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Brief paper

Some necessary and sufficient conditions for consensus of second-order multi-agent systems with sampled position data[☆]

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ABSTRACT

A novel distributed consensus protocol, where only causal sampled position data are used, is firstly designed for second-order linear multi-agent systems with a directed communication topology. In this context, a necessary and sufficient condition depending upon the coupling gains, sampling period, and spectrum of the Laplacian matrix, is established for achieving consensus. It is revealed that second-order consensus in such a multi-agent system cannot be reached without using past sampled position data. It is also found that a relatively small sampling period does not necessarily improve the consensus performance. Then, a delay-induced consensus protocol is proposed based on sampled position data and with the help of time delay. It is found that consensus under this designed protocol cannot be reached in the absence of time delay. More interestingly, the time delay should have both lower and upper bounds in order to achieve consensus. Finally, the effectiveness of the theoretical results is demonstrated through numerical simulations.

1. Introduction

Consensus refers to a group of agents under appropriate distributed control reaching an agreement on certain quantities of interest. It has been shown that a group of autonomous agents connected by a communication or sensing network can coordinate with their neighbors to perform some challenging tasks that cannot be accomplished otherwise. In the last decade, consensus via distributed cooperative control of multi-agent systems has received compelling attention (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, Fax, & Murray, 2007; Olfati-Saber & Murray, 2004; Ren & Beard, 2005, 2008). In classification, according to the absence or presence of a leader, consensus can be divided into leaderless consensus (Fan, Feng, Wang, & Song, 2013; Li, Ren, Liu, & Xie, 2013; Mei, Ren, & Ma, 2013; Wen, Duan, Yu, & Chen, 2012; You, Li, & Xie, 2013; Yu, Chen, & Cao, 2010) and consensus tracking (or leader-following consensus) (Hong, Chen, & Bushnell, 2008; Hong, Hu, & Cao, 2006; Hu, 2012; Huang, Duan, & Zhao, 2014; Song, Cao, & Yu, 2010; Song, Liu, Cao, & Yu, 2013).

Spurred by the pioneering works on the first-order consensus problem (Jadbabaie et al., 2003; Olfati-Saber & Murray, 2004), consensus has been extensively studied from various perspectives. In Ren and Beard (2005), a distributed consensus scheme was proposed for multiple vehicle systems, and it was found that first-order consensus can be reached if and only if the dynamically changing interaction topology has a directed spanning tree frequently enough as the system evolves. Moreover, the first-order consensus problem was investigated based on event-triggered control in the regard of reducing communication burden (Fan et al., 2013). Recently, the second-order consensus problem has come to receiving particular attention, where the second-order dynamics are governed by the position and velocity terms of the agents. A necessary and sufficient condition was proved for this case (Yu et al., 2010), which shows that both real and imaginary parts of the eigenvalues of the network Laplacian matrix are crucial for achieving consensus. Subsequently, many researchers have focused on investigating second-order consensus in multi-agent systems with sampled data, due to the fact that sampled-data control has many good properties, such as robustness with low cost. By using

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zero-order holds or direct discretization, some necessary and sufficient conditions were established for multi-agent systems with sampled-data control (Cao & Ren, 2010; Gao, Wang, Xie, & Wu, 2009; Liu, Xie, & Wang, 2010; Ma, Xu, & Lewis, 2014). On the other hand, for second-order continuous-time multi-agent systems, the consensus problem under a time-varying topology and sampled-data control was addressed in Gao and Wang (2011), with a necessary and sufficient condition developed for reaching consensus using sampled data (Yu, Zhou, Yu, Lü, & Lu, 2013).

In many real-world applications, the relative velocities of neighboring agents are more difficult to measure as compared to the relative positions (Hong et al., 2008, 2006). In general, a camera can be used for relative position measurements while more expensive sensors are required for relative velocity measurements (Yu, Chen, Cao, & Ren, 2013). Under this circumstance, a common solution is designing observers at the price of having some additional variables, which leads to the study of higher-order dynamical systems (Hong et al., 2008, 2006; Ren, 2008). Furthermore, a filter-based method was introduced for the second-order consensus problem with unavailable velocity measurements (Mei et al., 2013), where additional variables were also involved. Without designing observers or filters, a delay-induced method was proposed for second-order multi-agent systems in Yu, Chen et al. (2013), where a consensus protocol was designed based on both current and delayed position states. It was surprisingly found that consensus cannot be reached without delayed position information under the given protocol while it can be achieved with a relatively small time delay by appropriately choosing the coupling strength. However, all the position states in a certain time interval have to be kept in memory in the delay-induced consensus algorithm established in Yu, Chen et al. (2013). To overcome this shortage, an effective solution was developed in Yu, Zheng, Chen, Ren, and Cao (2011), where a distributed linear consensus protocol with second-order dynamics was designed according to the current and some sampled past position data. In this context, the position states only require to be kept in memory at some particular time instants. In other words, the controller design utilizes less information and saves more energy. It is noteworthy that the required current information is usually unavailable and, therefore, more sampled data without current ones were introduced into the protocol in Yu, Zhou et al. (2013). In other words, both sampled position and velocity data were utilized in the algorithmic design in Yu, Zhou et al. (2013). Note that the above consensus protocols are designed with or without using velocity measurements. However, when the velocity information is unavailable, position measurements are needed in real time. This inspires us to consider whether second-order consensus can still be achieved by using only past sampled position data for protocol design, even if both the velocity and real-time position data are unavailable.

The main contribution of this paper can be summarized as follows. (i) Two novel distributed consensus protocols are designed for multi-agent systems with second-order dynamics, which do not involve the velocity information as well as the current position information of the agents but can take full advantage of the sampled position data. (ii) A necessary and sufficient condition is established for achieving consensus under a directed communication topology. It is found that consensus with only causal sampled position information can be reached if and only if the sampling period is bounded by two positive critical values, as well as the coupling strengths are appropriately chosen. Compared with the recent results on reaching second-order consensus by using causal sampled position data in Ma et al. (2014), the proposed conditions can give the tolerance interval of the sampling period and provide guidelines for how to choose the coupling gains. (iii) A detailed discussion about how to choose the coupling gains and the sampling period is given for adequately reducing communication cost and saving energy. (iv) A necessary and sufficient condition is established for achieving consensus by utilizing only sampled position data and with the help of time delay. It is interesting to see that the time delay is necessary and lower-bounded by a critical value for reaching consensus. The proposed delay-induced protocol extends the continuous and undirected strategy in Yu, Chen et al. (2013) to the sampled and directed case. It should be noted that usually the latter is more practical and easier to implement. In either case, the sampling period subject to thresholds should be neither too small nor too large in order to achieve ideal consensus performance under the cooperative mechanisms proposed in this paper.

The remainder of this paper is organized as follows. Some preliminaries and the model formulation are given in Section 2. The main results are presented and proved in Section 3. Several simulation results are presented to verify the theoretical results in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries

In this section, some important preliminaries and the model formulation are introduced.

2.1. Notations and useful lemmas

Let $\mathbb{R}$ and $\mathbb{N}$ be the sets of real and natural numbers, respectively. Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ be the n-dimensional real vector space and $n \times n$ real matrix space, respectively. X^T denotes the transpose of matrix X. $\otimes$ denotes the Kronecker product. For a vector x, $\|x\|$ indicates its Euclidean norm. For a complex number y, $\|y\|$ denotes its modulus, and $\Re(y)$ and $\Im(y)$ represent its real part and imaginary part, respectively.

Consider a network consisting of N agents. Let a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ describe the communication topology among the agents, referred to as nodes, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix with non-negative elements $a_{ij},\ i, j \in \mathcal{V}$. An edge is denoted by an ordered pair of nodes (j, i) in $\mathcal{G}$ corresponding to an edge between agent i and agent j, which means that agent i can obtain information from agent j. As usual, self-loops are not allowed, i.e., $a_{ii} = 0$ for all $i \in \mathcal{V}$, and $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$. For an undirected graph, $a_{ij} = a_{ji}$ with (i, j) being an unordered pair, that is, agent i and agent j can exchange information. A path between node j and node i is a sequence of edges in the form $(j, j_{k1}), (j_{k1}, j_{k2}), \dots, (j_{kl}, i)$, with distinct nodes $j_{km} \in \mathcal{V}$, $m = 1, \dots, l$. An undirected graph is called connected if and only if there is a path between any pair of different nodes. A tree is a connected subgraph without loops (closed paths). A directed spanning tree is a directed tree consisting of all the nodes in $\mathcal{G}$, with exactly one node called root which has a directed path to every other node. The Laplacian matrix of the graph $\mathcal{G}$ associated with the adjacency matrix $\mathcal{A}$ is defined by $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij},\ i \neq j$, which satisfies that $\sum_{j=1}^{N} l_{ij} = 0$.

Lemma 1 (Ren & Beard, 2005). *A Laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 and all the other eigenvalues are positive if and only if the undirected network is connected. Also, a Laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 and all the other eigenvalues have positive real parts if and only if the directed network has a directed spanning tree.*

A polynomial is said to be stable if all its roots have negative real parts.

Lemma 2 (Duan, Chen, & Huang, 2008, Parks & Hahn, 1992). *Given a complex coefficient polynomial of order three in the form of $f(s) = s^3 + (p_2 + jq_2)s^2 + (p_1 + jq_1)s + p_0 + jq_0$. Then, $f(s)$ is stable if and only if $p_2 > 0$, $p_2q_2q_1 + p_2^2p_1 - q_1^2 - p_2p_0 > 0$, and*