Automatica 63 (2016) 338–348

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

An integrated control strategy to solve the disturbance decoupling problem for max-plus linear systems with applications to a high throughput screening system[☆]

Ying Shang [a,1], Laurent Hardouin [b], Mehdi Lhommeau [b], Carlos Andrey Maia [c]

[a] Department of Electrical and Computer Engineering, Southern Illinois University Edwardsville, Campus Box 1801, Edwardsville, IL 62026, USA

[b] Laboratoire Angevin de Recherche en Ingénierie des Systèmes of the University of Angers, 62 avenue Notre Dame du Lac, 49000 Angers, France

[c] Departamento de Engenharia Elétrica, Universidade Federal de Minas Gerais (UFMG), Av. Antônio Carlos 6627, Pampulha, 31270-010, Belo Horizonte, MG, Brazil

ARTICLE INFO

Article history:
Received 14 January 2014
Received in revised form
5 May 2015
Accepted 25 September 2015
Available online 11 November 2015

Keywords:
(Max, +)-algebra
Discrete-event systems
Geometrical control theory
Manufacturing systems

ABSTRACT

This paper presents the new investigations on the disturbance decoupling problem (DDP) for the geometric control of max-plus linear systems. The classical DDP concept in the geometric control theory means that the controlled outputs will not be changed by any disturbances. In practical manufacturing systems, solving for the DDP would require further delays on the output parts than the existing delays caused by the system breakdown. The new proposed modified disturbance decoupling problem (MDDP) in this paper ensures that the controlled output signals will not be delayed more than the existing delays caused by the disturbances in order to achieve the just-in-time optimal control. Furthermore, this paper presents the integration of output feedback and open-loop control strategies to solve for the MDDP, as well as for the DDP. If these controls can only solve for the MDDP, but not for the DDP, an evaluation principle is established to compare the distance between two output signals generated by controls solving for the MDDP and DDP, respectively. This distance can be interpreted as the number of tokens or firings that are needed in order for the controls to solve for the DDP. Moreover, another alternative approach is finding a new disturbance mapping in order to guarantee the solvability of the DDP by the same optimal control for the MDDP. The main results of this paper are illustrated by using a timed event graph model of a high throughput screening system in drug discovery.

1. Introduction

Max-plus linear systems (Baccelli, Cohen, Olsder, & Quadrat, 1992; Golan, 1999; Le Boudec & Thiran, 2002) are used to model timed discrete event systems. The main advantage of max-plus linear systems is incorporating the traditional linear system theory in modeling and analysis of the nonlinear synchronization behaviors in discrete event systems. They are suitable to describe algebraically the behaviors of timed event graphs (TEGs). A TEG is a subclass of timed Petri net models for discrete event systems in which each place only has a single upstream transition and a single downstream transition. Over the past three decades, many fundamental problems for max-plus linear systems have been studied by researchers, for example, controllability (Prou & Wagneur, 1999), observability (Hardouin, Maia, Cottenceau, & Lhommeau, 2010), and the model reference control problem (Maia, Hardouin, Santos-Mendes, & Cottenceau, 2005). However, the geometric theory for max-plus linear systems introduced in Cohen, Gaubert, and Quadrat (1999) has not been well established as the traditional linear systems (Basile & Marro, 1969; Wonham, 1979). Only a few existing research results on generalizing fundamental concepts and problems in geometric control are generalized to max-plus linear systems, such as computation of different controlled invariant sets (Katz, 2007; Di Loreto, Gaubert, Katz, &

☆ C.A. Maia is grateful for the support given by the Brazilian Agencies CNPq, FAPEMIG and CAPES. The material in this paper was partially presented at the 12th European Control Conference, July 17–19, 2013, Zurich, Switzerland and at the 53rd Conference on Decision and Control, December 15–17, 2014, Los Angeles, CA, USA. This paper was recommended for publication in revised form by Associate Editor Bart De Schutter under the direction of Editor Ian R. Petersen.

E-mail addresses: yshang@siue.edu (Y. Shang), laurent.hardouin@univ-angers.fr (L. Hardouin), mehdi.lhommeau@univ-angers.fr (M. Lhommeau), maia@cpdee.ufmg.br (C.A. Maia).

[1] Tel.: +1 618 650 2800; fax: +1 618 650 3374.

http://dx.doi.org/10.1016/j.automatica.2015.10.030
0005-1098/Published by Elsevier Ltd.

Loiseau, 2010; Maia, Andrade, & Hardouin, 2011) and the disturbance decoupling problem (Lhommeau, Hardouin, & Cottenceau, 2002b).

This paper reports recent investigations on the disturbance decoupling problem (DDP) for max-plus linear systems, which means the output signals remain unchanged in the presence of the disturbances. For max-plus linear systems, a disturbance is an event which blocks the occurrence of an event (in manufacturing setting, it could be a machine breakdown or a delay in a component supply) and the control consists in choosing the date of an input event (e.g. when a job should be started on a machine). Hence, solving for the DDP means finding input dates such that outputs will be delayed more than the delays caused by the disturbances. From a practical point of view, it should be more interesting to find a control such that the system is not delayed more than the delays caused by the disturbances. For example, when a system breakdown occurs, we can put the input parts of the manufacturing line as late as possible to reduce the unnecessary waiting time in the network, but not too late in order not to degrade the performance. Therefore, the modified disturbance decoupling problem (MDDP) in Hardouin, Lhommeau, and Shang (2011) and Shang, Hardouin, Lhommeau, and Maia (2013) is to find appropriate controls such that the output signals will not be delayed more than the outputs caused by the disturbances. In Hardouin et al. (2011), the solvability conditions for the DDP are presented, as well as the state feedback controls solving for the MDDP. In Shang et al. (2013), an open-loop control is presented to solve for the MDDP, and such a control can solve for the DDP at the same time if and only if the output images of the reachable space of the disturbances for the open-loop systems are subsets of the output images of the reachable space of the open-loop controls.

Hence, the aim of this paper is to present an integration of the output feedback controls (Hardouin et al., 2011) and the open-loop controls (Shang et al., 2013) such that the MDDP can be solved. Furthermore, it will be shown that this pair of controls solves for the DDP if and only if the output image of the reachable space of the disturbances for the open-loop system is a subset of the output image of the reachable space of the open-loop control. When this necessary and sufficient condition is not satisfied, the integrated strategy can only solve the MDDP, but not the DDP, then an evaluation principle is established to compare the distance, which is interpreted as the event delays between the output signals generated by the controllers and the output signals generated by the disturbances. Alternatively, if the disturbance is measurable,[2] a new control strategy is developed in order to guarantee the solvability of the DDP by using the same control as for the MDDP. This original control strategy yields a manner to modify how the disturbances act on the system such that the disturbance can be rejected. If you consider disturbances as component supply disruptions, the strategy gives the minimal number of rough parts you need on the shelf to be able to solve the DDP.

The remainder of this paper is organized as follows. Section 2 presents the mathematical preliminaries in max-plus algebra literature. Section 3 defines the max-plus linear system models and introduces the concepts of the DDP and the MDDP in max-plus linear systems. Section 4 presents the event domain approach to find the integrated controls solving for the MDDP and the DDP, respectively. If the integration of the state-feedback control and the open-loop control can only solve for the MDDP, not for the DDP, a distance evaluation is presented in Section 5 between the two output trajectories solving for the DDP and the MDDP, and a new disturbance mapping is established in Section 6 such that the integrated controls will solve for the DDP and the MDDP, simultaneously. The main results of this paper are illustrated by a high throughput screening system in drug discovery in Section 7. Section 8 concludes this paper with future research directions.

[2] This assumption is practically reasonable in manufacturing setting, because the component supply disruptions are generally known, and can be measured quite easily.

2. Mathematical preliminaries

A *semiring* is a set $\mathcal{S}$, equipped with two operations $\oplus, \otimes$, such that $(\mathcal{S}, \oplus)$ is a commutative monoid (the zero element will be denoted ε), $(\mathcal{S}, \otimes)$ is a monoid (the unit element will be denoted e), operation $\otimes$ is right and left distributive over $\oplus$, and ε is absorbing for the product (*i.e.* $\varepsilon \otimes a = a \otimes \varepsilon = \varepsilon, \forall a$). A semiring $\mathcal{S}$ is *idempotent* if $a \oplus a = a$ for all $a \in \mathcal{S}$. A non empty subset $\mathcal{B}$ of a semiring $\mathcal{S}$ is a subsemiring of $\mathcal{S}$ if for all $a, b \in \mathcal{B}$ we have $a \oplus b \in \mathcal{B}$ and $a \otimes b \in \mathcal{B}$.[3] In this paper, we denote $\overline{\mathbb{Z}}_{\max} = (\mathbb{Z} \cup \{-\infty, +\infty\}, \max, +)$ as the integer max-plus semiring.

In an idempotent semiring $\mathcal{S}$, operation $\oplus$ induces a partial order relation

$$a \succeq b \iff a = a \oplus b, \quad \forall a, b \in \mathcal{S}. \tag{1}$$

Then, $a \vee b = a \oplus b$. An idempotent semiring $\mathcal{S}$ is *complete* if sums of infinite numbers of terms are always defined, and if multiplication distributes over infinite sums too. In particular, the sum of all the elements of the idempotent semiring is denoted as $\top$ (for 'top'). A mapping $f : \mathcal{S} \to \mathcal{S}$, where $\mathcal{S}$ is a complete idempotent semiring, is residuated if and only if $f(\varepsilon) = \varepsilon$ and f is lower-semicontinuous, that is,

$$f\left(\bigoplus_{i \in I} a_i\right) = \bigoplus_{i \in I} f(a_i)$$

for any (finite or infinite) set I. The mapping f is said to be residuated and $f^\sharp$ is called its residual. It is straightforward that $L_a : \mathcal{S} \to \mathcal{S}, x \mapsto ax$ and $R_a : \mathcal{S} \to \mathcal{S}, x \mapsto xa$ are lower semi-continuous. Therefore these mappings are both residuated *i.e.*, $L_a(x) \preceq b$ (resp. $R_a(x) \preceq b$) admits the greatest solution $\top$, then the following notations are considered :

$$L_a^\sharp(b) = a \circ\!\!\!\backslash\, b = \bigoplus \{x | ax \preceq b\} \quad \text{and}$$
$$R_a^\sharp(b) = b \circ\!\!\!/\, a = \bigoplus \{x | xa \preceq b\}, \quad \forall a, b \in \mathcal{S},$$

where $L_a^\sharp$ is called the residual mappings, and is the unique mapping such that $L_a \circ L_a^\sharp \preceq Id$ and $L_a^\sharp \circ L_a \succeq Id$ where Id is the identity mapping (the same holds for R_a). The implicit equation $x = ax \oplus b$ admits $x = a^* b = (\bigoplus_{k \geq 0} a^k) b$ as smallest solution. All these results admit a natural extension to the matrix case, where the sum and product of matrices are defined with the same rules as in classical theory (see Baccelli et al., 1992).

Definition 1 (*Cohen, Gaubert, & Quadrat, 1996, 1997, 2006*)**.** Let $\mathcal{S}$ be a complete idempotent semiring and let C be a $n \times p$ matrix with entries in $\mathcal{S}$. We call null kernel of C as the set of elements $x \in \mathcal{S}^p$ such that $Cx = \varepsilon$, denoted as $\ker C$. We call equivalence kernel of L_C (denoted by $\ker_{\text{eq}} C$), the subset of all pairs of elements of $\mathcal{S}^p$ whose components are both mapped by L_C to the same element in $\mathcal{S}^n$, *i.e.*, the following definition

$$\ker_{\text{eq}} C := \left\{(s, s') \in \left(\mathcal{S}^p\right)^2 \mid Cs = Cs'\right\}. \tag{2}$$

Clearly $\ker_{\text{eq}} C$, is an equivalence relation on $\mathcal{S}^p$, *i.e.*, $Cs = Cs' \iff s' \equiv s \pmod{\ker_{\text{eq}} C}$ and furthermore it is a congruence and then we can define the quotient $\mathcal{S}^p / \ker_{eq} C$.

[3] As in the conventional algebra, the multiplication $\otimes$ is often omitted.