-based extremum seeking scheme for systems with unmodeled dynamics and unknown disturbances without using dither signals. The robust extremum seeking scheme is composed of a numerical gradient estimator, a numerical optimizer and an extended-state observer based state regulator. A conjugate gradient method is adopted to achieve extremum seeking. A nonlinear extended-state observer based state regulator is proposed to regulate the states. The robust extremum seeker is based on the numerical optimization based extremum seeking framework. Sufficient conditions are derived to ensure the convergence of the overall extremum seeking scheme. The ultimate convergence bound is quantified for a class of strongly convex functions. A numerical example is presented to verify the proposed method.

1. Introduction

Extremum seeking, a non-model-based real-time online optimization method for seeking the extremum of a performance function, has received increasing attention ever since the seminal works in Krstic and Wang (2000), Krstic (2000). Most existing extremum seeking schemes are based on the modulation of dither signals which makes the gradients extractable (e.g., see Durr, Stankovic, Ebenbauer, & Johansson, 2013; Guay & Dochain, 2015; Krstic, 2000; Krstic & Wang, 2000; Tan, Nesic, & Mareels, 2006; Ye & Hu, 2015a,b, and the references therein). However, in some practical applications, the dither signals place great challenges on the implementation of the controllers. To overcome this drawback, several methods that are free of dither signals have been developed (Fu & Ozguner, 2011; Haskara, Ozguner, & Winkelman, 2000; Khong, Nesic, Manzie, & Tan, 2013; Khong, Nesic, Tan, & Manzie, 2013; Nesic, Nguyen, Tan, & Manzie, 2013; Oliveira, Liu, & Peixoto, 2011; Pan, Kumar, & Liu, 2012; Teel & Popvic, 2001; Zhang & Ordonez, 2005, 2007, 2009). These methods fall basically into three categories. The methods in the first category of dither signal free extremum seeking scheme are based on sliding mode control (Haskara et al., 2000; Oliveira et al., 2011; Pan et al., 2012). These methods contain a reference model that determines the convergence speed. Finite time convergence is ensured by using a periodic switching control law. However, the controller is discontinuous on the sliding surface which may cause chattering. The methods in the second category are based on a sampled-data framework (Khong, Nesic, Manzie et al., 2013; Khong, Nesic, Tan et al., 2013; Nesic et al., 2013; Teel & Popvic, 2001). A sampler and a zero-order-hold are utilized such that nonlinear programming methods can be implemented in the extremum seeking loop. The main advantage of this extremum seeking scheme lies in the usage of existing nonlinear programming methods, which makes it able to deal with possibly non-convex optimization problems. The sampled-data extremum seeking scheme can deal with systems with equilibria or systems with periodic attractors (Khong, Nesic, Tan et al., 2013). The methods in the third category are based on the interconnection of numerical optimization and state regulation (Fu & Ozguner, 2011; Zhang & Ordonez, 2005, 2007, 2009). The numerical method is implemented to achieve optimization and the state regulator handles the system dynamics. State feedback linearizable systems and input–output feedback linearizable systems are mainly considered. Our interest in this paper is on the third category of extremum seeking methods, i.e., the numerical optimization based extremum seeking.

In Zhang and Ordonez (2007), the authors considered both linear time-invariant systems and feedback linearizable systems using the interconnected structure based extremum seeking scheme.

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The line search method and the trust region method were analyzed and their robustness against regulation error was considered. However, the line search method was implemented based on the exact gradients, which were thus required to be measurable at some time instants. The input disturbance was further discussed in Zhang and Ordonez (2009) where the authors utilized a sliding mode term to deal with the disturbance with a known bound. Approximation-based control laws were leveraged to approximate the unknown dynamics. The approximation error was treated as input disturbance and a sliding term was used to handle it. In Fu and Ozguner (2011), a sliding mode based gradient estimation method was proposed and a reference signal was constructed for the plant model by using the estimated gradient. To accommodate the disturbances, a signum function was employed in the control law. The methods are either discontinuous, which may lead to chattering, or continuous but not differentiable. In this paper, we propose an extremum seeking method by designing a continuous state regulator and an optimization algorithm that is robust against the state regulation error and sufficiently small gradient estimation error under the numerical optimization-based extremum seeking scheme (Zhang & Ordonez, 2007, 2009). Compared with the existing works, the main contributions of this paper are summarized as follows. (1) Systems with unmodeled dynamics and disturbances are considered and a robust state regulator is designed to handle the uncertainties and disturbances. Moreover, the proposed control law is continuous and differentiable which yields less chattering. (2) Different from the work in Zhang and Ordonez (2007), Zhang and Ordonez (2009), the effect of the gradient estimation error is analyzed for line search implementation and a conjugate gradient method is adopted to solve the optimization problem based on the approximated gradients. (3) Sufficient conditions to achieve extremum seeking are provided. The stability and robustness issues are addressed.

The rest of the paper is organized as follows. The problem is formulated in Section 2 and the robust extremum seeking method is presented in Section 3. In Section 4, a nonlinear extended-state observer based regulator is introduced and then it is shown that the conjugate gradient method can be used to seek for the extremum. A numerical example is given in Section 5 to validate the proposed method and brief conclusions are given in Section 6.

2. Problem formulation

Problem 1. Consider a first-order dynamic system with unmodeled dynamics and disturbances, described by

$$\dot{x}_i = u_i + f_i(\mathbf{x}) + d_i(t), \quad i \in \{1, 2, \ldots, N\}, \tag{1}$$
$$y_i = x_i,$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T \in \mathbf{R}^N$ is the concatenated vector of the state variable x_i and y_i is the system output, u_i is the control input. The performance function is

$$\theta = J(\mathbf{y}),$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_N]^T$ is the concatenated vector of y_i. The explicit expressions of $f_i(\mathbf{x})$, $d_i(t)$ and $J(\mathbf{y})$ are unknown. The functions $f_i(\mathbf{x})$, $d_i(t)$ are differentiable. Measurements of the system output $\mathbf{y}$ and the performance function θ are available. Design a control strategy such that the performance function is minimized.

The performance function is assumed to satisfy the following property.

Assumption 1. The performance function $J(\mathbf{y})$ is convex and continuously differentiable. Furthermore, there exists a global minimizer $\mathbf{y}^*$ which is isolated and $J(\mathbf{y}) > J(\mathbf{y}^*) > -\infty$ for all $\mathbf{y} \neq \mathbf{y}^*$.

3. Preliminary and integration of the numerical optimization-based extremum seeking

In this paper, the numerical optimization-based extremum seeking scheme is investigated for systems with unmodeled dynamics and disturbances. The main steps of the numerical optimization based extremum seeking scheme can be summarized as follows (Zhang & Ordonez, 2007, 2009).

Step 1: Initialize $\mathring{\mathbf{x}}_0$; set $t_0 = 0, \mathbf{x}(t_0) = \mathring{\mathbf{x}}_0, k = 0$.

Step 2: Estimate the gradient of $J(\mathbf{y})$ at $\mathbf{y} = \mathbf{x}(t_k)$ and denote the estimated gradient as $\mathbf{g}_k$.

Step 3: Let $\mathring{\mathbf{x}}_{k+1} = \mathbf{x}(t_k) + \alpha_k \mathbf{p}_k$ where α_k is the step size determined by (12)–(13) and $\mathbf{p}_k$ is the search direction produced by William and Zhang (2005)

$$\begin{aligned} &\mathbf{p}_{k+1} = -\mathbf{g}_{k+1} + \bar{\beta}_k \mathbf{p}_k, \qquad \mathbf{p}_0 = -\mathbf{g}_0 \\ &\bar{\beta}_k = \max\{\beta_k, \eta_k\}, \qquad \eta_k = \frac{-1}{\|\mathbf{p}_k\| \min(\eta, \|\mathbf{g}_k\|)} \\ &\beta_k = \frac{1}{\mathbf{p}_k^T \mathbf{q}_k} \left(\mathbf{q}_k - 2\mathbf{p}_k \frac{\|\mathbf{q}_k\|^2}{\mathbf{p}_k^T \mathbf{q}_k} \right)^T \mathbf{g}_{k+1}, \end{aligned} \tag{2}$$

where $\mathbf{q}_k = \mathbf{g}_{k+1} - \mathbf{g}_k$ and η is a positive constant.

Step 4: The extended-state observer based state regulator regulates $\mathbf{x}(t)$ to $\mathring{\mathbf{x}}_{k+1}$. Let $\mathbf{e}_{k+1}$ be the state regulation error, i.e., $\mathbf{e}_{k+1} = \mathbf{x}(t) - \mathring{\mathbf{x}}_{k+1}$. If the state regulation error $\|\mathbf{e}_{k+1}\| \le \kappa_{k1} \|\mathbf{g}_k\|$ for some predefined small positive κ_{k1} which gives a regulation error that satisfies the descent condition quantified in Theorem 3, let the current time instant be t_{k+1}, and the state at the current time instant be $\mathbf{x}(t_{k+1})$. Furthermore, let $k = k + 1$ and go to Step 2.

Remark 1. Several methods have been proposed to estimate the gradient without using dither signals and the explicit expression on the performance function (Fu & Ozguner, 2011; Hunnekens, Haring, Wouw, & Nijmeijer, 2014; Olver, 2014). One typical method is the finite difference method (Olver, 2014). The work in Hunnekens et al. (2014) provided a gradient estimation method by using first-order least square fits. The authors in Fu and Ozguner (2011) leveraged a sliding mode method to extract the gradient.

Remark 2. In Steps 1–4, a conjugate gradient method, which is implemented by line search, is utilized for minimum seeking. The conjugate gradient method was proposed in William and Zhang (2005) by using exact gradients. Since the gradients may not be available, approximated gradients are used in the extremum seeking loop. The main advantage of using this method is that it ensures a strict descent search direction (William & Zhang, 2005). In (2), $\mathbf{p}_k^T \mathbf{q}_k \neq 0$, $\|\mathbf{p}_k\| \neq 0$ if $\mathbf{g}_k \neq 0$ under the step size selection criteria (12)–(13). Furthermore, the stop criterion for the conjugate gradient method can be defined as $\|\mathbf{g}_k\| \le \kappa_2$ for some predefined small positive parameter κ_2. For more detailed implementation issues of the numerical optimization based extremum seeking, readers are referred to Zhang and Ordonez (2007), Zhang and Ordonez (2009).

The schematic outline of the numerical optimization-based extremum seeker is shown in Fig. 1. The detailed analysis will be presented in the next section.

4. Main results

In this section, an extended-state observer based state regulator will be designed for the numerical optimization-based extremum seeker. Furthermore, the robustness of the numerical optimization method against the state regulation error and the gradient estimation error will be analyzed.