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Brief paper

New stability and exact observability conditions for semilinear wave equations☆

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ABSTRACT

The problem of estimating the initial state of 1-D wave equations with globally Lipschitz nonlinearities from boundary measurements on a finite interval was solved recently by using the sequence of forward and backward observers, and deriving the upper bound for exact observability time in terms of Linear Matrix Inequalities (LMIs) (Fridman, 2013). In the present paper, we generalize this result to n-D wave equations on a hypercube. This extension includes new LMI-based exponential stability conditions for n-D wave equations, as well as an upper bound on the minimum exact observability time in terms of LMIs. For 1-D wave equations with locally Lipschitz nonlinearities, we find an estimate on the region of initial conditions that are guaranteed to be uniquely recovered from the measurements. The efficiency of the results is illustrated by numerical examples.

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1. Introduction

Lyapunov-based solutions of various control problems for finite-dimensional systems can be formulated in the form of Linear Matrix Inequalities (LMIs) (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). The LMI approach to distributed parameter systems is capable of utilizing nonlinearities and of providing the desired system performance (see e.g. Castillo, Witrant, Prieur, & Dugard, 2012, Fridman & Orlov, 2009b, Lamare, Girard, & Prieur, 2013). For 1-D wave equations, several control problems were solved by using the direct Lyapunov method in terms of LMIs (Fridman, 2013; Fridman & Orlov, 2009a). However, there have not been yet LMI-based results for n-D wave equations, though the exponential stability of the n-D wave equations in bounded spatial domains has been studied in the literature via the direct Lyapunov method (see e.g. Ammari, Nicaise, & Pignotti, 2010, Fridman, Nicaise, & Valein, 2010, Guo, Zhou, & Yao, 2014, Zuazua, 1990).

The problem of estimating the initial state of 1-D wave equations with globally Lipschitz nonlinearities from boundary measurements on a finite interval was solved recently by using the sequence of forward and backward observers, and deriving the upper bound for exact observability time in terms of LMIs (Fridman, 2013). In the present paper, we generalize this result to n-D wave equations on a hypercube. This extension includes new LMI-based exponential stability conditions for n-D wave equations. Their derivation is based on n-D extensions of the Wirtinger (Poincaré) inequality (Hardy, Littlewood, & Pólya, 1988) and of the Sobolev inequality with tight constants, which is crucial for the efficiency of the results. As in 1-D case, the continuous dependence of the reconstructed initial state on the measurements follows from the integral input-to-state stability of the corresponding error system, which is guaranteed by the LMIs for the exponential stability. Some preliminary results on global exact observability of multidimensional wave PDEs will be presented in Fridman and Terushkin (2015).

Another objective of the present paper is to study regional exact observability for systems with locally Lipschitz in the state nonlinearities. Here we restrict our consideration to 1-D case, and find an estimate on the region of initial conditions that are guaranteed to be uniquely recovered from the measurements. Note that our result on the regional observability cannot be extended to multi-dimensional case (see Remark 4 for explanation and for discussion on possible n-D extensions for different classes of nonlinearities). The efficiency of the results is illustrated by numerical examples.

The presented simple finite-dimensional LMI conditions complete the theoretical qualitative results of e.g. Ramdani, Tucsnak, and Weiss (2010) (where exact observability of linear systems in a Hilbert space was studied via a sequence of forward and backward observers) and Baroun, Jacob, Maniar, and Schnaubelt (2013)

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(where local exact observability of abstract semilinear systems was considered).

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with the norm $|\cdot|$, $\mathbb{R}^{n\times m}$ is the space of $n\times m$ real matrices. The notation $P>0$ with $P\in\mathbb{R}^{n\times n}$ means that P is symmetric and positive definite. For the symmetric matrix M, $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the minimum and the maximum eigenvalues of M respectively. The symmetric elements of the symmetric matrix will be denoted by $*$. Continuous functions (continuously differentiable) in all arguments, are referred to as of class C (of class C^1). $L^2(\Omega)$ is the Hilbert space of square integrable $f:\Omega\to\mathbb{R}$, where $\Omega\subset\mathbb{R}^n$, with the norm $\|f\|_{L^2}=\sqrt{\int_\Omega |f(x)|^2dx}$. For the scalar smooth function $z=z(t,x_1,\dots,x_n)$ denote by $z_t, z_{x_k}, z_{tt}, z_{x_kx_j}$ $(k,j=1,\dots,n)$ the corresponding partial derivatives. For $z:\Omega\to\mathbb{R}$ define $\nabla z=z_x^T=[z_{x_1}\dots z_{x_n}]^T$, $\Delta z=\sum_{p=1}^n z_{x_px_p}$. $\mathcal{H}^1(\Omega)$ is the Sobolev space of absolutely continuous functions $z:\Omega\to\mathbb{R}$ with the square integrable ∇z. $\mathcal{H}^2(\Omega)$ is the Sobolev space of scalar functions $z:\Omega\to\mathbb{R}$ with absolutely continuous ∇z and with $\Delta z\in L^2(\Omega)$.

2. Observers and exponential stability of n-D wave equations

2.1. System under study and Luenberger type observer

Throughout the paper we denote by Ω the n-D unit hypercube $[0,1]^n$ with the boundary Γ. We use the partition of the boundary:

$$\Gamma_D=\{x=(x_1,\dots,x_n)^T\in\Gamma:\exists p\in 1,\dots,n \text{ s.t. } x_p=0\}$$

$$\Gamma_{N,p}=\{x\in\Gamma:x_p=1\},\qquad \Gamma_N=\bigcup_{p=1,\dots,n}\Gamma_{N,p}.$$

Here subscripts D and N stand for Dirichlet and for Neumann boundary conditions respectively.

We consider the following boundary value problem for the scalar n-D wave equation:

$$\begin{aligned} z_{tt}(x,t)&=\Delta z(x,t)+f(z,x,t) && \text{in } \Omega\times(t_0,\infty),\\ z(x,t)&=0 && \text{on } \Gamma_D\times(t_0,+\infty),\\ \frac{\partial}{\partial\nu}z(x,t)&=0 && \text{on } \Gamma_N\times(t_0,\infty),\end{aligned}\tag{2.1}$$

where f is a C^1 function, ν denotes the outer unit normal vector to the point $x\in\Gamma$ and $\frac{\partial}{\partial\nu}z$ is the normal derivative. Let $g_1>0$ be the known bound on the derivative of $f(z,x,t)$ with respect to z:

$$|f_z(z,x,t)|\le g_1\quad\forall(z,x,t)\in\mathrm{R}^{n+2}.\tag{2.2}$$

Since Ω is a unit hypercube, the boundary conditions on Γ_N can be rewritten as

$$z_{x_p}(x,t)\Big|_{x_p=1}=0\quad\forall x_i\in[0,1],\ i\neq p,\ p=1,\dots,n.$$

Consider the following initial conditions:

$$z(x,t_0)=z_0(x),\qquad z_t(x,t_0)=z_1(x),\quad x\in\Omega.\tag{2.3}$$

The boundary measurements are given by

$$y(x,t)=z_t(x,t)\quad\text{on } \Gamma_N\times(t_0,\infty).\tag{2.4}$$

Similar to Fridman (2013), the boundary-value problem (2.1) can be represented as an abstract differential equation by defining the state $\zeta(t)=[\zeta_0(t)\ \zeta_1(t)]^T=[z(t)\ z_t(t)]^T$ and the operators

$$\mathcal{A}=\begin{bmatrix}0 & I\\ \Delta z & 0\end{bmatrix},\qquad F(\zeta,t)=\begin{bmatrix}0\\ F_1(\zeta_0,t)\end{bmatrix},$$

where $F_1:\mathcal{H}^1(\Omega)\times R\to L^2(\Omega)$ is defined as $F_1(\zeta_0,t)=f(\zeta_0(x),x,t)$ so that it is continuous in t for each $\zeta_0\in\mathcal{H}^1(\Omega)$. The differential equation is

$$\dot\zeta(t)=\mathcal{A}\zeta(t)+F(\zeta(t),t),\quad t\ge t_0\tag{2.5}$$

in the Hilbert space $\mathcal{H}=\mathcal{H}^1_{\Gamma_D}(\Omega)\times L^2(\Omega)$, where

$$\mathcal{H}^1_{\Gamma_D}(\Omega)=\left\{\zeta_0\in\mathcal{H}^1(\Omega)\,\middle|\,\zeta_{0|\Gamma_D}=0\right\}$$

and $\|\zeta\|^2_{\mathcal{H}}=\|\nabla\zeta_0\|^2_{L^2}+\|\zeta_1\|^2_{L^2}$. The operator $\mathcal{A}$ has the dense domain

$$\begin{aligned}\mathcal{D}(\mathcal{A})=\Big\{&(\zeta_0,\zeta_1)^T\in\mathcal{H}^1_{\Gamma_D}(\Omega)\times\mathcal{H}^1_{\Gamma_D}(\Omega)\,\Big|\,\Delta\zeta_0\in L^2(\Omega)\\ &\text{and } \frac{\partial}{\partial\nu}\zeta_{0|\Gamma_N}=-b\zeta_{1|\Gamma_N}\Big\},\end{aligned}$$

where $b=0$. Here the boundary condition holds in a weak sense (as defined in Sect. 3.9 of Tucsnak & Weiss, 2009), i.e. the following relation holds:

$$\begin{aligned}\langle\Delta\zeta_0,\phi\rangle_{L^2(\Omega)}+\langle\nabla\zeta_0,\nabla\phi\rangle_{[L^2(\Omega)]^n}&=-b\langle\zeta_0,\phi\rangle_{L^2(\Gamma_N)}\\ &\forall\phi\in\mathcal{H}^1_{\Gamma_D}(\Omega).\end{aligned}$$

The operator $\mathcal{A}$ is m-dissipative (see Proposition 3.9.2 of Tucsnak & Weiss, 2009) and hence it generates a strongly continuous semigroup. Due to (2.2), the following Lipschitz condition holds:

$$\|F_1(\zeta_0,t)-F_1(\bar\zeta_0,t)\|_{L^2}\le g_1\|\zeta_0-\bar\zeta_0\|_{L^2}\tag{2.6}$$

where $\zeta_0,\bar\zeta_0\in\mathcal{H}^1_{\Gamma_D}(\Omega)$, $t\in R$. Then by Theorem 6.1.2 of Pazy (1983), a unique continuous mild solution $\zeta(\cdot)$ of (2.5) in $\mathcal{H}$ initialized by

$$\zeta_0(t_0)=z_0\in\mathcal{H}^1_{\Gamma_D}(\Omega),\qquad \zeta_1(t_0)=z_1\in L^2(\Omega)$$

exists in $C([t_0,\infty),\mathcal{H})$. If $\zeta(t_0)\in\mathcal{D}(\mathcal{A})$, then this mild solution is in $C^1([t_0,\infty),\mathcal{H})$ and it is a classical solution of (2.1) with $\zeta(t)\in\mathcal{D}(\mathcal{A})$ (see Theorem 6.1.5 of Pazy, 1983).

We suggest a Luenberger type observer of the form:

$$\hat z_{tt}(x,t)=\Delta\hat z(x,t)+f\left(\hat z,x,t\right),\quad t\ge t_0,\ x\in\Omega\tag{2.7}$$

under the initial conditions $[\hat z(\cdot,t_0),\hat z_t(\cdot,t_0)]^T\in\mathcal{H}$ and the boundary conditions

$$\begin{aligned}\hat z(x,t)&=0 && \text{on } \Gamma_D\times(t_0,\infty)\\ \frac{\partial}{\partial\nu}\hat z(x,t)&=k\Big[y(x,t)-\hat z_t(x,t)\Big] && \text{on } \Gamma_N\times(t_0,\infty)\end{aligned}\tag{2.8}$$

where k is the injection gain.

The well-posedness of (2.7), (2.8) will be established by showing the well-posedness of the estimation error $e=z-\hat z$. Taking into account (2.1), (2.3) we obtain the following PDE for the estimation error $e=z-\hat z$:

$$e_{tt}(x,t)=\Delta e(x,t)+ge(x,t)\quad t\ge t_0,\ x\in\Omega\tag{2.9}$$

under the boundary conditions

$$\begin{aligned}e(x,t)&=0 && \text{on } \Gamma_D\times(t_0,\infty)\\ \frac{\partial}{\partial\nu}e(x,t)&=-ke_t(x,t) && \text{on } \Gamma_N\times(t_0,\infty).\end{aligned}\tag{2.10}$$

Here $ge=f(z,x,t)-f(z-e,x,t)$ and

$$g=g(z,e,x,t)=\int_0^1 f_z(z+(\theta-1)e,x,t)d\theta.$$

The initial conditions for the error are given by

$$\begin{aligned}e(x,t_0)&=z_1(x)-z(\cdot,t_0),\\ e_t(x,t_0)&=z_2(x)-z_t(\cdot,t_0).\end{aligned}$$

The boundary conditions on Γ_N can be presented as

$$e_{x_p}(x,t)\Big|_{x_p=1}=-ke_t(x,t)\quad\forall x_i\in[0,1],$$

$$i\neq p,\quad p=1,\dots,n.$$

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