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Brief paper

A stochastic unknown input realization and filtering technique☆

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ABSTRACT

This paper studies the state estimation problem of linear discrete-time systems with unknown inputs which can be treated as a wide-sense stationary process with rational power spectral density, while no other prior information needs to be known. We propose an autoregressive (AR) model based unknown input realization technique which allows us to recover the input statistics from the output data by solving an appropriate least squares problem, then fit an AR model to the recovered input statistics and construct an innovations model of the unknown inputs using the eigensystem realization algorithm. An augmented state system is constructed and the standard Kalman filter is applied for the state estimation. A reduced order model filter is also introduced to reduce the computational cost of the Kalman filter. A numerical example is given to illustrate the procedure.

1. Introduction

In this paper, we consider the state estimation problem for systems with unknown inputs. The main contribution of our work is that when no prior information of the unknown inputs is known, we recover the statistics of the unknown inputs from the measurements, and then construct an innovations model of the unknown inputs from the recovered statistics such that the standard Kalman filter can be applied for the state estimation. The innovations model is constructed by fitting an autoregressive (AR) model to the recovered input correlation data from which a state space model is constructed using the balanced realization technique. The method is tested on the stochastically perturbed heat transfer problem.

For stochastic systems, the state estimation problem with unknown inputs is known as unknown input filtering (UIF) problem, and many UIF approaches are based on the Kalman filter (Darouach, Zasadzinski, Onana, & Nowakowski, 1995; Hou & Patton, 1998; Koenig & Mammar, 2003). When the dynamics of the unknown inputs is available, for example, if it can be assumed to be a wide-sense stationary (WSS) process with known mean and covariance, one common approach called Augmented State Kalman Filter (ASKF) is used, where the states are augmented with the unknown inputs (Hsieh, 2013). To reduce the computational complexity of ASKF, optimal two-stage Kalman filters (OTSKF) and optimal three-stage Kalman filters have been developed to decouple the augmented filter into two parallel reduced-order filters by applying a U-V transformation (Ben Hmida, Khemiri, Ragot, & Gossa, 2012; Hsieh & Chen, 1999; Kanev & Verhaegen, 2005). When no prior information about the unknown input is available, an unbiased minimum-variance (UMV) filtering technique has been developed (Gillijns & De Moor, 2007; Hsieh, 2009). The problem is transformed into finding a gain matrix such that the trace of the estimation error matrix is minimized. Certain algebraic constraints must be satisfied for the unbiased estimator to exist.

In this paper, we address the state estimation problem of systems when the unknown inputs can be treated as a WSS process with rational power spectral density (PSD), while no other information about the unknown inputs is known. We propose a new unknown input filtering approach based on the system realization techniques. Instead of constructing the observer gain matrix which needs to satisfy certain constraints, we apply the standard Kalman filtering using the following procedure: (1) recover the statistics of the unknown inputs from the measurements by solving an appropriate least squares problem, (2) find a spectral factorization of unknown input process by fitting an autoregressive (AR) model, (3) construct an innovations model of the unknown inputs via the eigensystem realization algorithm (ERA) (Juang, 1994) to the recovered input correlation data, and (4) apply the ASKF for state estimation. To reduce the computational cost of the ASKF, we apply the Balanced Proper Orthogonal Decomposition (BPOD) technique

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(Rowley, 2005) to construct a reduced order model (ROM) for filtering.

The main advantage of the AR model based algorithm we propose is that the performance of the algorithm is better than the ASKF, OTSKF and UMV algorithms when the unknown inputs can be treated as WSS processes with rational PSDs. The AR model based algorithm we propose constructs one particular realization of the true unknown input model, and the performance of the AR model based algorithm is the same as OTSKF when the assumed unknown input model used in OTSKF is accurate, and is better than UMV algorithm in the sense that the error covariances are smaller. With the increase of the sensor noise, we have seen that the performance of AR model based algorithm gets much better than the UMV algorithm.

The paper is organized as follows. In Section 2, the problem is formulated, and general assumptions are made about the system and the unknown inputs. In Section 3, the AR model based unknown input realization approach is proposed. The unknown input statistics are recovered from the measurements, then a linear model is constructed using an AR model and the ERA is used to generate a balanced minimal realization of the unknown inputs. After an innovations model of the unknown inputs is constructed, the ASKF is applied for state estimation in Section 4. Also, a ROM constructed using the BPOD is introduced to reduce the computational cost of Kalman filter. Section 5 presents a numerical example consisting of a stochastically perturbed heat transfer problem that utilizes the proposed technique.

2. Problem formulation

Consider a complex valued linear time-invariant (LTI) discrete time system:

$$x_k = Ax_{k-1} + Bu_{k-1}, \qquad y_k = Cx_k + v_k, \tag{1}$$

where $x_k \in \mathbb{C}^n$, $y_k \in \mathbb{C}^q$, $v_k \in \mathbb{C}^q$, $u_k \in \mathbb{C}^p$ are the state vector, the measurement vector, the measurement white noise with zero mean and known covariance Ω, and the unknown stochastic inputs respectively. The process u_k is used to model the presence of the external disturbances, process noise, and unmodeled terms. Here, $A \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}^{n\times p}$, $C \in \mathbb{C}^{q\times n}$ are known.

Denote $h_i = CA^{i-1}B$, $i = 1, 2, \dots$ as the Markov parameters of system (1). We use x^* to denote the complex conjugate transpose of x, and x^T to denote the transpose of x. Denote $\bar{h}_i$ as the matrix h_i with complex conjugated entries, and $h_i^* = (\bar{h}_i)^T$. $\|A\| = (\sum_{i,j=1}^{n} |a_{i,j}|^2)^{1/2}$ denotes the Frobenius norm of matrix A, and $\|x\|_2 = (|x_1|^2 + |x_2|^2 + \cdots + |x_n|^2)^{1/2}$ denotes the Euclidean norm of vector x.

The following assumptions are made about system (1):

- A1. A is a stable matrix, and (A, C) is detectable.
- A2. $\operatorname{rank}(B) = p$, $\operatorname{rank}(C) = q$, $p \le q$ and $\operatorname{rank}(CAB) = \operatorname{rank}(B) = p$.
- A3. u_k and v_k are uncorrelated.
- A4. We further assume that the unknown input u_k can be treated as a WSS process:

$$\xi_k = A_e\xi_{k-1} + B_e\nu_{k-1}, \qquad u_k = C_e\xi_k + \mu_k, \tag{2}$$

where ν_k, μ_k are uncorrelated white noise processes.

Remark 1. A2 is a weaker assumption than the so-called "observer matching" condition used in unknown input observer design. The observer matching condition requires $\operatorname{rank}(CB) = \operatorname{rank}(B) = p$, which in practice, may be too restrictive. A2 implies that if there are p inputs, then there should be at least p controllable and observable modes. A4 implies that u_k is a WSS process with a rational power spectrum.

In this paper, we consider the state estimation problem when the system (2), i.e., (A_e, B_e, C_e) is unknown. Given the output data y_k, we want to construct an innovations model for the unknown stochastic input u_k, such that the output statistics of the innovations model and system (2) are the same. Given such a realization of the unknown input, we apply the standard Kalman filter for the state estimation, augmented with the unknown input states.

3. AR model based unknown input realization technique

In this section, we propose an AR model based unknown input realization technique which can construct an innovations model of the unknown inputs such that the ASKF can be applied for state estimation. First, a least squares problem is formulated based on the relationship between the inputs and outputs to recover the statistics of the unknown inputs. Then an AR model is constructed using the recovered input statistics, and a balanced realization model is then constructed using the ERA.

3.1. Extraction of input autocorrelations via a least squares problem

Consider system (1) with zero initial conditions, the output y_k can be written as:

$$y_k = \sum_{i=1}^{\infty} h_i u_{k-i} + v_k. \tag{3}$$

For a LTI system, under assumption A1 that A is stable, the output $\{y_k\}$ is a WSS process when $\{u_k\}$ is WSS. The output autocorrelation can be written as:

$$\begin{aligned} R_{yy}(m) &= E[y_k y_{k+m}^*] \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} h_i u_{k-i} u_{k+m-j}^* h_j^* + R_{vv}(m) \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} h_i R_{uu}(m+i-j) h_j^* + R_{vv}(m), \end{aligned} \tag{4}$$

where $m = 0, \pm1, \pm2, \dots$ is the time-lag between y_k and y_{k+m}. Here, assumption A3 is used.

We denote $\hat{R}_{yy}(m) = R_{yy}(m) - R_{vv}(m)$, where $R_{vv}(m) = \Omega$ for $m = 0$, and $R_{vv}(m) = 0$, otherwise. Therefore, the relationship between input and output autocorrelation function is given by:

$$\hat{R}_{yy}(m) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} h_i R_{uu}(m+i-j) h_j^*. \tag{5}$$

To solve for the unknown input autocorrelations $R_{uu}(m)$, first we need to use a theorem from linear matrix equations (Roth, 1934).

Theorem 2. *Consider the matrix equation*

$$AXB = C, \tag{6}$$

where A, B, C, X are all matrices. If $A \in \mathbb{C}^{m\times n} = (a_1, a_2, \dots, a_n)$, where a_i are the columns of A, then define $\operatorname{vec}(A) \in \mathbb{C}^{mn\times 1}$ and the Kronecker product $A \otimes B$ as:

$$\operatorname{vec}(A) = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \qquad A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \cdots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}. \tag{7}$$

If A is an $m \times n$ matrix and B is a $p \times q$ matrix, then the Kronecker product $A \otimes B$ is an $mp \times nq$ block matrix.

The matrix equation (6) can be transformed into one vector equation:

$$(B^T \otimes A)\operatorname{vec}(X) = \operatorname{vec}(C), \tag{8}$$

where $B^T \otimes A$ is the Kronecker product of B^T and A.