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Brief paper

A robust estimator for stochastic systems under unknown persistent excitation ☆

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ABSTRACT

A robust estimator for uncertain stochastic systems under unknown persistent disturbance is presented. The given discrete-time stochastic formulation neither requires a known bound on the magnitude of the unknown excitation nor assumes stability of the system. However, the proposed estimator assumes certain structural conditions on system uncertainties. Though the proposed estimator is developed based on stochastic Lyapunov analysis, its structure and performance are comparable to that of unbiased minimum-variance filters based on the disturbance decoupling technique. Unlike unbiased minimum-variance filters, implementation of the developed estimator only requires adding an auxiliary term to the nominal steady-state Kalman filter, and it does not involve any similarity transformation or propagation of matrix difference equations.

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1. Introduction

The robust estimation problem involves recovering unmeasured state variables when the available plant model and the measurement equation are uncertain. Four main approaches exist to deal with the robust Kalman filter problem and they are based on (i) H_∞ filtering, (ii) set-valued estimation, (iii) guaranteed-cost filtering, and (iv) regularized least-squares. All these approaches are compared in Sayed (2001), where relevant concerns on parameterizations, stability, robustness, and online implementation of each approach are addressed. In H_∞ filtering, estimators are designed to minimize the worst-case H_∞ norm of the transfer function from the noise inputs to the estimation error output (Bernstein, Haddad, & Mustafa, 1991; Fu, de Souza, & Xie, 1992; Wang & Unbehauen, 1999). Since H_∞ filtering is a worst-case design method, while guaranteeing the worst-case performance, it generally sacrifices the average filter performance. A robust state estimator for a class of uncertain systems where the noise and uncertainty are modeled deterministically via an integral quadratic constraint is presented in Savkin and Petersen (1995). This approach, known as set-valued state estimation (Bertsekas & Rhodes, 1971), involves finding the set of all states consistent with given output measurements for a system with norm-bounded noise input (James & Petersen, 1998). The robust estimation approach known as guaranteed-cost filtering is presented in Petersen and McFarlane (1994) and Xie, Soh, and de Souza (1994). While the H_∞ formulation and guaranteed-cost filtering involve de-regularization, a robust estimator design based on the regularized least-squares approach is presented in Sayed (2001). Petersen and Savkin provide a comprehensive research monograph on robust filtering for both discrete and continuous time systems from a deterministic as well as H_∞ point of view (Petersen & Savkin, 1999).

An alternative approach to robust estimator design consists of representing the modeling errors and the unknown external disturbances as unknown inputs and using the disturbance decoupling principle to render an estimator that is immune to the unknown inputs. In the deterministic context this problem has been extensively studied, and the conditions for the stability of the obtained observers, commonly known as *unknown input observers* (UIOs), are well known (Charandabi & Marquez, 2014; Darouach, Zasadzinski, & Xu, 1994; Hou & Muller, 1994; Kudva, Viswanadham, & Ramakrishna, 1980; Sundaram & Hadjicostis, 2007, 2008). However, compared to the deterministic counterpart, fewer results are obtained for stochastic systems with unknown inputs. In Kitanidis (1987), Kitanidis introduced an unbiased minimum-variance estimator for determining mean areal precipitation in the presence of unknown inputs. This estimator, which later came to be known as the Kitanidis filter, required that the filter gain is selected such that the typical disturbance decoupling structural condition is satisfied and the estimation error covariance is minimized.

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While the Kitanidis filter uses a single gain to satisfy both the structural condition and the minimum-variance condition, Darouach and Zasadzinski relax these design criteria by dividing the estimator gain into two separate terms, one satisfying the structural condition and canceling the unknown inputs, and the other ensuring the minimum-variance criterion (Darouach & Zasadzinski, 1997). Global optimality of the estimators given in Kitanidis (1987) and Darouach and Zasadzinski (1997) is proved in Kerwin and Prince (2000). The robust two-stage Kalman filter given in Hsieh (2000) further extends the Kitanidis filter by estimating the unknown inputs after assuming its dynamics.

Unlike the UIO-based filters, most of the robust Kalman filter schemes only consider minor system parametric uncertainties or asymptotically decaying disturbance. On the other hand, the unbiased minimum-variance filters have shown to yield desirable performance, but they require discarding the nominal estimator and designing a brand-new estimator. Also, the UIO-based unbiased minimum-variance filters often require the propagation of a matrix difference equation, access to delayed measurements, and either a similarity transformation or an extension of the estimator dimensionality. Therefore, we propose a robust estimator that can guarantee asymptotically unbiased estimates and exponentially bounded mean-square error in the presence of persistently exciting external disturbances. The given formulation neither requires a known upper bound on unknown excitation nor assumes the stability of the unknown system. Performance of the proposed estimator does not depend on any design parameters and is consistent across all admissible system uncertainties and external disturbances. The proposed filter builds on a nominal estimator and it only requires the addition of an auxiliary-input to the nominal estimator to account for the unknown inputs. Based on the stochastic Lyapunov stability analysis, we present a systematic approach for selecting the auxiliary-inputs such that the estimation error is exponentially mean-square bounded and it asymptotically tracks a desired optimal error. However, similar to the UIO-based filters, the proposed estimator uses a linear image of the system measurements to fully cancel the effects of unknown inputs. Thus, the implementation of the proposed estimator requires that the system satisfies a structural condition as well as a detectability condition. We show that the above conditions can be easily checked using a linear matrix inequality (LMI) and then examines their relation to the traditional disturbance decoupling structural condition as well as the detectability condition of the disturbance decoupled system.

The structure of this paper is as follows. Formulation of the problem and the development of a nominal estimator are first given in Sections 2 and 3, respectively. Afterwards, development of the proposed robust estimator based on stochastic Lyapunov stability analysis is presented in Section 4. Generalization of the developed robust estimator and its relation to UIO-based filters are given in Section 5. Finally, concluding remarks are given in Section 6.

2. Problem formulation

Consider a stochastic system of the following form:

$$\begin{aligned} \mathbf{z}(k+1) &= F\,\mathbf{z}(k) + \Gamma\,\boldsymbol{\mu}(k) + G\,\boldsymbol{\omega}(k), \\ \mathbf{y}(k) &= H\mathbf{z}(k) + \boldsymbol{\nu}(k). \end{aligned} \tag{1}$$

Here $\mathbf{z}(k) \in \mathbb{R}^n$ is the state vector, $\mathbf{y}(k) \in \mathbb{R}^m$ is the output vector, $\boldsymbol{\mu}(k) \in \mathbb{R}^r$ is the unknown excitation, and $\boldsymbol{\omega}(k)$ is an n-dimensional Gaussian white process noise sequence, i.e., $\boldsymbol{\omega}(k) \sim \mathcal{N}(\mathbf{0}, Q)$, while $\boldsymbol{\nu}(k)$ is an m-dimensional Gaussian white measurement noise sequence, i.e., $\boldsymbol{\nu}(k) \sim \mathcal{N}(\mathbf{0}, R)$.

Since all allowable system uncertainties can be lumped into the unknown input, $\boldsymbol{\mu}(k)$, here we assume that the elements of the system matrices F, H, Γ, and G are considered as exactly known. Furthermore, the noise covariances $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are assumed known. We also assume without loss of generality that the matrix Γ is full column rank. This assumption can always be satisfied by an appropriate transformation and renaming of the unknown input signals.

Remark 1. The allowable system uncertainties are restricted by the space spanned by the columns of Γ. For example, any uncertainty in matrix F that is not in the range-space of Γ is not an allowed system uncertainty.

Though the existence of the proposed estimator requires some structural conditions on H and Γ, it neither requires any bounds on unknown excitations nor assumes stability of F. Indeed, the performance guarantee of the proposed estimator holds even if the system matrix F is unstable and $\boldsymbol{\mu}(k)$ is unbounded. At this point, the only *a priori* system assumption is

Assumption 1. The n-dimensional pair (F, H) is detectable.

Detectability of (F, H) is a weak assumption, and it is required for the design on a nominal estimator even when there are no unknown inputs, i.e., when $\boldsymbol{\mu}(k) = \mathbf{0}$.

3. Nominal estimator

Consider a nominal system of the following form:

$$\begin{aligned} \mathbf{z}_m(k+1) &= F\mathbf{z}_m(k) + G\boldsymbol{\omega}(k), \\ \mathbf{y}_m(k) &= H\mathbf{z}_m(k) + \boldsymbol{\nu}(k). \end{aligned} \tag{2}$$

For the system in (2), an optimal estimator such as a steady-state Kalman filter of the following form may be implemented:

$$\hat{\mathbf{z}}_m(k+1) = F\hat{\mathbf{z}}_m(k) + K_{ss}\left[\mathbf{y}_m(k+1) - HF\hat{\mathbf{z}}_m(k)\right], \tag{3}$$

where $K_{ss} = P_{ss}H^T\left(HP_{ss}H^T + R\right)^{-1}$ is the observer gain and the steady-state error covariance, P_{ss}, can be obtained by solving the discrete-time algebraic Riccati equation:

$$P_{ss} = FP_{ss}F^T + GQG^T - FP_{ss}H^T\left[HP_{ss}H^T + R\right]^{-1}HP_{ss}F^T. \tag{4}$$

Now the corresponding estimator error dynamics may be written as

$$\tilde{\mathbf{z}}_m(k+1) = F\tilde{\mathbf{z}}_m(k) + G\boldsymbol{\omega}(k) - K_{ss}\left[\mathbf{y}_m(k+1) - HF\hat{\mathbf{z}}_m(k)\right], \tag{5}$$

where $\tilde{\mathbf{z}}_m(k) = \mathbf{z}_m(k) - \hat{\mathbf{z}}_m(k)$. It can be shown that $\tilde{\mathbf{z}}_m(k)$ is unbiased and the steady-state value of the error covariance is given as Jazwinski (1998)

$$\lim_{k\to\infty} E\left[\tilde{\mathbf{z}}_m(k)\tilde{\mathbf{z}}_m^T(k)\right] = P_{ss}. \tag{6}$$

Remark 2. Note that if the unknown inputs are fully known, then the optimal estimator is the Kalman filter and its steady-state performance would be comparable to that of the nominal estimator in (3). Therefore, we adopt the nominal estimator error dynamics in (5) as the desired estimator performance, and the goal of the proposed design methodology is to introduce an extra term to the nominal estimator such that we asymptotically recover the desired performance. This is done at the cost of introducing additional noise into the system and increasing the overall estimation error covariance.

4. Robust estimator formulation

Here we propose a robust estimator of the following form:

$$\hat{\mathbf{z}}(k+1) = F\hat{\mathbf{z}}(k) + K_{ss}\left[\mathbf{y}(k+1) - HF\hat{\mathbf{z}}(k)\right] + \odot, \tag{7}$$

where $\odot$ is an auxiliary input signal that we will design to account for the unknown input. Define $\tilde{\mathbf{z}}(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k)$. Now the