Automatica 63 (2016) 321–329

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A unified filter for simultaneous input and state estimation of linear discrete-time stochastic systems ☆

Sze Zheng Yong [a,1], Minghui Zhu [b], Emilio Frazzoli [a]

[a] Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

[b] Department of Electrical Engineering, Pennsylvania State University, 201 Old Main, University Park, PA 16802, USA

ARTICLE INFO

Article history:
Received 11 October 2013
Received in revised form
8 July 2015
Accepted 6 October 2015

Keywords:
Optimal filtering
State estimation
Input estimation
Filter stability
Recursive filter

ABSTRACT

In this paper, we present a unified optimal and exponentially stable filter for linear discrete-time stochastic systems that simultaneously estimates the states and unknown inputs in an unbiased minimum-variance sense, without making any assumptions on the direct feedthrough matrix. We also provide the connection between the stability of the estimator and a system property known as strong detectability, and discuss the global optimality of the proposed filter. Finally, an illustrative example is given to demonstrate the performance of the unified unbiased minimum-variance filter.

1. Introduction

The term filter or estimator is commonly used to refer to systems that extract information about a quantity of interest from measured data corrupted by noise. Kalman filtering provides the tool needed for obtaining that reliable estimate when the system is linear and when the disturbance inputs are well modeled by a zero-mean, Gaussian white noise. However, in many instances, the exogenous input (e.g., the inputs of other autonomous vehicles) cannot be modeled as a Gaussian stochastic process rendering the estimates unreliable. Nonetheless, we want to be able to estimate the states and inputs of other vehicles based on noisy measurements for purposes of collision avoidance, route planning, etc. Similar problems can be found across a wide range of disciplines, from the real-time estimation of mean areal precipitation during a storm (Kitanidis, 1987) to fault detection and diagnosis (Patton, Clark, & Frank, 1989) to input estimation in physiological systems (De Nicolao, Sparacino, & Cobelli, 1997).

Literature review. Much of the research focus has been on state estimation of systems with unknown inputs without actually estimating the inputs. An optimal filter that estimates a minimum-variance unbiased (MVU) state estimate for a system with unknown inputs is first developed for linear systems without direct feedthrough in Kitanidis (1987). This design was extended to a more general parameterized solution by Darouach and Zasadzinski (1997), and eventually to state estimation of systems with direct feedthrough in Cheng, Ye, Wang, and Zhou (2009), Darouach, Zasadzinski, and Boutayeb (2003) and Hou and Patton (1998). Similarly, while H_∞ filters (e.g., Li & Gao, 2013; Wang, Dong, Shen, & Gao, 2013; Wang, Liu, & Liu, 2008) can deal with non-Gaussian disturbance inputs in minimizing the worst-case state estimation error, the unknown input is not estimated. However, the problem of estimating the unknown input itself is often as important as the state information, and should also be considered.

Palanthandalam-Madapusi and Bernstein (2007) proposed an approach to reconstruct the unknown inputs, in a process that is decoupled from state estimation with an emphasis on unbiasedness, but neglecting the optimality of the estimate. On the other hand, Gillijns and De Moor (2007a) and Hsieh (2000) developed simultaneous input and state filters that are optimal in the minimum-variance unbiased sense, for systems without direct feedthrough. Extensions to systems with a full rank direct feedthrough matrix were proposed by Fang, Shi, and Yi (2011), Gillijns and De Moor (2007b) and Yong, Zhu, and Frazzoli (2013b). In an attempt to deal with systems with a rank deficient direct feedthrough matrix, Hsieh (2009) allowed the input estimate to be

☆ This work was supported by NSF grant CNS-1239182. M. Zhu is partially supported by ARO W911NF-13-1-0421 (MURI) and NSF grant CNS-1505664. The material in this paper was partially presented at the 52nd IEEE Conference on Decision and Control, December 10–13, 2013, Florence, Italy. This paper was recommended for publication in revised form by Associate Editor Huijun Gao under the direction of Editor Ian R. Petersen.

E-mail addresses: szyong@mit.edu (S.Z. Yong), muz16@psu.edu (M. Zhu), frazzoli@mit.edu (E. Frazzoli).

[1] Tel.: +1 339 200 9664; fax: +1 617 324 6819.

http://dx.doi.org/10.1016/j.automatica.2015.10.040
0005-1098/

biased. Thus, the problem of finding a simultaneous state and input filter for systems with rank deficient direct feedthrough matrix, that is both unbiased and has minimum variance remains open. Moreover, a unified MVU filter that works for all cases remains elusive.

Another set of relevant literature pertains to the stability of the state and input filters, since optimality does not imply stability and vice versa. However, to the best of our knowledge, the literature on this subject is limited to linear time-invariant systems (Cheng et al., 2009; Fang, Shi, & Yi, 2008; Fang et al., 2011). Yet another related literature is on state and input observability and detectability conditions, also known as strong or perfect observability and detectability, as this will be shown to be related to the stability of the filter dynamics for both linear time-varying and time-invariant systems with unknown inputs. Some conditions for state and input observability were derived in Hautus (1983) and Palanthandalam-Madapusi and Bernstein (2007).

Contributions. We introduce a unified filter for simultaneously estimating both states and unknown inputs such that the estimates are unbiased and have minimum variance with no restrictions on the direct feedthrough matrix of the linear discrete-time stochastic system, which is a generalization of the estimators in the literature, specifically of Gillijns and De Moor (2007a,b) and Yong et al. (2013b), and the Kalman filter. Furthermore, we derive sufficient conditions for the filter stability of linear time-varying systems with unknown inputs, an important problem that has been previously unexplored; while for linear time-invariant systems, necessary and sufficient conditions for convergence of the filter gains to a steady-state solution are provided. The key insight we gained is that the exponential stability of the filter is directly related to the strong detectability of the time-varying system, without which unbiased state and input estimates cannot be obtained even in the absence of stochastic noise. We shall also show that the proposed filter is globally optimal (i.e., optimal over the class of all linear state and input estimators as in Kerwin and Prince (2000)).

In connection to the existing literature, this paper presents a combination of several ideas from Cheng et al. (2009) and Gillijns and De Moor (2007a,b) and our recent work (Yong et al., 2013b) into a unified filter in a manner that provably preserves and extends the nice properties of these filters. However, there are a number of distinctions between our filter and the above referenced filters. In particular, we show that the state-only filter in Cheng et al. (2009) implicitly estimates the unknown inputs in a suboptimal manner and so does the approach for input estimation in Gillijns and De Moor (2007b). In contrast, our filter uses the approaches of our previous work in Yong et al. (2013b) and of generalized least squares estimation, which lead to the desired optimality of the input estimates. In addition, we gave sufficient conditions for filter stability of linear time-varying systems, which cannot be carried over from the existing literature (including Cheng et al., 2009; Gillijns & De Moor, 2007a,b) for linear time-invariant systems. Preliminary versions of the results appeared in a conference paper (Yong et al., 2013b) and on arXiv (Yong, Zhu, & Frazzoli, 2013a) (in which more details on input and state observability/detectability are provided and a suboptimal filter variant is described).

Notation. We first summarize some notations used throughout the paper. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{C}$ the field of complex numbers and $\mathbb{N}$ nonnegative integers. For a random vector, $v \in \mathbb{R}^n$, the expectation is denoted by $\mathbb{E}[v]$. Given a matrix $M \in \mathbb{R}^{p\times q}$, its transpose, inverse, Moore–Penrose pseudoinverse, range, trace and rank are given by $M^\top$, M^{-1}, $M^\dagger$, $\mathrm{Ra}(M)$, $\mathrm{tr}(M)$ and $\mathrm{rk}(M)$. For a symmetric matrix S, $S \succ 0$ ($S \succeq 0$) indicates S is positive (semi-)definite.

2. Problem statement

Consider the linear time-varying discrete-time system

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k + G_k d_k + w_k \\ y_k &= C_k x_k + D_k u_k + H_k d_k + v_k \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector at time k, $u_k \in \mathbb{R}^m$ is a known input vector, $d_k \in \mathbb{R}^p$ is an unknown input vector, and $y_k \in \mathbb{R}^l$ is the measurement vector. The process noise $w_k \in \mathbb{R}^n$ and the measurement noise $v_k \in \mathbb{R}^l$ are assumed to be mutually uncorrelated, zero-mean, white random signals with known bounded covariance matrices, $Q_k = \mathbb{E}[w_k w_k^\top] \succeq 0$ and $R_k = \mathbb{E}[v_k v_k^\top] \succ 0$, respectively. The matrices A_k, B_k, C_k, D_k, G_k and H_k are known and bounded. Note that no assumption is made on H_k to be either the zero matrix (no direct feedthrough), or to have full column rank when there is direct feedthrough. Without loss of generality, we assume that $\max_k(\mathrm{rk}[G_k^\top\ H_k^\top]) = p$, $n \geq l \geq 1$, $l \geq p \geq 0$, $m \geq 0$, the current time variable r is strictly nonnegative and x_0 is independent of v_k and w_k for all k.

The estimator design problem, addressed in this paper, can be stated as follows:

Given a linear discrete-time stochastic system with unknown inputs (1), design a globally optimal and stable filter that simultaneously estimates system states and unknown inputs in an unbiased minimum-variance manner.

3. Preliminary material

3.1. System transformation

We first decouple the output equation into two components, one with a full rank direct feedthrough matrix and the other without direct feedthrough. In this form, the filter can be designed leveraging existing approaches for both cases (e.g., Gillijns & De Moor, 2007a; Yong et al., 2013b).

Let $p_{H_k} := \mathrm{rk}(H_k)$. Using singular value decomposition, we rewrite the direct feedthrough matrix H_k as $H_k = \begin{bmatrix} U_{1,k} & U_{2,k} \end{bmatrix} \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{1,k}^\top \\ V_{2,k}^\top \end{bmatrix}$, where $\Sigma_k \in \mathbb{R}^{p_{H_k}\times p_{H_k}}$ is a diagonal matrix of full rank, $U_{1,k} \in \mathbb{R}^{l\times p_{H_k}}$, $U_{2,k} \in \mathbb{R}^{l\times(l-p_{H_k})}$, $V_{1,k} \in \mathbb{R}^{p\times p_{H_k}}$ and $V_{2,k} \in \mathbb{R}^{p\times(p-p_{H_k})}$, while $U_k := \begin{bmatrix} U_{1,k} & U_{2,k} \end{bmatrix}$ and $V_k := \begin{bmatrix} V_{1,k} & V_{2,k} \end{bmatrix}$ are unitary matrices. When there is no direct feedthrough, Σ_k, $U_{1,k}$ and $V_{1,k}$ are empty matrices,[2] and $U_{2,k}$ and $V_{2,k}$ are arbitrary unitary matrices.

Then, as suggested in Cheng et al. (2009), we define two orthogonal components of the unknown input given by

$$d_{1,k} = V_{1,k}^\top d_k, \qquad d_{2,k} = V_{2,k}^\top d_k. \tag{2}$$

Since V_k is unitary, $d_k = V_{1,k} d_{1,k} + V_{2,k} d_{2,k}$ and the system (1) can be rewritten as

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k + G_k V_{1,k} d_{1,k} + G_k V_{2,k} d_{2,k} + w_k \\ &= A_k x_k + B_k u_k + G_{1,k} d_{1,k} + G_{2,k} d_{2,k} + w_k \end{aligned} \tag{3}$$

$$\begin{aligned} y_k &= C_k x_k + D_k u_k + H_k V_{1,k} d_{1,k} + H_k V_{2,k} d_{2,k} + v_k \\ &= C_k x_k + D_k u_k + H_{1,k} d_{1,k} + v_k, \end{aligned} \tag{4}$$

where $G_{1,k} := G_k V_{1,k}$, $G_{2,k} := G_k V_{2,k}$ and $H_{1,k} := H_k V_{1,k} = U_{1,k}\Sigma_k$. Next, we decouple the output y_k using a nonsingular transformation $T_k = \begin{bmatrix} T_{1,k}^\top & T_{2,k}^\top \end{bmatrix}^\top$

$$T_k = \begin{bmatrix} I_{p_{H_k}} & -U_{1,k}^\top R_k U_{2,k} (U_{2,k}^\top R_k U_{2,k})^{-1} \\ 0 & I_{(l-p_{H_k})} \end{bmatrix} \begin{bmatrix} U_{1,k}^\top \\ U_{2,k}^\top \end{bmatrix} \tag{5}$$

[2] We adopt the convention that the inverse of an empty matrix is also an empty matrix and assume that operations with empty matrices are possible.