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Brief paper

Estimation and synthesis of reachable set for switched linear systems[☆]

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ABSTRACT

This paper focuses on the problems of reachable set estimation and state-feedback controller design for discrete-time switched linear systems under bounded peak disturbances. For the reachable set estimation problem, a Lyapunov-based inequality is developed based on the multiple Lyapunov strategy. By choosing appropriate Lyapunov functions, the ellipsoidal reachable set estimation conditions of discrete-time switched linear systems are obtained. In order to make the estimated ellipsoids as small as possible, three optimization approaches are proposed. Specifically, the Genetic Algorithm is used to search for the optimal parameters satisfying the obtained reachable set estimation conditions. In addition, the state-feedback controller design problem for discrete-time switched linear systems is considered. The function of the controller is to manipulate the reachable set of the closed-loop system to lie within a given ellipsoid or make the reachable set small. Finally, the effectiveness of the obtained results is verified through some numerical examples.

1. Introduction

Switched systems have been extensively studied in the past few decades. Typically, a switched system consists of a number of subsystems and there is a switching signal determining which subsystem is active. Owing to the multi-system feature, switched systems have strong modeling capacity to practical systems. Moreover, in control synthesis, switching control is regarded as an effective control strategy to improve system performance. To date, many basic problems concerning switched systems have been considered in the literature, see, e.g., Daafouz, Riedinger, and Iung (2002), Liberzon (2003), Liberzon, Hespanha, and Morse (1999), Lin and Antsaklis (2009) and Zhang, Wang, and Chen (2009) for stability analysis and Chen, Zhang, Karimi, and Zhao (2011), Zhang and Shi (2009) and Zhao and Hill (2008) for H_∞ controller design.

As a fundamental concept in control theory, the reachable set has received many researchers' attention. The reachable set of a system is defined as the set containing all system states reachable from the origin for a prescribed class of system disturbances. For continuous-time switched linear systems, a preliminary analysis of reachable set was given in Sun, Ge, and Lee (2002). In that work, the switching signal and system disturbances were considered to be arbitrary, that is, switching may occur at any time and there is no restriction in the system disturbances. Under such assumptions, the reachable set of continuous-time switched linear systems can be recursively determined by conditions involving the system matrices. A computational method was also developed in Sun et al. (2002) to tackle the problem of reducing computing burdens of the proposed algorithm. For discrete-time switched linear systems, the corresponding results were presented in Ge, Sun, and Lee (2001). In addition, the structure of the reachable set of switched linear systems was studied in Petreczky (2006) by using a differential geometric approach.

Besides the reachable set of a system under arbitrary disturbances, research efforts have also been focused on the reachable set of a system under bounded peak disturbances. For this problem, new challenges often arise since it is generally difficult to obtain the exact characterization of a reachable set. Usually, we have to determine a region that is as small as possible such that this region bounds the reachable set. This is referred to as the reachable set estimation problem. For systems with bounded peak disturbances, one of the recent methods for studying the reachable set estimation problem is the so-called ellipsoidal technique, which aims to determine ellipsoids containing the concerned reachable set. Such ellipsoids can be obtained by using the Lyapunov

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method and linear matrix inequality (LMI) technique (Boyd, El-Ghaoui, Feron, & Balakrishnan, 1994). Very recently, the problem of reachable set estimation of linear systems with both bounded peak disturbances and time delays has received growing attention. In Fridman and Shaked (2003), bounded and time-varying delays were considered in continuous-time linear systems, and the Lyapunov–Razumikhin method was applied to determine the bounding ellipsoids. In order to improve the results given in Fridman and Shaked (2003), the Lyapunov–Krasovskii functional method was developed in Kim (2008), where delay-dependent conditions were presented in terms of matrix inequalities involving only one non-convex parameter. In Zuo, Ho, and Wang (2010), a maximal Lyapunov–Krasovskii functional method was developed for the reachable set estimation of polytopic uncertain systems. When the lower bound of the delay is not zero, a delay decomposition strategy was applied to estimate the reachable set in Nam and Pathirana (2011). In addition, the reachable set bounding problem of discrete-time linear systems was studied in Lam, Zhang, Chen, and Xu (2015) and That, Nam, and Ha (2013). In Chen and Lam (2015) and Feng and Lam (2015), the reachable set estimation problem was considered for singular systems and periodic systems. Besides the reachable set, in Nam, Pathirana, and Trinh (2014), the authors investigated the convergence property of time-delay systems with bounded disturbances under non-zero initial conditions.

On the other hand, it is necessary to consider the control synthesis problem for meeting the design requirements related to the reachable set. Two problems are worth considering in this aspect. The first one is to design a controller such that the reachable set of the closed-loop system is contained in a given ellipsoid. In practice, the given ellipsoid may be imposed for safety reasons or due to other special requirements. The second problem is to design a controller such that the reachable set of the closed-loop system is contained in an ellipsoid that is as small as possible. In Zhang, Lam, and Xu (2014), the controller synthesis problem for distributed delay systems has been studied. However, to the best of the authors' knowledge, the reachable set estimation and controller design problems have not been considered for systems with arbitrary switching. This paper aims to solve this open problem by employing multiple Lyapunov functions.

The organization of this paper is as follows. Section 2 gives the problem formulation. The main results of the reachable set estimation and controller design for discrete-time switched linear systems are presented in Section 3. In this section, the ellipsoidal reachable set estimation conditions are first presented. In order to make the bounding ellipsoids as small as possible, three optimization approaches are developed. Genetic Algorithm is adopted to obtain the optimal parameters in the reachable set estimation conditions. In addition, the state-feedback controller design problem is also studied in this section. Some numerical examples are given in Section 4 to verify the obtained results. Finally, Section 5 gives the conclusions of this paper.

Notations: The superscript "T" represents the matrix transposition, $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. In symmetric block matrices, the notation "$*$" is used as an ellipsis for terms that are induced by symmetry. The notation $P > 0 (P \geq 0)$ indicates that P is a real symmetric and positive (semi-) definite matrix. The identity matrix and zero matrix are represented by I and $\mathbf{0}$ respectively. If their dimensions are not stated explicitly, matrices are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the following discrete-time switched linear system:

$$x_{k+1} = A_{\sigma(k)}x_k + B_{\omega,\sigma(k)}\omega_k, \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state vector, switching signal $\sigma(k)$ is a piecewise constant function of the time and it takes its value in the set $\mathcal{I} = \{1, \ldots, N\}$, $N \geq 1$ is the number of subsystems. A_i, and $B_{\omega,i}$, $i \in \mathcal{I}$ are constant system matrices with appropriate dimensions. $\omega_k \in \mathbb{R}^{n_\omega}$ is the bounded peak disturbance vector satisfying

$$\omega_k^T\omega_k \leq \overline{\omega}^2, \quad \forall k \geq 0, \tag{2}$$

$\overline{\omega} > 0$ is a scalar.

Remark 1. It should be pointed out that ω_k is the exogenous disturbance, the peak of ω_k is normally independent of switching signal $\sigma(k)$ in practice. However, even if in the case that the peak of ω_k is dependent on the switching signal $\sigma(k)$, that is; $\omega_{\sigma(k),k}^T\omega_{\sigma(k),k} \leq \overline{\omega}_{\sigma(k)}^2$, $\forall k \geq 0$, where $\overline{\omega}_i > 0$, $i \in \mathcal{I}$ are scalars. We can make the exogenous disturbance independent of the switching signal through the following transformation. Let $w_k = \omega_{\sigma(k),k}\overline{\omega}/\overline{\omega}_{\sigma(k)}$ and $\overline{B}_{\omega,\sigma(k)} = B_{\omega,\sigma(k)}\overline{\omega}_{\sigma(k)}/\overline{\omega}$, then system (1) can be rewritten as $x_{k+1} = A_{\sigma(k)}x_k + \overline{B}_{\omega,\sigma(k)}w_k$, where $w_k^Tw_k \leq \overline{\omega}^2$, $\forall k \geq 0$. Thus without loss of generality, we assume that the peak of the exogenous disturbance ω_k is independent of the switching signal $\sigma(k)$.

The aim of the reachable set estimation problem is to determine a region as small as possible to bound the reachable set of system (1) under bounded peak disturbances satisfying (2). The concerned reachable set is defined as

$$\Re_x \triangleq \{x_k \mid x_0 = 0,\ x_k,\ \omega_k \text{ satisfy } (1), (2), k \geq 0\}. \tag{3}$$

It is well known that in the system analysis and synthesis of switched systems, common quadratic Lyapunov function method will lead to a certain level of conservatism. In order to reduce the conservatism, multiple Lyapunov function method, which is a type of non-quadratic Lyapunov function method, is widely used in the system analysis and synthesis of switched systems. In our work, the multiple Lyapunov function method is adopted to solve the reachable set estimation and controller design problems for switched systems. The reachable set is bounded within some ellipsoids. We make use of the bounding ellipsoid in the following form:

$$E(P) \triangleq \left\{x \in \mathbb{R}^{n_x} \mid x^TPx \leq 1,\ P > 0\right\}. \tag{4}$$

Lemma 1 is a basic reachable set estimation tool for discrete-time switched systems and it will be used in later development.

Lemma 1. *Consider system (1) under bounded peak disturbance (2). Let $\{V_i(x_k),\ i \in \mathcal{I}\}$ be a set of Lyapunov functions satisfying $V_i(0) = 0$ and $V_i(x_k) > 0$, $\forall x_k \neq 0$, $i \in \mathcal{I}$. If there exist scalars $0 < \alpha_{j,i} < 1$ such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$,*

$$V_j(x_{k+1}) - \alpha_{j,i}V_i(x_k) - \frac{1-\alpha_{j,i}}{\overline{\omega}^2}\omega_k^T\omega_k \leq 0, \tag{5}$$

then system (1) is globally uniformly asymptotically stable (GUAS) and we have $V_i(x_k) \leq 1$, $\forall i \in \mathcal{I}$ for all x_0 satisfying $V_i(x_0) \leq 1$, $\forall i \in \mathcal{I}$.

Proof. On one hand, under non-zero initial conditions, condition (5) guarantees the global uniform asymptotic stability of system (1). Specifically, when $\omega_k = 0$, condition (5) can be rewritten as

$$V_j(x_{k+1}) - \alpha_{j,i}V_i(x_k) \leq 0, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I}. \tag{6}$$

Noting that $0 < \alpha_{j,i} < 1$, then we have

$$V_j(x_{k+1}) - V_i(x_k) \leq -\left(1 - \alpha_{j,i}\right)V_i(x_k) < 0, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I}, \tag{7}$$

for $x_k \neq 0$, which implies that system (1) is GUAS (Daafouz et al., 2002).