Automatica 63 (2016) 374–383

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Fault-tolerant finite time consensus for multiple uncertain nonlinear mechanical systems under single-way directed communication interactions and actuation failures ☆

Yujuan Wang [a,b], Yongduan Song [a,b,1], Miroslav Krstic [c], Changyun Wen [d]

[a] Key Laboratory of Dependable Service Computing in Cyber Physical Society, Ministry of Education, Chongqing University, Chongqing, 400044, China
[b] School of Automation, Chongqing University, Chongqing, 400044, China
[c] Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla CA 92093, USA
[d] School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, 639798, Singapore, Singapore

ARTICLE INFO

Article history:
Received 9 February 2015
Received in revised form
29 September 2015
Accepted 12 October 2015

Keywords:
Nonlinear multi-agent systems
Unknown actuation faults
Finite-time control
Directed interaction topology

ABSTRACT

This paper investigates the problem of finite time consensus for a group of uncertain nonlinear mechanical systems under single-way directed communication topology and actuation failures. Due to the existence of the unknown inherent nonlinear dynamics and the undetectable actuation faults, the resultant control gain of the system becomes unknown and time-varying, making the control impact on the system uncertain and the finite time control synthesis nontrivial. The underlying problem becomes further complex as the communication among the agents is not only local but also one-way directed. In this work, three major steps are employed to circumvent the aforementioned difficulties, leading to a robust adaptive fault-tolerant finite time consensus solution. Firstly, by deriving a useful property on the newly constructed Laplacian matrix, the technical difficulty in finite time control design and stability analysis is circumvented; Secondly, to deal with the time-varying and uncertain control gain, the concept of virtual parameter estimation error is introduced and incorporated into a skillfully chosen Lyapunov function; Thirdly, to facilitate the global stability analysis of the proposed adaptive fault-tolerant finite time consensus scheme for multiple nonlinear systems, an important lemma (Lemma 7) containing a useful inequality is derived. In addition, the finite convergence time for each agent to reach the required consensus configuration is explicitly established and recipes for control parameter selection to make the residual errors as small as desired are provided. The effectiveness of the proposed control scheme is confirmed by numerical simulation.

1. Introduction

In cooperative control of multi-agent systems (MAS), it is often required that the consensus be reached in finite time as such feature offers numerous benefits including faster convergence rate, better disturbance rejection, and robustness against uncertainties (Bhat & Bernstein, 2000; Haimo, 1986). Finite time consensus control for single or double integrator linear MAS has been well addressed, such as Chen, Lewis, and Xie (2011), Li, Du, and Lin (2011), Wang and Xiao (2010), Xiao, Wang, Chen, and Gao (2009), Zhang and Yang (2013) and so forth. The finite-time consensus problem for nonlinear MAS has also been extensively studied. For instance, the finite-time consensus problem for first-order MAS with unknown nonlinear dynamics was addressed in Cao and Ren (2014) with the assumption that the nonlinear dynamics satisfies the Lipschitz condition. Distributed finite-time consensus for first-order nonlinear systems was investigated in Li and Qu (2014) by using nonsmooth analysis, where the proposed controller is discontinuous and the nonlinear function is assumed to be uniformly bounded. The finite-time synchronization for a class of second-order nonlinear MAS is studied in Du, He, and Cheng (2014) by using homogeneity technique, where the Lipschitz condition

☆ This work was supported in part by the Major State Basic Research Development Programs 973 (No. 2012CB215202; No. 2014CB249200) and the National Natural Science Foundation of China (No. 61134001). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonis Papachristodoulou under the direction of Editor Christos G. Cassandras.

E-mail addresses: iamwyj123456789@163.com (Y. Wang), ydsong@cqu.edu.cn (Y. Song), krstic@ucsd.edu (M. Krstic), ecywen@ntu.edu.sg (C. Wen).

[1] Tel.: +86 2365102056; fax: +86 2365102056.

http://dx.doi.org/10.1016/j.automatica.2015.10.049
0005-1098/© 2015 Elsevier Ltd. All rights reserved.

on the nonlinear term is imposed. It is noted that under Lipschitz or homogeneity condition, the finite-time consensus problem for nonlinear MAS can be handled by methods similar to the consensus problem for linear systems. More recently, efforts have been made by using adaptive method in addressing finite time consensus for MAS with nonlinearities subject to bilateral graph topology (Huang, Wen, Wang, & Song, 2015; Yu, Shen, & Xia, 2013). However, it is worth noting that the uncertainties are assumed to exhibit the linear parametric property, without which the adaptive methods for finite time consensus (Huang et al., 2015; Yu et al., 2013) are inapplicable.

Another area of interest is how to achieve consensus with the least possible topological requirement. Note that all of the aforementioned results are derived with somewhat special topological requirements such as undirected connected graph (Du et al., 2014; Huang et al., 2015; Li et al., 2011; Yu et al., 2013), digraph but detailed-balanced (Chen et al., 2011; Zhang & Yang, 2013), or strongly connected digraph (Wang & Xiao, 2010; Xiao et al., 2009). Compared with the undirected topology, the finite-time consensus of MAS with directed topology is much more challenging mainly because the Laplacian matrix under directed graph is no longer symmetric. It should be mentioned that the digraph with detail-balanced condition largely reduces the complexity for convergence analysis as the corresponding Laplacian matrix under this condition can be made symmetric by using the "diagonal matrix multiplication" method (Chen et al., 2011; Zhang & Yang, 2013).

To our best knowledge, leaderless fault-tolerant finite-time consensus control for MAS with non-parametric uncertainties and undetectable actuation faults under directed topology is still an open problem. In this paper, we attempt to provide a solution to this problem. Compared with the existing related works, this work differs in four aspects: (1) unknown and time-varying control gains are explicitly tackled; (2) the nonlinearities and uncertainties do not have to satisfy linear parametric property; (3) additive and loss of effectiveness actuation faults are addressed; and (4) the finite time consensus is achieved with sufficient precision under single-way directed communication constraints. The main contributions of this work are summarized as follows. Firstly, the finite-time consensus control scheme is derived for a group of uncertain nonlinear mechanical systems under the directed communication topology. Such solution is made possible by developing an important graph theory result on the newly defined Laplacian matrix in Lemmas 5 and 6. It is shown that all the internal signals are ensured to be uniformly bounded and the consensus configuration error uniformly converges to a small residual set in finite time. Secondly, by introducing the skillfully defined weighting parameter estimate error and by establishing an important inequality in Lemma 7, the lumped uncertainties in the system with unknown and time-varying control gains are compensated gracefully. Thirdly, the unknown and undetectable actuation failures, including both loss of effectiveness faults and additive faults, are explicitly accommodated. This is the first literature report addressing finite time consensus with fault-tolerant capability for MAS in the presence of one-way communication interactions and unknown time-varying control gains.

The rest of this paper is organized as follows. In Section 2, the problem formulation and some useful preliminaries are addressed. The adaptive fault-tolerant finite-time control scheme is developed in Section 3. In Section 4 numerical simulation is conducted and Section 5 concludes the paper.

2. Problem formulation and preliminaries

Throughout this paper, the initial time t_0 is set as $t_0 = 0$ without loss of generality; $1_n(0_n) \in R^n$ denotes a vector with each entry being 1 (0); $\otimes$ denotes the Kronecker product; For a vector $X = [x_1, \ldots, x_n]^T$, $|X| = [|x_1|, \ldots, |x_n|]^T$ with $|\cdot|$ the absolute value of a real number, $X^h = [x_1^h, \ldots, x_n^h]^T$ with $h \in R$, and $\|X\|$ denotes the Euclidean norm; $J = \{1, \ldots, n\}$ denotes the set of node indexes.

2.1. Problem formulation

The MAS considered in this paper is a group of nonlinear mechanical systems modeled by

$$\begin{aligned} &\dot{r}_k(t) = v_k(t), \\ &g_k(t)\dot{v}_k(t) = u_{ak} + F_k(r_k, v_k) + D_k(t), \quad k \in J, \end{aligned} \tag{1}$$

where $r_k = [r_{k1}, \ldots, r_{kl}]^T \in R^l$, $v_k = [v_{k1}, \ldots, v_{kl}]^T \in R^l$ and $u_{ak} = [u_{ak1}, \ldots, u_{akl}]^T \in R^l$ represents, respectively, the position, velocity and control input of the kth subsystem; $g_k \in R^{l\times l}$ is unknown and time-varying, $F_k(\cdot)$ and $D_k(\cdot)$ denote system nonlinearities and external disturbance acting on the kth subsystem, respectively.

In contrast to most existing works on finite-time distributed control that are based on healthy actuation of MAS, in this work actuators with undetectable faults are considered. In this case, the actual control input u_{ak} does not behave in the way as designed by u_k. Instead, it acts according to (Wang, Song, & Lewis, 2015)

$$u_{ak} = \rho_k(t, t_{\rho k})u_k + u_{rk}(t, t_{rk}), \quad k \in J \tag{2}$$

where $\rho_k = \mathrm{diag}\{\rho_{k1}, \ldots, \rho_{kl}\} \in R^{l\times l}$ indicates the actuation effectiveness of the kth agent; $u_{rk}(\cdot) \in R^l$ is the uncontrollable additive actuation fault; $t_{\rho k} = [t_{\rho k1}, \ldots, t_{\rho kl}]^T$ and $t_{rk} = [t_{rk1}, \ldots, t_{rkl}]^T$, with $t_{\rho ki}$ and t_{rki} $(i = 1, 2, \ldots, l)$ denoting, respectively, the time instant at which the loss of actuation effectiveness fault and the additive actuation fault occurs in the ith actuator of the kth agent.

The objective in this paper is to design a distributed adaptive fault-tolerant controller such that the synchronization of the multiple mechanical nonlinear systems under one-way directed communication interactions is achieved in finite time. Namely, the agreement configuration among the subsystems (agents) is reached with adjustable bounded error in finite time. Before designing controller, the following assumptions are in order.

Assumption 1. The directed communication network G is strongly connected.

Assumption 2. The matrix $g_k = \mathrm{diag}\{g_{ki}\}$, where g_{ki} $(i = 1, \ldots, l)$ is unknown and time-varying yet bounded away from zero, that is, there exist unknown constants $\underline{g}$ and $\bar{g}$ such that $0 < \underline{g} \le |g_{ki}(\cdot)| \le \bar{g} < \infty$ and $g_{ki}(\cdot)$ is sign-definite (without loss of generality, we assume $\mathrm{sgn}(g_{ki}) = +1$ in this paper).

Assumption 3. For the nonlinear term $F_k(\cdot)$, there exist a bounded constant matrix $c_{fk} = \mathrm{diag}\{c_{fki}, \ldots, c_{fkl}\} \in R^{l\times l}$ $(0 < c_{fki} < \infty$ are unknown constants) and a function vector $\phi_k(r_k, v_k) = [\phi_{k1}, \phi_{k2}, \ldots, \phi_{kl}]^T \in R^l$ $(\phi_{ki}(r_{ki}, v_{ki}) \ge 0$ is known scalar function), such that $|F_k(\cdot)| \le c_{fk}\phi_k(\cdot)$. It is assumed that if r_{ki} and v_{ki} are bounded, so is $\phi_{ki}(\cdot)$. For the external disturbance $D_k(\cdot)$, there exists an unknown constant vector $d_k^{\max} = [d_{k1}^{\max}, \ldots, d_{kl}^{\max}]^T$ $(0 \le d_{ki}^{\max} < \infty$ is unknown and bounded constant $(i = 1, \ldots, l))$, such that $|D_k(\cdot)| \le d_k^{\max}$.