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Brief paper

Design of interval observer for a class of uncertain unobservable nonlinear systems ☆

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ABSTRACT

This paper investigates the interval observer design for a class of nonlinear continuous systems, which can be represented as a superposition of a uniformly observable nominal subsystem with a Lipschitz nonlinear perturbation. It is shown in this case there exists an interval observer for the system that estimates the set of admissible values for the state consistent with the output measurements. An illustrative example of the observer application is given with simulation results.

1. Introduction

The state estimation problem of uncertain nonlinear systems is studied in this work. In particular we are interested in the case when the model is nonlinear parameterized by a vector of unknown parameters θ and the model equations do not belong to a canonical form. Usually in such a case it is necessary to apply a transformation of coordinates representing the system in a canonical form with posterior design of an observer (Besançon, 2007; Nijmeijer & Fossen, 1999). The presence of unknown parameters may seriously complicate the design of a required transformation of coordinates, since the transformation has to be dependent on θ. In this case the initial problem of the state estimation can be replaced with a relaxed one dealing with approximation of the interval of admissible values of the state vector.

Suppose that the unknown (may be time-varying) parameters θ belong to a compact set $\Theta \subset \mathbb{R}^p$, then the plant dynamics under consideration is given by

$$\begin{cases} \dot{x} = f(x) + B(x,\theta)u + \delta f(x,\theta), \\ y = h(x) + \delta h(x,\theta), \end{cases} \tag{1}$$

where x belongs to an open subset Ω of $\mathbb{R}^n$ (it is assumed that $0 \in \Omega$) and the initial state value belongs to a compact set $I_0(x_0) = [\underline{x}_0, \overline{x}_0]$; $y \in \mathbb{R}$ and $u \in \mathbb{R}^m$ represent respectively the output and the input. The vector fields f and h are smooth, and δf, δh and B are assumed to be locally Lipschitz continuous.

Despite of the existence of many solutions for observer design (Besançon, 2007; Nijmeijer & Fossen, 1999), a design of state estimators for (1) is rather complicated since the system is intrinsically nonlinear and it has uncertain terms in the state and in the output equations. Therefore, the whole system (1) may be even not observable, which means that an exact estimation is not possible. Under this situation, we can relax the estimation goal making an evaluation of the interval of admissible values for the state applying the theory of set-membership or interval estimation (Gouzé, Rapaport, & Hadj-Sadok, 2000; Mazenc & Bernard, 2010; Walter, Norton, Piet-Lahanier, & Milanese, 1996). Contrarily to the conventional case, where a pointwise value of the state is the objective for estimation, in the interval estimation two bounds on the set of admissible values are calculated and the width of the estimated interval is dependent of the model uncertainty.

Recently the interval observers have been proposed for a special class of nonlinear systems (Raïssi, Efimov, & Zolghadri, 2012), the model (1) is a generalization of that case. Applying a coordinate transformation to a canonical form computed for the known nominal system, we are going to estimate the interval value of the state of the uncertain system (1) improving the result

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from Raïssi et al. (2012). Another solution has been presented in Meslem and Ramdani (2011), where a hybrid interval observer design is presented for a class of continuous-time nonlinear systems. In the present work we are going to avoid the complexity of the hybrid system framework developing a continuous-time interval observer. For upper-triangular systems, an iterative design procedure for robust interval observers is proposed in Mazenc and Bernard (2012), which is started from the assumption that for each subsystem a robust interval observer has been designed. The result of this paper is an extension of our recent work in Zheng, Efimov, and Perruquetti (2013), and can be considered as a complementary method for such an observer syntheses for a nonlinear system. Comparing with the existing results in the literature, the present paper considers an interval observer design for more general uncertain nonlinear systems, which may be not observable. When an exact estimation for such systems becomes impossible, the main contribution of this paper is to present a method to obtain an interval estimation.

The outline of this paper is as follows. Some preliminary results and notations are given in Section 2. The precise problem formulation is presented in Section 3. The main results are described in Section 4. An example of computer simulation is given in Section 5.

2. Preliminaries

2.1. Notations

- $\mathbb{R}$ denotes the set of real numbers and $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$
- $L_f h(x) = \frac{\partial}{\partial x} h(x).f(x)$ denotes the Lie derivative of h along the vector field f, and $L_f^n h = L_f(L_f^{n-1} h)$ is the nth Lie derivative of h along the vector field f
- $a\mathcal{R}b$ represents the element-wise relation $\mathcal{R}$ (a and b are vectors or matrices): for example $a < b$ (vectors) means $\forall i$: $a_i < b_i$
- for a matrix $P = P^T$, the relation $P \preceq 0$ means that the matrix is negative semidefinite
- for a matrix $\mathcal{A} \in \mathbb{R}^{n\times n}$, define $\mathcal{A}^+ = \max\{0, \mathcal{A}\}$[1] and $\mathcal{A}^- = \mathcal{A}^+ - \mathcal{A}$. For a vector $x \in \mathbb{R}^n$, define $x^+ = \max\{0, x\}$ and $x^- = x^+ - x$
- for a matrix (function) $\mathcal{A}$ the symbol $\mathcal{A}_i$ denotes its ith column, for a vector (function) b the symbol b_i denotes its corresponding element
- a matrix $\mathcal{A} \in \mathbb{R}^{n\times n}$ is called Metzler if all its elements outside the main diagonal are nonnegative.
- a Lebesgue measurable function $u : \mathbb{R}_+ \to \mathbb{R}^m$ belongs to the space $\mathcal{L}_\infty$ if $ess\,\sup_{t\geq 0} \|u(t)\| < +\infty$.

2.2. Backgrounds on cooperative/comparison systems

The notions of Comparison systems and Cooperative systems have appeared separately, but they concern the same class of systems:

- *Comparison systems*: when dealing with a qualitative property involving solutions of a complex system, it is sometimes of interest to obtain a simpler system whose solutions overvalue the solutions of the initial system in some sense. For ODE (Ordinary Differential Equation), the contributions of Kamke (1932), Müller (1926) and Wazewski (1950) are probably the most important in this field: they give necessary and sufficient hypotheses ensuring that the solution of $\dot{x} = f(t, x)$, with initial state x_0 at time t_0 and function f satisfying the inequality $f(t, x) \leq g(t, x)$ is overvalued by the solution of the so-called "comparison system" $\dot{z} = g(t, z)$, with initial state $z_0 \geq x_0$ at time t_0, or, in other words, conditions on function g that ensure $x(t) \leq z(t)$ for $t \geq t_0$. These results were extended to many different classes of dynamical systems (Bitsoris, 1978; Dambrine, 1994; Dambrine, Goubet, & Richard, 1995; Dambrine & Richard, 1993, 1994; Grujič, Martynyuk, & Ribbens-Pavella, 1987; Laksmikantham & Leela, 1969; Matrosov, 1971; Perruquetti, Richard, & Borne, 1995; Perruquetti, Richard, Grujić, & Borne, 1995; Tokumaru, Adachi, & Amemiya, 1975).
- *Cooperative systems*: this class of systems includes those involving in $\mathbb{R}^n$ preserving positive order relation on initial data and input signals Smith (1995), *i.e.* if the initial conditions and properly rescaled inputs are positive, then so is the corresponding solution.

From these results one can deduce the following corollary:

Corollary 1 (*Smith, 1995*). *Assume that:*

(H1) *A is a Metzler matrix,*

(H2) $b(t) \in \mathbb{R}_+^n$, $\forall t \geq t_0$, *where t_0 represents the initial time,*

(H3) *the system*

$$\frac{dx(t)}{dt} = Ax + b(t), \tag{2}$$

possesses, for every $x(t_0) \in \mathbb{R}_+^n$, a unique solution $x(t)$ for all $t \geq t_0$.

Then, for any $x(t_0) \in \mathbb{R}_+^n$, the inequality

$$x(t) \geq 0$$

holds for every $t \geq t_0$.

In other word, under conditions of Corollary 1, $\mathbb{R}_+^n$ is positively invariant w.r.t (2).

2.3. Canonical representation of a nonlinear system

Based on the studied system (1), one obtains the *nominal drift-system* by setting $u = 0$, $\delta f = 0$, $\delta h = 0$ in (1):

$$\begin{cases} \dot{x} = f(x), \\ y = h(x). \end{cases} \tag{3}$$

For a nonlinear system, "observability" depends on the considered state (local property) and control: this is the main reason why many different concepts related to observability exists (Besançon, 2007; Gauthier, Hammouri, & Othman, 1992; Nijmeijer & Fossen, 1999). This paper assumes that the nominal system (3) satisfies the observability rank condition, *i.e.* the following change of coordinates:

$$\Phi_{(3)} : \mathbb{R}^n \to \mathbb{R}^n$$

$$x \mapsto \left(h(x), L_f h(x), \ldots, L_f^{n-1} h(x)\right)^T, \tag{4}$$

defines a local diffeomorphism from Ω onto $\Phi_{(3)}(\Omega)$. With this diffeomorphism $\zeta = \Phi_{(3)}(x)$, it follows that, the system (3) can be rewritten as:

$$\begin{cases} \dot{\zeta} = \tilde{A}\zeta + \tilde{b}\varphi(\zeta), \\ y = \tilde{C}\zeta, \end{cases} \tag{5}$$

where

$$\tilde{A} = \begin{pmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & 0 \end{pmatrix}, \tag{6}$$

[1] The $\max\{\cdot\}$ operation is applied element-wise.