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Brief paper

Repetitive learning position control for full order model permanent magnet step motors[☆]

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ABSTRACT

We provide a novel theoretical solution to the yet unsolved problem of tracking, via state feedback, periodic reference signals (with known period) for the rotor position of full order model uncertain permanent magnet step motors with non-sinusoidal flux distribution and uncertain position-dependent load torque. The resulting control is of simple structure and incorporates three repetitive learning estimation schemes generalizing the classical integral actions. Realistic simulation results illustrate the effectiveness of the proposed approach.

1. Introduction

Permanent magnet motors replace DC motors in a wide range of drive applications (machine tools and industrial robots): high efficiency, high torque to inertia ratio, high power density, absence of rotor windings, absence of external rotor excitation are definite advantages. However, the high-precision position tracking control problem for permanent magnet motors turns to be rather difficult to be solved. This is due to the non-sinusoidal flux distribution in the air-gap, which causes speed oscillations (ripples) and deteriorates the system performance especially at low speeds. Even though improvements in motor design can be effective in ripple minimization (Petrović, Ortega, Stanković, & Tadmor, 2000), both production process complexity and machine costs increase. Compensation of torque pulsations by feedback actions thus becomes an attractive solution (Jahns & Soong, 1996).

In the case of periodic position reference signals with known period T_*, the undesirable disturbances become periodic with the same period T_*. Consequently, learning control techniques can be successfully used to reduce the position tracking error. In this context, adaptive learning controls (see Sencer & Shamoto, 2014 for an adaptive/sliding mode approach) have been presented in Marino, Tomei, and Verrelli (2008) for current-fed motors (see experimental analyses and comparisons in Bifaretti, Iacovone, Rocchi, Tomei, & Verrelli, 2011) and extended in Marino, Tomei, and Verrelli (2012b) to voltage-fed motors. Exponential convergence of the position tracking error to an arbitrarily small residual set (containing the origin) is achieved, even though the estimation of a possibly large number of Fourier coefficients may be involved in the approximation of the uncertain reference input. On the other hand, iterative/repetitive learning controls (see Ahn, Chen, & Moore, 2007; Dixon, Zergeroglu, Dawson, & Costic, 2002; Xu, 2004; Xu & Tan, 2003 for the fundamental ideas) have been proposed in Bifaretti, Tomei, and Verrelli (2011) and Chen, Yung, and Cheng (2006) (see also Luo, Chen, Ahn, & Pi, 2011; Luo, Chen, & Pi, 2010 for a space-learning control design minimizing cogging effects and Betin, Pinchon, & Capolino, 2000; Holtz, 1996; Mohamed, 2007; Qian, Panda, & Xu, 2004, 2005; Tsui, Cheung, & Yuen, 2009; Xu, Panda, Pan, Lee, & Lam, 2004 for experimental applications of standard iterative learning control techniques to torque and speed control in permanent magnet synchronous motors). Even though asymptotic position tracking is guaranteed, their design is however restricted to a simplified current-fed motor model, which constitutes a second order system with matching uncertainties.

In this paper, novel repetitive learning control techniques (see Marino, Tomei, & Verrelli, 2012a; Tomei & Verrelli, 2015 for recent advances, even though they do not apply to the nonlinear system in exam) are used to innovatively generalize the result in

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Bifaretti, Tomei et al. (2011) to voltage-fed motors. The resulting control is of simple structure and incorporates three repetitive learning estimation schemes which generalize the classical integral actions. It is shown that, with a proper choice of the control gains, asymptotic convergence to zero of the rotor position tracking error is achieved through a resulting input signal which is a continuous time function. When compared to Marino et al. (2012a) and Tomei and Verrelli (2015), technical difficulties here appear: (i) an uncertain function multiplying the rotor speed derivative in the motor model here replaces the uncertain constant multiplying the output derivative in Marino et al. (2012a) and Tomei and Verrelli (2015); (ii) uncertain terms in the current dynamics here replace the known terms in the filter state dynamics of Marino et al. (2012a) and Tomei and Verrelli (2015). The resulting innovative control design and stability analysis consequently involve: (i) more than one learning estimation scheme (just like in Marino et al., 2012b in which, however, a "separation-like principle" can be invoked due to persistency of excitation); (ii) the use of an uncertain periodic function in the quadratic-integral Lyapunov-like function (see also Jin & Xu, 2013 for a similar idea). Realistic simulation results finally illustrate the effectiveness of the proposed approach.

2. Dynamic model and problem statement

Under the assumptions of negligible stator self inductance variations with position and negligible mutual inductance between stator windings, a permanent magnet step motor (see Khorrami, Krishnamurthy, & Melkote, 2003) with two phases in the (d, q) reference frame rotating at speed $N_r\omega$ and identified by the angle $N_r\theta$ in the fixed (a, b) reference frame attached to the stator (θ is the rotor position, ω is the rotor speed and N_r is the number of rotor teeth) are given by

$$\begin{aligned}
\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} &= \omega(t)\\
h_p(\theta(t))\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} &= -\alpha_p(\theta(t)) - \beta_p(\theta(t))\omega(t) + c_p(\theta(t))i_d(t) + i_q(t)\\
\frac{\mathrm{d}i_d(t)}{\mathrm{d}t} &= -\frac{R}{L_0}i_d(t) + N_r i_q(t)\omega(t) - \frac{\omega(t)}{L_0}\eta_d(\theta(t)) + \frac{1}{L_0}u_d(t)\\
\frac{\mathrm{d}i_q(t)}{\mathrm{d}t} &= -\frac{R}{L_0}i_q(t) - N_r i_d(t)\omega(t) - \frac{\omega(t)}{L_0}\eta_q(\theta(t)) + \frac{1}{L_0}u_q(t)
\end{aligned} \tag{1}$$

where

$$h_p(\theta) = \frac{J}{i_f N_r}\left[\sum_{j=1}^{n} jL_{mj}\cos[(1-j)N_r\theta]\right]^{-1}$$

$$\alpha_p(\theta) = \frac{h_p(\theta)}{J}\left[T_L(\theta) + \frac{N_r i_f^2}{2}\sum_{j=4}^{n} jL_{fj}\sin[jN_r\theta]\right]$$

$$\beta_p(\theta) = \frac{Dh_p(\theta)}{J}$$

$$c_p(\theta) = \frac{h_p(\theta)i_f N_r}{J}\sum_{j=2}^{n} jL_{mj}\sin[(1-j)N_r\theta]$$

$$\eta_d(\theta) = -i_f N_r \sum_{j=2}^{n} jL_{mj}\sin[(j-1)N_r\theta]$$

$$\eta_q(\theta) = i_f N_r \sum_{j=1}^{n} jL_{mj}\cos[(j-1)N_r\theta]$$

and: (i_d, i_q) are the stator current vector (d, q) components; (u_d, u_q) are the stator voltage vector (d, q) components [which constitute the control inputs]; $n \geq 4$ is an uncertain positive integer; D is the friction coefficient; J is the rotor inertia; $T_L(\cdot)$ is the load torque; i_f is the fictitious rotor current provided by the permanent magnet; R and L_0 are the stator windings resistance and the self inductance, respectively; the harmonics $\sum_{j=1}^{n} L_{mj}\cos[jN_r\theta]$ and $\sum_{j=1}^{n} L_{mj}\cos\left[jN_r\theta - \frac{\pi}{2}\right]$ model the non-sinusoidal flux distribution in the air-gap; the term $\frac{N_r i_f^2}{2}\sum_{j=4}^{n} jL_{fj}\sin[jN_r\theta]$ represents the disturbance torque due to cogging; the parameters L_{mj}, $2 \leq j \leq n$ (which are zero under the standard assumption of sinusoidal flux distribution) are much smaller than L_{m1} (see Khorrami et al., 2003); hence the direct-axis current i_d does not significantly contribute to torque production, while the quadrature-axis current i_q is assigned to produce the required torque. Since, as in Marino et al. (2012b), all the (constant) system parameters along with the load torque function are here allowed to be uncertain excepting for the number of rotor teeth N_r and the stator windings self inductance L_0, all the previously defined functions are crucially uncertain. The control problem is stated as follows. Under the assumptions: (M.1) N_r and L_0 are known parameters; (M.2) $\alpha_p(\theta)$ is a class $\mathcal{C}^{s_\alpha}$ function on $\mathbb{R}$ ($s_\alpha \geq 2$); (M.3) there exist known positive reals h_m, h_M, k_h, $k_{\alpha i}$, $k_{\beta i}$, η_{dM}, η_{qM}, c_M, $\bar{\eta}_{dM}$, $\bar{\eta}_{qM}$, R_m, R_M $\in \mathbb{R}^+$ ($i = 1, 2$) such that for all $\theta \in \mathbb{R}$ [$i = 1, 2$]: (i) $h_m \leq h_p(\theta) \leq h_M$; (ii) $\left|\frac{\partial h_p(\theta)}{\partial\theta}\right| \leq k_h$; (iii) $\left|\frac{\partial^{i-1}\alpha_p(\theta)}{\partial\theta^{i-1}}\right| \leq k_{\alpha i}$; (iv) $\left|\frac{\partial^{i-1}\beta_p(\theta)}{\partial\theta^{i-1}}\right| \leq k_{\beta i}$; (v) $|\eta_d(\theta)| \leq \eta_{dM}$; (vi) $|\eta_q(\theta)| \leq \eta_{qM}$; (vii) $|c_p(\theta)| \leq c_M$; (viii) $\left|\frac{\partial\eta_d(\theta)}{\partial\theta}\right| \leq \bar{\eta}_{dM}$; (ix) $\left|\frac{\partial\eta_q(\theta)}{\partial\theta}\right| \leq \bar{\eta}_{qM}$; (x) $R_m \leq R \leq R_M$, we address the problem of designing a state feedback control for system (1) in order to guarantee the rotor position tracking of a reference signal $\theta^*(t)$ which is assumed to belong to the following class: (M.4) $\theta^*(t)$ is a class $\mathcal{C}^{s_\theta}$ ($s_\theta \geq 3$) periodic function of known period T_* (i.e. $\theta^*(t) = \theta^*(t+T_*)$, $\forall t \geq -T_*$), with bounded time derivatives $|\theta^{*(i)}(t)| \leq c_{\theta i}$ ($i = 1, 2$) for all $t \in [0, T_*)$.

3. Repetitive control design

3.1. Preliminary computations

Since a non-zero i_d only contributes to torque ripples, it is desirable to set the i_d-reference $i_d^* = 0$ while choosing, as aforementioned, the i_q-reference i_q^* to produce the desired torque reference (see for instance Chen & Paden, 1993). We define the rotor position and speed tracking errors (k_θ is a positive control parameter): $\tilde{\theta} = \theta - \theta^*$, $\tilde{\omega} = \omega - \omega^* \doteq \omega + k_\theta\tilde{\theta} - \dot{\theta}^*$ so that $\dot{\tilde{\theta}} = -k_\theta\tilde{\theta} + \tilde{\omega}$. Furthermore, we express the uncertain function $f_c(\theta, \omega) = \alpha_p(\theta) + \beta_p(\theta)\omega$ as

$$\begin{aligned}
f_c(\theta, \omega) &= q_{0c}(\theta^*, \dot{\theta}^*, \ddot{\theta}^*) - h_p(\theta)\ddot{\theta}^* + g_c(\tilde{\theta}, \tilde{\omega}, t)\\
&\quad + h_p(\theta)k_\theta\tilde{\omega} - h_p(\theta)k_\theta^2\tilde{\theta}
\end{aligned}$$

with

$$q_{0c}(\theta^*, \dot{\theta}^*, \ddot{\theta}^*) = \alpha_p(\theta^*) + \beta_p(\theta^*)\dot{\theta}^* + h_p(\theta^*)\ddot{\theta}^*$$

$$\begin{aligned}
g_c(\tilde{\theta}, \tilde{\omega}, t) &= -h_p(\theta)k_\theta\tilde{\omega} + h_p(\theta)k_\theta^2\tilde{\theta} + \alpha_p(\theta) - \alpha_p(\theta^*)\\
&\quad + \beta_p(\theta^*)(\tilde{\omega} - k_\theta\tilde{\theta}) + [\beta_p(\theta) - \beta_p(\theta^*)]\omega\\
&\quad + [h_p(\theta) - h_p(\theta^*)]\ddot{\theta}^*.
\end{aligned}$$

By virtue of assumption (M.4), $q_c(t) = q_{0c}(\theta^*(t), \dot{\theta}^*(t), \ddot{\theta}^*(t))$ is a class $\mathcal{C}^{p_c}$-periodic function of known period T_* with $p_c = \min\{s_\theta - 2, s_\alpha\}$: it constitutes the uncertain periodic input reference for the current $i_q(t)$ achieving, for $i_d = 0$ and compatible initial conditions $\theta(0) = \theta^*(0)$, $\omega(0) = \dot{\theta}^*(0)$, perfect tracking. From assumptions (M.3) and (M.4), a known bound $B_q = k_{\alpha 1} + k_{\beta 1}c_{\theta 1} + h_M c_{\theta 2}$