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Disturbance observer-based disturbance attenuation control for a class of stochastic systems☆

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ABSTRACT

This paper studies a class of stochastic systems with multiple disturbances which include the disturbance with partially-known information and the white noise. A disturbance observer is constructed to estimate the disturbance with partially-known information, based on which, a disturbance observer-based disturbance attenuation control (DOBDAC) scheme is proposed by combining pole placement and linear matrix inequality (LMI) methods.

1. Introduction

Disturbance observer-based control (DOBC) was established in the late of 1980s for linear frequency-domain systems. The basic idea is that the disturbance can be estimated by a disturbance observer and then compensated in the feed-foreword channel immediately. In Chen (2004), a DOBC method for the nonlinear disturbance observer design was proposed using an algebraic structure for a class of single-input and single-output (SISO) nonlinear systems. In Guo and Chen (2005), new composite control laws based on the disturbance observer are proposed to estimate and reject disturbances for multiple-input and multiple-output (MIMO) nonlinear systems. In recent years, composite DOBC and PI control (Yang & Tsubakihara, 2008), composite DOBC and H_∞ control (Wei & Guo, 2010), composite DOBC and terminal sliding mode (TSM) control (Wei & Guo, 2009), composite DOBC and robust adaptive control (Guo & Wen, 2011) have been presented for nonlinear systems subject to multiple disturbances.

On the other hand, stochastic systems have received much attention in theory and practice, see for instance Krstic and Deng (1988) and Wu, Yang, and Shi (2010). The robust stochastic stability problem was studied and some useful robust stochastic stability conditions were proposed in Krstic and Deng (2000) and Liu, Zhang, Shi, and Karimi (2014). Moreover, the idea of combining stochastic noise with set-membership noise and using state dependent Brownian noise was proposed in Kurzanskii (1988). However, most of the aforementioned stochastic control methods only focus on single type of disturbance or integrate multiple disturbances into a new equivalent disturbance. The characteristic of multiple disturbances and influence on system performance are not given enough attention in the literature.

The purpose of this paper is to present a disturbance observer-based disturbance attenuation control (DOBDAC) for a class of stochastic systems, which can be considered as an extension of the existing DOBC method to the stochastic case. The technical difficulties come from the coupling of the state and the disturbance estimate error, which leads to the invalidity of certainty equivalence principle. To separate the disturbance observer design from the controller design, the composite pole placement and LMI methods are proposed.

Notations. For matrices M_1, M_2, we denote by $M_1 < 0$ to state that M_1 is a negative definite matrix. $\text{diag}\{M_1, M_2\}$ stands for a block diagonal matrix; $*$ represents the corresponding elements in the symmetric matrix; $\lambda_{\min}$ and $\lambda_{\max}$ are minimum and maximum of matrix eigenvalues respectively; $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+)$ denotes the family

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of all nonnegative functions $V(x,t)$ on $\mathbb{R}^n \times \mathbb{R}_+$ which are $\mathcal{C}^2$ in x and $\mathcal{C}^1$ in t; $\mathcal{C}^k$ denotes k-times continuously differentiable; $\mathcal{K}$ denotes the set of all functions: $\mathbb{R}_+ \to \mathbb{R}_+$, which are continuous, strictly increasing and vanishing at zero; $\mathcal{K}_\infty$ denotes the set of all functions which are of class-$\mathcal{K}$ and unbounded.

2. Problem statement

Consider the following stochastic system with multiple disturbances

$$\dot{x}(t) = Ax(t) + B_0(u(t) + D_0(t)) + B_1\xi_1(t) + B_2x(t)\xi_2(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state and the control input, respectively. The additive noise $\xi_1(t) \in \mathbb{R}$ and multiplicative noise $\xi_2(t) \in \mathbb{R}$ are white noises. In many cases (such as reaction wheels Hasha, 1986, etc.), the stochastic disturbance $D_0(t) \in \mathbb{R}^m$ represents a class of signals with known frequency and unknown amplitude and phase, which can be formulated by the following exogenous system

$$\begin{aligned} D_0(t) &= D_{01}(t) + D_{02}(t), \qquad D_{02}(t) = S\xi_3(t), \\ D_{01}(t) &= Cz(t), \qquad \dot{z}(t) = Gz(t) + H\delta(t), \end{aligned} \tag{2}$$

where $z(t) \in \mathbb{R}^{2r}$, $\xi_3(t) \in \mathbb{R}^r$ are white noises. $\delta(t) \in \mathbb{R}^r$ is a bounded nonrandom disturbance that results from the perturbations and uncertainties. $\xi_1(t)$, $\xi_2(t)$ and $\xi_3(t)$ are independent. $A, B_0, B_1, B_2, S, C, G$ and H are known matrices.

It is assumed that system (2) is stable, the matrices $G \in \mathbb{R}^{2r\times 2r}$ and $H \in \mathbb{R}^{2r\times r}$ can be respectively described as $G = \mathrm{diag}\{G_1, G_2, \ldots, G_r\}$ and $H = [H_1, H_2, \ldots, H_r]$ with $G_i = \begin{bmatrix} 0 & 2\pi\omega_i \\ -2\pi\omega_i & 0 \end{bmatrix}$ and $H_i = [b_{i1}, b_{i2}]^T$, where $\omega_i > 0$ $(i = 1, 2, \ldots, r)$ represents the frequency of disturbances, and b_{i1}, b_{i2} are constant parameters.

Then, substituting (2) into (1) yields

$$\begin{aligned} \dot{x}(t) = {} & Ax(t) + B_0u(t) + B_0Cz(t) + B_1\xi_1(t) \\ & + B_2x(t)\xi_2(t) + B_0S\xi_3(t). \end{aligned} \tag{3}$$

Denoting $\zeta(t) = [\xi_1^T(t), \xi_3^T(t)]^T$ and $F = [B_1, B_0S]$, system (3) can be formulated as

$$\dot{x}(t) = Ax(t) + B_0Cz(t) + B_0u(t) + F\zeta(t) + B_2x(t)\xi_2(t). \tag{4}$$

According to Øksendal (2003, P61), by replacing $\zeta(t)$ with $\frac{dW_1(t)}{dt}$ and $\xi_2(t)$ with $\frac{dW_2(t)}{dt}$, the general equivalent equations of (4) and (2) are formally obtained as follows:

$$\begin{aligned} dx(t) = {} & Ax(t)dt + B_0u(t)dt + B_0D_{01}(t)dt + FdW_1(t) \\ & + B_2x(t)dW_2(t), \end{aligned} \tag{5}$$

$$dz(t) = Gz(t)dt + H\delta(t)dt, \qquad D_{01}(t) = Cz(t), \tag{6}$$

where $W_1(t)$ and $W_2(t)$ are independent standard Wiener processes, and the underlying complete probability space is taken to be the quartet $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions.

Assumption 1. For systems (5) and (6), the pair (A, B_0) is controllable and the pair (G, B_0C) is observable.

As a special case of Mao and Yuan (2006), the following definition and criterion of stability for stochastic systems are given.

Consider a nonlinear stochastic differential equations (SDE)

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t), \quad t \ge t_0(\ge 0) \tag{7}$$

where $f : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^{n\times m}$ are locally Lipschitz in $x(t) \in \mathbb{R}^n$ with $f(0,t) = 0$, $g(0,t) = 0$. $B(t)$, $t \ge 0$, is an m-dimensional independent standard Wiener process (or Brownian motion).

Definition 1 (*Mao & Yuan, 2006, P157*)**.** Let $p > 0$. System (7) is said to be asymptotically bounded in pth moment if there is a positive constant H such that

$$\lim_{t\to\infty} \sup \mathbb{E}|x(t; t_0, x_0)|^p \le H \tag{8}$$

for all $(t_0, x_0) \in \mathbb{R}_+ \times \mathbb{R}^n$. When $p = 2$ we say Eq. (7) is asymptotically bounded in mean square.

Definition 2 (*Deng & Krstic, 2001*)**.** The equilibrium $x(t) = 0$ of (7) is said to be globally stable in probability if for every $\epsilon > 0$, there exists a class $\mathcal{K}$ function $\gamma(.)$ such that

$$P\{\|x(t)\| < \gamma(x_0)\} \ge 1 - \epsilon, \quad \forall t \ge 0,\ \forall x_0 \in R^n \setminus \{0\}. \tag{9}$$

The equilibrium $x(t) = 0$ of (7) is said to be globally asymptotically stable in probability if it is globally stable in probability and

$$P\{\lim_{t\to\infty} \|x(t)\| = 0\} = 1, \quad \forall x_0 \in R^n. \tag{10}$$

Lemma 1 (*Mao & Yuan, 2006, P157–P158*)**.** *Assume that there exist functions $V \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+)$ and $\kappa \in \mathcal{K}_\mathcal{V} \subset \mathcal{K}_\infty$ and positive numbers p, β, λ such that*

$$\kappa(|x|^p) \le V(x,t), \quad \text{and} \quad \mathcal{L}V(x,t) \le -\lambda V(x,t) + \beta, \tag{11}$$

for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}_+$. Then

$$\lim_{t\to\infty} \sup \mathbb{E}|x(t; t_0, x_0)|^p \le \kappa^{-1}\left(\frac{\beta}{\lambda}\right) \tag{12}$$

for all $(x,t) \in \mathbb{R}^n \times \mathbb{R}_+$. That is, system (7) is asymptotically bounded in pth moment.

Lemma 2 (*Deng & Krstic, 2001*)**.** *For system (7), if there exist a $\mathcal{C}^2$ function $V(x)$, and class $\mathcal{K}_\infty$ functions α_1 and α_2 and class $\mathcal{K}$ functions α_3, such that for all $x(t) \in \mathbb{R}^n$, $t \ge 0$*

$$\alpha_1(\|x(t)\|) \le V(x,t) \le \alpha_2(\|x(t)\|), \tag{13}$$

$$\begin{aligned} \mathcal{L}V(x,t) &= \frac{\partial V}{\partial x} f(x,t)dt + \frac{1}{2} Tr\left\{ g(x,t)^T \frac{\partial^2 V}{\partial x^2} g(x,t) \right\} \\ &\le -\alpha_3(\|x(t)\|). \end{aligned} \tag{14}$$

Then, the equilibrium $x = 0$ is globally asymptotically stable in probability.

3. Main results

In this section, system states $x(t)$ are supposed to be available for measurement. A disturbance observer-based disturbance attenuation control (DOBDAC) scheme is proposed by combining pole placement and linear matrix inequality (LMI) methods.

3.1. Stochastic disturbance observer(SDO)

The stochastic disturbance observer (SDO) is constructed as

$$\begin{aligned} & d\nu(t) = (G + LB_0C)\hat{z}(t)dt + L(Ax(t) + B_0u(t))dt, \\ & \hat{D}_{01}(t) = C\hat{z}(t), \qquad \hat{z}(t) = \nu(t) - Lx(t), \end{aligned} \tag{15}$$

where $\hat{z}(t)$ is the estimation of $z(t)$ in (6), and $\nu(t)$ is the auxiliary vector as the state of stochastic disturbance observer. The estimation error is denoted as $e_z(t) = z(t) - \hat{z}(t)$. Based on (5), (6) and (15), it is shown that the error dynamics satisfy

$$\begin{aligned} de_z(t) = {} & (G + LB_0C)e_z(t)dt + LFdW_1(t) \\ & + LB_2x(t)dW_2(t) + H\delta(t)dt. \end{aligned} \tag{16}$$

Since (G, B_0C) is observable, the poles can be placed at an arbitrarily chosen location by adjusting L in (16) to satisfy the performance requirement for SDO.