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Observer-based $\mathcal{H}_\infty$ control of networked systems with stochastic communication protocol: The finite-horizon case☆

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ABSTRACT

This paper is concerned with the $\mathcal{H}_\infty$ control problem for a class of linear time-varying networked control systems (NCSs) with stochastic communication protocol (SCP). The sensor-to-controller network (the controller-to-actuator network) is considered where only one sensor (one actuator) obtains access to the communication network at each transmission instant. The SCP is applied to determine which sensor (actuator) should be given the access to the network at a certain instant. The aim of the problem addressed is to design an observer-based controller such that the $\mathcal{H}_\infty$ performance of the closed-loop system is guaranteed over a given finite horizon. For the purpose of simplifying the NCS model, a new Markov chain is constructed to model the SCP scheduling of communication networks. Then, both the methods of stochastic analysis and completing squares are utilized to establish the sufficient conditions for the existence of the desired controller. The controller parameters are characterized by solving two coupled backward recursive Riccati difference equations subject to the scheduled SCP. Finally, a numerical example is given to illustrate the effectiveness of the proposed controller design scheme.

1. Introduction

Networked control systems (NCSs) are control systems in which the signal transmission between system components (e.g. sensors, actuators and controller) is implemented through the communication networks. Since NCSs possess many advantages such as low cost, simple installation, reduced system wiring and high reliability, they have found successful applications in a wide range of areas including environmental monitoring, industrial automation, smart grids and distributed/mobile communications. Accordingly, the control and filtering issues of NCSs have gained ever-increasing research attention, see e.g. Caballero-Águila, Hermoso-Carazo, and Linares-Pérez (2015), Sahebsara, Chen, and Shah (2008) and Tian, Yue, and Peng (2010). For instance, the reliable control problem has been investigated in Tian et al. (2010) for unreliable NCSs with probabilistic actuator failures, measurement distortions, random network-induced delays and packet dropouts. The design problem of the optimal $\mathcal{H}_\infty$ filtering has been dealt with in Sahebsara et al. (2008) for NCSs with multiple packet dropouts.

In reality, almost all systems have certain time-varying parameters since the system parameters may be changeable in time due to a variety of reasons such as temperature fluctuation, operating point shifting, graduate aging of system components, etc. Because of the time-varying nature of the underlying systems, one would be naturally more interested in analyzing their *transient* dynamics over a finite horizon than the traditional *steady-state* behaviors over the infinite horizon, see e.g. Liang, Sun, and Liu (2014), Jiménez-Lizárraga, Basin, Rodríguez, and Rodríguez (2015) and Shi and Chen (2013). In recent years, considerable research attention has been devoted to the $\mathcal{H}_\infty$ control/filtering problems for time-varying systems, see e.g. Dong, Wang, Ding, and Gao (2015), Ding, Wang, Lam, and Shen (2015), Ding, Wang, Shen, and Dong (2015), Fridman and Shaked (2000), Hu, Wang, Gao, and Stergioulas (2012), Liang et al. (2014) and Sheng, Zhang, and Gao (2014) and the references therein. From a technical point of view,

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there are generally two effective approaches to solving the $\mathcal{H}_\infty$ control/filtering problems for time-varying systems: the so-called recursive linear matrix inequality (RLMI) approach (Ding, Wang, Shen et al., 2015; Dong et al., 2015; Hu et al., 2012; Liang et al., 2014) and the Riccati differential/difference equation (RDE) approach (Ding, Wang, Lam et al., 2015). For example, in Dong et al. (2015), a finite-horizon $\mathcal{H}_\infty$ fault estimator has been designed for a class of nonlinear stochastic time-varying systems with both randomly occurring faults and fading channels based on the RLMI approach. The probability-guaranteed $\mathcal{H}_\infty$ finite-horizon filtering problem has been considered in Hu et al. (2012) for a class of nonlinear time-varying systems with uncertain parameters and sensor saturations by using the RLMI approach. The $\mathcal{H}_\infty$ control problem has been investigated in Ding, Wang, Lam et al. (2015) for discrete time-varying nonlinear systems with both randomly occurring nonlinearities and fading measurements over a finite-horizon by using the backward recursive RDE approach.

In most existing literature concerning the control problems of NCSs, it has been assumed that all the sensors (or actuators) could simultaneously get access to the communication network to transmit/receive signals. However, this assumption is generally unrealistic since real-world networks unavoidably suffer from limited bandwidth which is likely to give rise to data collisions in case of simultaneous multiple accesses. As such, many communication protocols have been introduced in industry in order to prevent the data from collisions by determining which sensors (or actuators) should obtain access to the communication networks. These protocols include, but are not limited to, the Round-Robin protocol (Ugrinovskii & Fridman, 2014), the try-once-discard protocol (Walsh, Ye, & Bushnell, 2002) and the stochastic communication protocol (SCP) (Long & Yang, 2014; Tabbara & Nešić, 2008; Zhang, Yu, & Feng, 2011). So far, the analysis and synthesis problems of NCSs subject to various communication protocols have begun to stir some initial research interest. For example, in Ugrinovskii and Fridman (2014), the distributed $\mathcal{H}_\infty$ estimation problem has been studied for sensor networks subject to the Round-Robin protocol scheduling by using the time-delay system approach. The optimal linear estimation problem has been investigated in Zhang et al. (2011) for networked systems subject to a random media access control (MAC) protocol. In Donkers, Heemels, Bernardini, Bemporad, and Shneer (2012), the stability issue has been investigated for NCSs with time-varying transmission intervals, time-varying transmission delays, packet dropouts subject to various communication protocols (e.g. Round-Robin protocol, try-once-discard protocol and SCP) by using a switching system approach.

The SCP serves as a widely used model describing a certain class of carrier-sense multiple access with collision avoidance (CSMA/CA) protocols. The CSMA/CA protocols have been implemented in a variety of communication systems (e.g. IEEE 802.11-based wireless local area networks and IEEE 802.15.4-based wireless sensor networks). Recently, the analysis issue of NCSs subject to SCP has drawn some refreshed research attention, see e.g. Donkers et al. (2012), Long and Yang (2014) and Tabbara and Nešić (2008). In particular, a linear time-invariant (LTI) continuous-time NCS with the SCP has been modeled in Donkers et al. (2012) by utilizing the properties of the Markov process. It should be mentioned that the communication protocol would inevitably complicate the dynamics analysis of the NCS especially when the NCS exhibits the time-varying. To this end, a seemingly interesting research problem is to investigate the control problem for the time-varying NCS with SCP constraints owing to its clear engineering insight in both control and communication areas. Nevertheless, this is a non-trivial problem with three challenges identified as follows: (1) how to develop a recursive algorithm accounting for the time-varying nature of the SCP-constrained NCS? (2) how to obtain the sufficient conditions for the existence of the desired time-varying controllers? and (3) how to examine the impact from the SCP on the control performance of the overall system? It is, therefore, the main purpose of this paper to offer satisfactory answers to the aforementioned three questions.

In response to the above discussion, in this paper, we aim to investigate the finite-horizon $\mathcal{H}_\infty$ control problem for the NCS with the SCP constraints. More specifically, the objective of this paper is to design an observer-based controller for the NCS subject to SCP such that the $\mathcal{H}_\infty$ performance of the closed-loop system is guaranteed over a given finite horizon. *The main contributions of this paper are highlighted as follows.* (1) *The control problem is, for the first time, investigated for time-varying systems with the SCP.* (2) *Both sensor-to-controller network and controller-to-actuator network featured with the SCPs are simultaneously considered in the controller design.* (3) *A novel coupled RDE approach is developed to solve the addressed finite-horizon $\mathcal{H}_\infty$ control problem.* (4) *The impact from the SCP on the structure of the controller gain matrix is revealed.*

The rest of this paper is organized as follows. In Section 2, the NCS with time-varying parameters and two communication networks are introduced and the problem under consideration is formulated. In Section 3, the design problem of observer-based controller is solved in terms of the solution to two coupled backward recursive RDEs. Furthermore, a numerical simulation example is given in Section 4 to illustrate the effectiveness of the controller design scheme. Finally, we conclude the paper in Section 5.

Notations: The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and set of all $n \times m$ real matrices. The notation $X \geq Y$ ($X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (positive definite). Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ will, respectively, denote the expectation of the stochastic variable x and expectation of x conditional on y. 0 represents the zero matrix of compatible dimensions. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. The shorthand diag$\{\cdots\}$ stands for a block-diagonal matrix. $\|v\|$ refers to the Euclidean norm of a vector v. M^T and $M^\dagger \in \mathbb{R}^{n\times m}$ represent the transpose and the Moore–Penrose pseudo inverse of $M \in \mathbb{R}^{m\times n}$. $\|M\|_F$ denotes the Frobenius norm of the matrix M. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions. Let a be an integer and b be a positive integer. The function mod(a, b) represents the unique nonnegative remainder on division of the integer a by the positive integer b. The floor function $\lfloor b \rfloor$ denotes the largest integer not greater than b. The Kronecker delta function $\delta(a)$ is a binary function that equals 1 if $a = 0$ and equals 0 otherwise.

2. Problem formulation and preliminaries

In this section, we introduce some preliminaries related to the communication of NCSs and then describe the problem setup.

2.1. Stochastic communication protocol (SCP)

Consider a NCS with N transmission nodes labeled as $\{1, 2, \ldots, N\}$. The main idea of the SCP for *discrete-time systems* is that *only one node* is selected to transmit/receive data via the communication network at each transmission instant. Let $\xi(k)$ denote the selected node obtaining access to the network at time k. Then, as shown in Donkers et al. (2012), under the SCP scheduling, $\xi(k) \in \{1, 2, \ldots, N\}$ can be regarded as a stochastic process which