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Dilated LMI characterization for the robust finite time control of discrete-time uncertain linear systems[☆]

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ABSTRACT

This paper provides new dilated linear matrix inequalities (LMIs) characterizations for the finite time boundedness (FTB) and the finite time stability (FTS) analysis of discrete-time uncertain linear systems. The dilated LMIs are later used to design a robust controller for the finite time control of discrete-time uncertain linear systems. The relevant feature of the proposed approach is the decoupling between the Lyapunov and the system matrices, that allows considering a parameter-dependent Lyapunov function. In this way, the conservativeness with respect to previous results is decreased. Numerical examples are used to illustrate the results.

1. Introduction

The idea of finite time stability (FTS) allows analyzing or designing control systems that present some constraints on the state response without being necessarily stable. In fact, when the behavior of the system over a fixed time interval is of interest, a system could be defined *stable* when, given some initial conditions, the state remains within some desired bounds in such time interval, and *unstable* when it does not (Amato & Ariola, 2005; Amato, Ariola, & Cosentino, 2010b; Amato, Ariola, & Dorato, 2001). Another concept which is strongly related to FTS is finite time boundedness (FTB), which takes into account norm bounded disturbances affecting the system (Amato et al., 2001). FTB implies FTS, but the converse is not true.

These concepts have been developed considerably in the last decade (Amato, Ambrosino, Ariola, Cosentino, & De Tommasi, 2014). In particular, the development of results based on differential/difference linear matrix inequalities (DLMIs) has attracted a lot of interest, mainly due to the possibility of providing necessary and sufficient conditions for the FTS/FTB of linear systems (Amato, Ariola, & Cosentino, 2010a; Amato et al., 2010b; Ichihara & Katayama, 2009). In these works, it has been shown that the FTS of a time-varying linear system is guaranteed if and only if either a certain inequality involving the state transition matrix is satisfied, or a symmetric matrix function solving a certain Lyapunov differential/difference inequality exists. However, due to computational complexity issues, simpler sufficient (not necessary) conditions for FTS are used to address the problem of designing controllers guaranteeing some finite-time performance.

The problem of designing a robust finite time control law for uncertain systems has been solved only in the continuous-time case (Amato, Ariola, & Cosentino, 2011; Amato et al., 2001), and it seems that the discrete-time case represents still an open problem. In this paper, using the *dilation* technique suggested by de Oliveira, Bernussou, and Geromel (1999), we provide new dilated linear matrix inequalities (LMIs) characterizations for the FTB and the FTS analysis. The dilated LMIs are used to design a controller for the robust finite time control of discrete-time uncertain linear systems. The relevant feature of the proposed approach is the

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decoupling between the Lyapunov and the system matrices, that allows considering a Lyapunov function that depends on the parametric uncertainty. In this way, the conservativeness with respect to previous results, e.g. Amato et al. (2010b), is decreased. It is worth highlighting that in Amato et al. (2010b), time-varying linear systems *without uncertainty* are considered, in which case a time-varying Lyapunov matrix allows to obtain less conservative results than the ones obtained using a constant Lyapunov matrix. However, in the present work, the considered systems are subject to parametric uncertainty too, in which case using a Lyapunov function that associates a different Lyapunov matrix to each possible value of the *unknown* parametric uncertainty seems to be the most effective choice for reducing conservativeness with respect to parameter-independent Lyapunov functions.

2. Preliminaries

Let us recall the definitions of FTS and FTB in the case of discrete-time LTV systems (Amato & Ariola, 2005; Amato et al., 2001)

Definition 1. The discrete-time LTV system

$$x(k+1) = A(k)x(k) \tag{1}$$

is said to be finite time stable (FTS) with respect to (c_1, c_2, N, R), with $c_2 > c_1 > 0,\ N > 0$ and $R \succ 0$, if

$$x(0)^T Rx(0) \le c_1 \Rightarrow x(k)^T Rx(k) < c_2 \quad \forall k \in \{1, \dots, N\}. \tag{2}$$

Definition 2. The discrete-time LTV system

$$\begin{cases} x(k+1) = A(k)x(k) + G(k)w(k) \\ w(k+1) = F(k)w(k) \end{cases} \tag{3}$$

is said to be finite time bounded (FTB) with respect to (c_1, c_2, N, R, d), with $c_2 > c_1 > 0,\ N > 0,\ R \succ 0$, and $d > 0$ if

$$\begin{cases} x(0)^T Rx(0) \le c_1 \\ w(0)^T w(0) \le d \end{cases} \Rightarrow x(k)^T Rx(k) < c_2 \quad \forall k \in \{1, \dots, N\}. \tag{4}$$

Notice that FTS can be recovered as a special case of FTB when $w = 0$. Finally, the following lemmas provide sufficient conditions for the FTB and the FTS of discrete-time LTV systems.

Lemma 1. *Given $\rho \ge 1$, the discrete-time LTV system (3) is FTB with respect to (c_1, c_2, N, R, d) if there exist positive scalars λ_1, λ_2 and two positive definite matrix-valued functions $Q_1(\cdot) : k \in \{0, 1, \dots, N\} \mapsto Q_1(k) \in \mathbb{S}^{n_x \times n_x}$ and $P_2(\cdot) : k \in \{0, 1, \dots, N\} \mapsto P_2(k) \in \mathbb{S}^{n_w \times n_w}$ such that*

$$\begin{pmatrix} -\rho Q_1(k) & Q_1(k)A(k)^T & O & O \\ A(k)Q_1(k) & -Q_1(k+1) & G(k) & O \\ O & G(k)^T & -\rho P_2(k) & F(k)^T P_2(k+1) \\ O & O & P_2(k+1)F(k) & -P_2(k+1) \end{pmatrix} \prec 0$$

$$\forall k \in \{0, 1, \dots, N-1\} \tag{5}$$

$$Q_1(0) \succ \lambda_1 R^{-1} \tag{6}$$

$$Q_1(k) \prec R^{-1} \quad \forall k \in \{1, \dots, N\} \tag{7}$$

$$P_2(0) \prec \lambda_2 I \tag{8}$$

$$\begin{pmatrix} \dfrac{c_2}{\rho^N} - \lambda_2 d & \sqrt{c_1} \\ \sqrt{c_1} & \lambda_1 \end{pmatrix} \succ 0. \tag{9}$$

Proof. The proof is inspired by the one provided in Amato and Ariola (2005) for the case of discrete-time LTI systems, and is omitted here due to space limitation.

Notice that by considering $F(k) = G(k) = O$ and $d = 0$, conditions for analyzing the FTS of discrete-time LTV systems can be obtained. The analysis of FTS, with respect to FTB, is less demanding from a computational point of view, due to the lower size of the matrix inequalities and the lower number of variables that should be found (FTS analysis involves finding a positive scalar λ_1 and a positive definite matrix-valued function $Q_1(\cdot)$, contrarily to FTB analysis which involves finding in addition λ_2 and $P_2(\cdot)$). It can be shown that the sufficient condition for FTS of discrete-time LTV systems provided by Amato et al. (2010b) are a particular case obtained when $\rho = 1$ and $c_1 = 1$.

3. Main results

The following theorems provide new dilated LMIs for analyzing the finite time boundedness and the finite time stability properties of discrete-time LTV systems.

Theorem 1. *Given $\rho \ge 1$, the discrete-time LTV system (3) is FTB with respect to (c_1, c_2, N, R, d) if there exist positive scalars λ_1 and λ_2, two positive definite matrix-valued functions $Q_1(\cdot) : k \in \{0, 1, \dots, N\} \mapsto Q_1(k) \in \mathbb{S}^{n_x \times n_x}$ and $P_2(\cdot) : k \in \{0, 1, \dots, N\} \mapsto P_2(k) \in \mathbb{S}^{n_w \times n_w}$, and two matrix-valued functions $H_1(\cdot) : k \in \{0, 1, \dots, N-1\} \mapsto H_1(k) \in \mathbb{R}^{n_x \times n_x}$ and $H_2(\cdot) : k \in \{1, \dots, N\} \mapsto H_2(k) \in \mathbb{R}^{n_w \times n_w}$ such that (6)–(9) and Eq. (10) in Box I $\forall k \in \{0, 1, \dots, N-1\}$ hold.*

Proof. The proof is inspired by the results obtained in de Oliveira et al. (1999). We first show that (5) implies (10). In fact, if (5) holds, we can choose $H_1(k) = H_1(k)^T = Q_1(k)$ and $H_2(k+1) = H_2(k+1)^T = P_2(k+1)$ in (10) and recover (5).

It remains to show that (10) implies (5). To do so, let us assume that (10) holds, and let us notice that, for a given k, it can be rewritten as

$$\begin{pmatrix} \rho Q_1(k) & O & O & O \\ O & -Q_1(k+1) & G(k) & O \\ O & G(k)^T & -\rho P_2(k) & O \\ O & O & O & P_2(k+1) \end{pmatrix}$$

$$+ He \left\{ \begin{pmatrix} -\rho I & O \\ A(k) & O \\ O & F(k)^T \\ O & -I \end{pmatrix} \Xi(k) \begin{pmatrix} I & O & O & O \\ O & O & O & I \end{pmatrix} \right\} \prec 0 \tag{11}$$

$$\Xi(k) = \begin{pmatrix} H_1(k) & O \\ O & H_2(k+1) \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} -\rho\,(He\{H_1(k)\} - Q_1(k)) & H_1(k)^T A(k)^T & O & O \\ A(k)H_1(k) & -Q_1(k+1) & G(k) & O \\ O & G(k)^T & -\rho P_2(k) & F(k)^T H_2(k+1) \\ O & O & H_2(k+1)^T F(k) & P_2(k+1) - He\{H_2(k+1)\} \end{pmatrix} \prec 0. \tag{10}$$

Box I.