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Brief paper

Input-to-state stability of impulsive stochastic delayed systems under linear assumptions[☆]

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ABSTRACT

In this paper, the input-to-state stability (ISS), integral-ISS (iISS) and stochastic-ISS (SISS) are investigated for impulsive stochastic delayed systems. By means of the Lyapunov–Krasovskii function and the average impulsive interval approach, the conditions for ISS-type properties are derived under linear assumptions, respectively, for destabilizing and stabilizing impulses. It is shown that if the continuous stochastic delayed system is ISS and the impulsive effects are destabilizing, then the hybrid system is ISS with respect to a lower bound of the average impulsive interval. Moreover, it is unveiled that if the continuous stochastic delayed system is not ISS, the impulsive effects can successfully stabilize the system for a given upper bound of the average impulsive interval. An improved comparison principle is developed for impulsive stochastic delayed systems, which facilitates the derivations of our results for ISS/iISS/SISS. An example of networked control systems is provided to illustrate the effectiveness of the proposed results.

1. Introduction

The input-to-state stability (ISS) and integral input-to-state stability (iISS), originally introduced in Sontag (1989), Sontag (2008), have received much attention due to their wide usages in characterizing the effects of external inputs on a control system. The ISS/iISS implies that no matter what the size of the initial state is, the state will eventually approach to a neighborhood of the origin whose size is proportional to the magnitude of the input. Recently, various extensions of the ISS have been made for different types of dynamical systems, such as discrete-time systems (Jiang & Wang, 2001; Liu & Hill, 2010), impulsive systems (Chen & Zheng, 2009, 2011; Hespanha, Liberzon, & Teel, 2008) and hybrid systems (Cai & Teel, 2013; Liu, Liu, & Xie, 2011; Pepe & Jiang, 2006; Sun & Wang, 2012; Teel, 1998; Vu, Chatterjee, & Liberzon, 2007). Among them, impulsive dynamical systems have attracted considerable interests in science and engineering (Tang, Gao, Zhang, & Kurths, 2015; Zhang, Tang, Wu, & Fang, 2014), since they provide a natural framework for the mathematical modeling of many real world systems (Lakshmikantham, Bainov, & Simeonov, 1989; Yang, 2001). These systems have found important applications in various fields, such as sampled-data control systems, complex networks and mechanical systems (Chen & Zheng, 2011; Naghshtabrizi, Hespanha, & Teel, 2010; Nešić & Teel, 2004b).

Time-delays are frequently encountered in many practical engineering systems, which may induce instability, oscillation and poor performance of systems. Recently, great efforts have been devoted to extend the ISS from impulsive delay-free systems to impulsive time-delay systems (Chen & Zheng, 2009, 2011; Liu et al., 2011; Sun & Wang, 2012). On the other hand, control systems are often subjected to stochastic perturbations, which may arise from external random fluctuations in the process of transmission and/or other probabilistic factors. Recent studies on ISS and stochastic-ISS (SISS) for stochastic systems are fruitful, and significant contributions for the ISS and SISS properties of stochastic systems can be referred to Huang and Mao (2009), Kang, Zhai, Liu, Zhao, and Zhao (2014), Zhao, Feng, and Kang (2012) and the references therein.

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Up to now, the constraint imposed on uniform upper bound or lower bound for each impulsive interval is an indispensable assumption in existing research efforts on ISS/iISS for impulsive systems (Chen & Zheng, 2009; Liu et al., 2011; Sun & Wang, 2012). Different from these works, the average impulsive interval has been proposed first in Hespanha et al. (2008) and it is used for ISS of impulsive delay-free systems, which corresponds to the average dwell time in switched systems introduced in Hespanha and Morse (1999). The average impulsive interval is more general than the commonly used assumption of the uniform upper bound or lower bound for each impulsive interval. On the other hand, almost all research efforts on ISS-type properties for stochastic systems have been devoted to delay-free systems (Zhao et al., 2012), though time-delays widely exist in applications. Recently, ISS is examined for stochastic delayed systems with Markovian switching in Huang and Mao (2009), Kang et al. (2014) by using the Lyapunov–Razumikhin method, which is more conservative than the Lyapunov–Krasovskii function (Lu, Ho, & Cao, 2010). However, from the above discussions, little attention has been paid to the investigation on a comprehensive analysis of ISS-type properties (including ISS, iISS and SISS performance) for impulsive stochastic delayed systems, which substantially limits the application of the results in control systems.

In this paper, we aim to investigate the ISS-type properties for impulsive stochastic delayed systems under linear assumptions. Our proposed model can be used to investigate the stability and control of networked control systems (Antunes, Hespanha, & Silvestre, 2013; Chen & Zheng, 2011; Hespanha et al., 2008). The scheduling techniques like round robin and try-once-discard protocols (Hespanha et al., 2008) in networked control systems shown as an example in Section 4 fit into the framework of our model. The contributions of this paper reside in both *model* and *method*, which are summarized as follows: (1) the presented model generalizes some recent well-studied models (Chen & Zheng, 2009; Hespanha et al., 2008; Pepe & Jiang, 2006); (2) a comprehensive study is carried out by considering ISS, iISS and SISS problems of the impulsive stochastic delayed system, where both destabilizing and stabilizing impulses are considered, respectively; (3) a comparison principle is developed for impulsive delayed systems, which facilitates the examination of the ISS, iISS and SISS of impulsive stochastic delayed systems. The rest of this paper is organized as follows. In Section 2, the impulsive stochastic delayed system is presented, together with some definitions and lemmas. In Section 3, several new criteria are obtained to ensure the ISS-type properties of impulsive stochastic delayed systems. Finally, one example is provided to illustrate the effectiveness of the proposed results.

Notations: $(\Omega, \mathcal{F}, P)$ denotes a complete probability space with some filtration $\{\mathcal{F}_t\}_{t\geq t_0}$ satisfying the usual conditions, and $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_m(t))^T$ stands for an m-dimensional $\mathcal{F}_t$-adapted Brownian motion. For any matrix A, $\lambda_{\max}(A)$ and A^T denote the largest eigenvalue of A and the transpose of A, respectively. Let $\tau > 0$, $\mathbb{R}^+ = (0, +\infty)$ and $\mathbb{N}$ be the set of positive integers. $\mathbb{R}^d$ and $\mathbb{R}^{d\times m}$ represent, respectively, d-dimensional real space and $d \times m$-dimensional real matrix space. $\tau(t) : [t_0, \infty) \to [0, \tau]$ is a continuous function. For vector $x \in \mathbb{R}^d$, $|x|$ means the Euclidean norm of x. Let $\mathcal{C}^{1,2}$ denote the family of all nonnegative functions $V(t, x)$ on $[t_0 - \tau, \infty) \times \mathbb{R}^d$ that are continuously once differentiable in t and twice in x. Let $PC([a, b]; \mathbb{R}^d)$ denote the class of piecewise continuous functions from $[a, b]$ to $\mathbb{R}^d$, if the function has at most a finite number of jumps discontinuous on $(a, b]$ and are continuous from the right for all points in $[a, b)$. For function $\psi : \mathbb{R} \to \mathbb{R}$, denote $\psi(t^-) = \lim_{s\to 0^-} \psi(t + s)$, and the Dini derivative of $\psi(t)$ is defined as $D^+\psi(t) = \limsup_{s\to 0^+}(\psi(t+s) - \psi(t))/s$. Moreover, let $\mathcal{K}$ represent the class of continuous strictly increasing function $\kappa : \mathbb{R}^+ \to \mathbb{R}^+$ with $\kappa(0) = 0$. $\mathcal{K}_\infty$ is the subset of $\mathcal{K}$ functions that are unbounded. A function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is said to belong to the class of $\mathcal{KL}$, if $\beta(., t)$ is of class $\mathcal{K}$ for each fixed $t > 0$ and $\beta(s, t)$ decreases to 0 as $t \to +\infty$ for each fixed $s \geq 0$. The class $\mathcal{CK}$ ($\mathcal{CK}_\infty$) function and $\mathcal{VK}$ ($\mathcal{VK}_\infty$) function are the subsets of class $\mathcal{K}$ ($\mathcal{K}_\infty$) functions, which are concave and convex, respectively. Let set $\mathcal{A}^c$ be the complete set of $\mathcal{A}$, and $\alpha^{-1}(x)$ denote the inverse function of $\alpha(x)$.

2. Preliminaries

In this section, some preliminaries about models and definitions are given and a comparison principle is developed for our mathematical analysis of ISS of the proposed model.

In this paper, we consider the following impulsive nonlinear stochastic delayed system:

$$\begin{cases} dx(t) = f(t, x_t, u(t))dt + g(t, x_t, u(t))d\omega(t), \\ \qquad\qquad\qquad\qquad\qquad t \neq t_k,\ t \geq t_0; \\ x(t_k) = I(x(t_k^-), u(t_k^-)), \quad t = t_k,\ k \in \mathbb{N}; \\ x(t) = \xi(t), \quad t_0 - \tau \leq t \leq t_0, \end{cases} \tag{1}$$

where $u(t) \in PC([t_0, +\infty); \mathbb{R}^m)$ denotes the disturbance input; $\{t_k,\ k \in \mathbb{N}\}$ is a strictly increasing sequence such that $t_k \to \infty$ as $k \to \infty$; $x(t^-) = \lim_{s\uparrow t} x(s)$; x_t is defined by $x_t = x(t - \tau(t))$, $0 \leq \tau(t) \leq \tau$; the initial function of $\xi(t)$ is $\mathcal{F}_{t_0}$-adapted random variable such that $\mathbf{E}\|\xi(t)\|^2 < \infty$ and $\|\xi(t)\| = \sup_{t_0-\tau\leq t\leq t_0} |\xi(t)|$; The mappings $f : [t_0, +\infty) \times \mathbb{R}^d \times \mathbb{R}^m \to \mathbb{R}^d$, $g : [t_0, +\infty) \times \mathbb{R}^d \times \mathbb{R}^m \to \mathbb{R}^{d\times m}$ and $I : \mathbb{R}^d \times \mathbb{R}^m \to \mathbb{R}^d$ are all Borel-measurable functions.

As a standard hypothesis, f, g and I satisfy the Lipschitz condition and the linear growth condition. By Mao (2007), system (1) has a unique solution $x(t) = x(t, t_0, x(t_0))$ for $t \in [t_0 - \tau, t_1)$. At $t = t_1$, there exists an impulse, which makes the solution $x(t_1^-) = x(t_1^-, t_0, x(t_0))$ jump to $x(t_1) = I(t_1^-, x(t_1^-))$. By following the same procedure for $t \in [t_0 - \tau, t_1)$, there exists a unique solution $x(t) = x(t, t_1, x(t_1))$ for $t \in [t_1, t_2)$ (Lakshmikantham et al., 1989; Yang, 2001). By repeating the above derivations, it can be obtained that for any $\xi(t) \in PC([t_0 - \tau, t_0]; \mathbb{R}^d)$, system (1) has a unique global solution $x(t)$. In addition, suppose that $f(t, 0, 0) \equiv 0$, $g(t, 0, 0) \equiv 0$ and $I(0, 0) \equiv 0$ for all $t \geq t_0$, then system (1) admits a trivial solution $x(t) \equiv 0$.

For each $V \in \mathcal{C}^{1,2}$, we define an operator $\mathcal{L}V : [t_0, \infty) \times \mathbb{R}^d \to \mathbb{R}^d$ for system (1) as follows (Mao, 2007),

$$\mathcal{L}V(t, x_t) = V_t(t, x(t)) + V_x(t, x(t))f(t, x_t, u(t)) + \frac{1}{2}trace[g^T(t, x_t, u(t))V_{xx}(t, x(t))g(t, x_t, u(t))].$$

The following definitions and lemmas are needed for deriving the main results.

Definition 1. System in (1) is said to be

(i) input-to-state stable (ISS), if there exist functions $\beta \in \mathcal{KL}$ and $\alpha, \gamma \in \mathcal{K}_\infty$ such that

$$\alpha(\mathbf{E}|x(t)|) \leq \beta(\mathbf{E}\|\xi\|, t - t_0) + \sup_{t_0\leq s\leq t} \gamma(|u(s)|);$$

(ii) integral input-to-state stable (iISS), if there exist functions $\beta \in \mathcal{KL}$ and $\alpha, \gamma \in \mathcal{K}_\infty$ such that

$$\alpha(\mathbf{E}|x(t)|) \leq \beta(\mathbf{E}\|\xi\|, t - t_0) + \int_{t_0}^{t} \gamma(|u(s)|)ds + \sum_{t_0<t_k\leq t} \gamma(|u(t_k^-)|);$$