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Coverage control for heterogeneous mobile sensor networks on a circle[☆]

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ABSTRACT

The coverage control problem for a network of heterogeneous mobile sensors with first-order dynamics is addressed in this paper. The goal of the problem is to minimize a coverage cost function which is defined to be the largest arrival time from the mobile sensor network to the points on a circle. The heterogeneity of the network is considered in terms of different maximum velocities of the mobile sensors, which in turn imposes different constraints on the sensors' control inputs. A necessary and sufficient condition for the global minimization of the coverage cost function is firstly derived via a partition of the circle. Then, a distributed coverage control scheme with input saturation is developed to drive the sensors to the optimal configuration such that the necessary and sufficient condition is satisfied. Under the distributed coverage control scheme, the mobile sensors' spatial order on the circle is preserved throughout the network's evolution and thus collision between mobile sensors is avoided. Finally, simulation results are presented to illustrate the effectiveness of the proposed distributed control scheme.

1. Introduction

This paper considers the coverage problem of a circle using a network of mobile sensors with non-identical maximum velocities. The goal is to deploy the sensors on the circle such that the largest arrival time from the mobile sensor network to any point on the circle is minimized. This problem is motivated by the facts that in practice the assumption of mobile sensors with identical moving speed often cannot be satisfied and events taking place in the mission domain only last for a finite time period (Bisnik, Abouzeid, & Isler, 2007; Chen & Zhang, 2013; Seyboth, Wu, Qin, Yu, & Allgöwer, 2014). When the sensing range of mobile sensors is negligible with respect to the length of a circle, reduction of the largest arrival time from a sensor network to the points on the circle will increase the possibility of capturing the events taking place on the circle before they fade away.

In the past decade, much effort has been devoted to the coverage control problem for mobile sensor networks, where the goal is to drive the sensors to the optimal locations such that the overall sensing performance of the sensor network is optimized (Cortés & Bullo, 2005; Cortés, Martínez, Karatus, & Bullo, 2004; Hu & Xu, 2013; Lekien & Leonard, 2009; Sayyaadi & Moarref, 2011; Song, Feng, Fan, & Wang, 2011; Song, Liu, Feng, Wang, & Gao, 2013; Zhong & Cassandras, 2011). In Cortés et al. (2004), gradient descent coverage control laws based on Voronoi partition are developed for mobile sensors with limited sensing and communication capabilities to minimize a locational optimization function. This work is extended in Sayyaadi and Moarref (2011) to address an optimal deployment problem, where the optimal deployment of mobile sensors is achieved only when the sensors' duty to capability ratios reach a consensus. Assuming that all vehicles move at the same constant speed, the maximum traveling time it takes for the vehicles to arrive at an arbitrary point in a two-dimensional mission domain is minimized in Hu and Xu (2013) via optimizing the vehicles' locations.

The coverage problem for mobile sensors in a one-dimensional mission space has received increasing attention in recent years due to its wide potential applications such as environmental boundary monitoring and target tracking (Martínez & Bullo, 2006; Song & Hong, 2011; Susca, Bullo, & Martínez, 2008). A benchmark problem of one-dimensional coverage is the uniform coverage problem in which the distance between neighboring agents is required to reach a consensus. It has been shown that the sensing performance of a homogeneous sensor network is maximized when the sensors are uniformly deployed on a line or circle

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provided that the information density of all points on the line or circle is identical (Carli & Bullo, 2009; Martínez, Bullo, Cortés, & Frazzoli, 2007; Song & Feng, 2014). In contrast, when the density of information is not uniform over the mission space, uniform coverage is generally undesirable and more sensors should be deployed in areas with high information density. In Leonard and Olshevsky (2013), distributed coverage control laws are developed for a network of mobile agents to optimally sense a nonuniform field which is defined to be the density of information at each point on a line.

Problems that are closely related to the coverage control problem on a circle include circular formation and multi-agent consensus. In the circular formation problem, a team of mobile agents is required to form a formation on a circle and the desired distance between neighboring agents is generally prescribed *a priori* (El-Hawwary & Maggiore, 2013; Sepulchre, Paley, & Leonard, 2007, 2008; Wang, Xie, & Cao, 2013, 2014). In contrast, in the coverage control problem the desired distance between sensors is unknown beforehand and depends on the coverage cost function to be optimized. The multi-agent consensus problem has been studied extensively in literature in which all agents reach an agreement on a variable of interest (Ding, 2013; Hong, Hu, & Gao, 2006; Olfati-Saber, Fax, & Murray, 2007). Recently, the consensus problem for multi-agent systems with input saturation has attracted much interest due to the fact that input saturation is ubiquitous in real-world applications (Li, Xiang, & Wei, 2011; Meng, Zhao, & Lin, 2013; Ren, 2008; Su, Chen, Lam, & Lin, 2013; Wang & Gao, 2013; Yang, Meng, Dimarogonas, & Johansson, 2014).

In this paper, a distributed coverage control scheme is developed for heterogeneous mobile sensor networks on a circle to minimize the coverage cost function while preserving the mobile sensors' order on the circle. The difficulties caused by the heterogeneity of mobile sensors' maximum velocities are twofold. Firstly, for a network of mobile sensors with identical maximum speed it has been shown in Song and Feng (2014) that the optimal configuration is uniform deployment of the sensors on the circle. However, it is still unclear under what conditions the coverage cost function is minimized when a network of heterogeneous mobile sensors is deployed. Secondly, it is noted that different constraints are imposed on the mobile sensors' control inputs due to the existence of non-identical maximum velocity for each sensor. This further complicates the proof of mobile sensors' order preservation and convergence analysis of the distributed coverage control scheme. This paper extends the preliminary results in Song, Liu, and Feng (2014) by taking into consideration input constraints and order preservation of the mobile sensors.

The main contributions of this paper can be summarized as follows. Firstly, it is shown that the mobile sensors' order is preserved throughout the network's evolution under the proposed coverage control scheme and thus collision between the sensors is avoided during the coverage task. The idea is to first prove that the spatial order of the sensors is preserved under the coverage control laws without input constraints. Then, we show that the order preservation property is not affected by the introduction of saturation constraints on the sensors' control inputs. Secondly, a necessary and sufficient condition for the minimization of the coverage cost function is derived by partitioning the circle into subregions such that each sensor is assigned to a subregion and the shortest arrival time from each sensor to an arbitrary point located in its subregion is less than that from the other sensors in the network. It is shown that the coverage cost function is globally minimized if and only if the proportion of the counterclockwise distance from each sensor to its right neighbor and the sum of the two sensors' maximum velocities reaches a consensus. Finally, it is observed that the necessary and sufficient condition is satisfied when all sensors' control inputs converge to zero provided that the sensors' order is preserved. Using tools from graph theory and matrix analysis, we show that under the distributed coverage control laws the sensors' control inputs reach a consensus as time goes to infinity and the consensus value is equal to zero.

The rest of the paper is organized as follows. In Section 2, preliminaries and notations are presented. Problem formulation is given in Section 3. Distributed coverage control laws which preserve the order of the mobile sensors are proposed in Section 4 and convergence analysis of the coverage control laws is given in Section 5. In Section 6, simulation results are provided to illustrate the main results. Finally, Section 7 concludes the paper.

2. Preliminaries and notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph of order n with the set of nodes $\mathcal{V}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A subgraph $(\mathcal{V}_1, \mathcal{E}_1)$ of $\mathcal{G}$ is a graph with $\mathcal{V}_1 \subset \mathcal{V}$ and $\mathcal{E}_1 \subset \mathcal{E} \cap (\mathcal{V}_1 \times \mathcal{V}_1)$. An edge of $\mathcal{G}$ is denoted by (i, j) which indicates node j can receive information from node i. The in-neighbors of node j are $\mathcal{N}_j^{\text{in}} = \{i : (i, j) \in \mathcal{E}\}$. The adjacency matrix of $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix of $\mathcal{G}$ is given by $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ with $l_{ii} = \sum_{k=1,k\neq i}^{n} a_{ik}$ and $l_{ij} = -a_{ij},\ j \neq i$. A directed graph is said to be strongly connected if there exists a directed path from every node to every other node. A directed graph is said to have a spanning tree if there exists one node which has a directed path to every other node. For a given matrix $E = [e_{ij}] \in \mathbb{R}^{n\times n}$, the corresponding directed graph denoted by $\mathcal{G}(E)$ is a directed graph with n nodes and $(j, i) \in \mathcal{G}(E)$ if and only if $e_{ij} \neq 0$. A matrix C is nonnegative and denoted by $C \geq 0$ if all of its entries are nonnegative. Similarly, C is positive and denoted by $C > 0$ if each of its entry is positive.

Throughout this paper, $\mathbb{Z}$ and $\mathbb{R}_+$ represent the sets of integers and strictly positive real numbers, respectively. Denote $\prod_{i=1}^{n} P_i = P_1P_2 \dots P_n$. A standard saturation function $\text{sat}(\cdot) : \mathbb{R} \to \mathbb{R}$ is defined by $\text{sat}(x) = \text{sign}(x)\min\{1, |x|\}$. For a vector $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, $\text{sat}(x) = [\text{sat}(x_1), \text{sat}(x_2), \dots, \text{sat}(x_n)]^T$. $m(E)$ denotes the Lebesgue measure of a set $E \subseteq \mathbb{R}$. $\mathbf{1}_n$ is a $n \times 1$ column vector of ones.

3. Problem formulation

Consider a network of mobile sensors $i,\ i \in \mathcal{I}_n = \{1, \dots, n\}$ initially located on a unit circle. Let $\mathbb{S}$ be the set of all points on the circle. Denote the position of an arbitrary point q on the circle as the angle measured counterclockwise from the positive horizontal axis. The counterclockwise distance from sensor i to point $q \in \mathbb{S}$ can be defined as $\bar{d}(q_i, q) = (q - q_i) \mod 2\pi$, where q_i is the position of sensor i and $x \mod 2\pi$ is the positive remainder of the division of x by 2π. Then, the clockwise distance from sensor i to point q is $2\pi - \bar{d}(q_i, q)$. The distance between sensor i and point q is defined by $d(q_i, q) = \min\{\bar{d}(q_i, q), 2\pi - \bar{d}(q_i, q)\}$. In this paper, q is not required to be constrained in $[0, 2\pi]$ and points q and $q + 2k\pi,\ k \in \mathbb{Z}$ refer to the same point on the circle.

For our subsequent analysis, label the sensors counterclockwise in accordance with their initial locations on the circle and assume that the sensors' initial positions do not coincide with each other, that is,

$$0 \leq q_1(0) < \cdots < q_i(0) < q_{i+1}(0) < \cdots < q_n(0) < 2\pi. \tag{1}$$

A mobile sensor network's order is preserved if the inequalities $q_1(k) < \cdots < q_i(k) < q_{i+1}(k) < \cdots < q_n(k) < 2\pi + q_1(k)$ always hold. In this work, each sensor can only communicate with its right neighbor and left neighbor on the circle. Then, the network topology is fixed and strongly connected if the sensors' order is preserved throughout the entire evolution.