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A generalized reaching law with different convergence rates☆

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ABSTRACT

Traditionally, the convergence rate of the sliding variable given by a discrete reaching law has been fixed in nature. This paper presents a generalized algorithm for discrete time sliding mode control systems which offers a flexible convergence rate for the sliding variable. For analysis, we use the recently developed band approach method which gives us the values of the parameters in the algorithm for the desired convergence to take place, once the ultimate band is chosen appropriately.

1. Introduction

Since the advent of digital computer, most continuous systems are treated in their discretized form. Hence, a major focus is on the discrete sliding mode, which has been dealt in various literatures (Bandyopadhyay & Janardhanan, 2006; Bartolini, Ferrara, & Utkin, 1995; Bartoszewicz, 1998; Gao, Wang, & Homaifa, 1995). All these works mainly deal with proposing a reaching law and deriving the control and the ultimate band in terms of the controller parameters used in the reaching law. The most popular among these reaching laws are the ones attributed to Bartolini et al. (1995) and Gao et al. (1995).

In the recent work (Chakrabarty & Bandyopadhyay, 2014), a new approach to analyze Gao's discrete reaching law has been developed and utilized to find out the controller parameter values once the desired ultimate band value is chosen from a specified range. This approach is proved to be better than Gao's approach in Gao et al. (1995), since the ultimate band obtained this way is much lesser than that proposed by existing analysis. Also, one is at liberty to choose the value of the ultimate band beforehand and then calculate the controller parameters accordingly.

1.1. Motivation

The new approach in Chakrabarty and Bandyopadhyay (2014), which we refer as the band approach method, has not been used to analyze any other dynamics than the Gao's reaching law until recently in Chakrabarty and Bandyopadhyay (2013), where the sliding variable dynamics as generated by the system is found out to be different than the reaching law used to derive the control. The dynamics was not only a function of the sliding variable but also had the states of the system mixed in it. The band approach method was applied to analyze this dynamics and the ultimate band was found out, for which the controller parameters were computed. This work put forward a more general dynamics than the Gao's reaching law, which was subjected to the band approach analysis. This inspired the authors to work further and search for a more generalized dynamics that this analysis method can handle. This led to the proposal of the generalized reaching law in Chakrabarty and Bandyopadhyay (2015). In the mentioned paper, a discrete time reaching law as

$$s(k+1) = f_1(s(k)) + f_2(\xi(k), k) + f_3\mathrm{sgn}(s(k)) + d(k) \quad (1)$$

was proposed, where f_1 and f_2 are functions in the variables mentioned, f_3 can either be a constant or a varying gain depending on a particular problem. Here, $s(k) \in \mathbb{R}$ and $\xi(k) \in \mathbb{R}^q$ are taken with k denoting the sample count, where $\xi(k)$ can be any variable other than $s(k)$, which is known at all k. The uncertainty $d(k)$ is assumed to vary only at the sampling instants and bounded. The above reaching law (1) is general in the sense that it includes functions of variables other than the sliding variable. Also, $f_1(s(k))$ is not necessarily a linear function as in Gao's reaching law

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(Gao et al., 1995). This led us to choose the functions appropriately so as to design a discrete reaching law which would have a flexible convergence rate.

The same reaching law (1) is used in the work in Chakrabarty and Bandyopadhyay (2015) to show convergence of the sliding variable inside an ultimate bound, when the system is affected by a disturbance which is bounded by known but varying functions. Different choice of the functions in the generalized reaching law (1) made it possible to establish a relationship between the ultimate bound and the parameters in the reaching law.

One needs to mention at this point that there had been several works in the literature which deal with proposing new reaching laws or modifying the already popular reaching laws (Bartoszewicz & Latosinski, 2014; Bartoszewicz & Lesniewski, 2014a,b; Niu, Ho, & Wang, 2010; Yuan, Shen, Xiao, & Wang, 2012; Zhu, Wang, Jiang, & Wang, 2009). However, there had not been any proposal for any generalized reaching algorithm, and any analysis to show that it would result in varying convergence rate of the sliding variable by simple choices of the functions in the generalized reaching law. The work in this paper is novel in that regard.

2. Main analysis

The requirement for steady convergence with rate one can be defined as the convergence which assures $|s(k+1)| < |s(k)|$. However, one can easily extend the idea of convergence to higher rates n, which would mean that $|s(k+1)| < |s(k)|^{1/n}$ will be assured for $n \in \mathbb{I}$.

In this section, a generalized convergence scheme of the sliding variable is proposed with the help of the reaching law (1) for different $n \in \mathbb{I}$. Thereby, the relationships of the controller parameters with a chosen ultimate band are also proposed.

Let $f_2(\xi(k), k) \equiv 0$ and $f_1(s(k)) = f_0|s(k)|^{1/n}\text{sgn}(s(k))$ in the reaching law (1), where f_0 is a constant and n is a positive integer. Hence, the reaching law (1) becomes

$$s(k+1) = f_0|s(k)|^{1/n}\text{sgn}(s(k)) + f_3\text{sgn}(s(k)) + d(k) \tag{2}$$

where $d(k) \in [-d_m, d_m]$ is assumed.

In the next subsection, it will be shown how the reaching law in (2) gives us a general convergence rate of $|s(k+1)| < |s(k)|^{1/n}$, where convergence will be faster as n is increased, i.e., the reaching phase will be minimized.

2.1. Main analysis with band approach method

In the sequel, the proofs for the lemmas and the theorem will be shown for $s(k) > 0$. The analysis would lead to the same results if it was done for $s(k) < 0$ because of the symmetric nature of the reaching law (2).

Lemma 1. *Suppose the following relations hold:*

(i) $B_d \in [d_m, 2d_m)$ (3a)

(ii) $f_0 \in [0, 1)$ (3b)

(iii) $f_3 = (1-f_0)B_d - d_m$ (3c)

Then the reaching law (2) satisfies $|s(k+1)| < |s(k)|^{1/n}$ whenever $|s_k|^{1/n} > B_d$.

Proof. Using (3c) and $f_0 < 1$ in the region $s(k) > B_d^n$ and considering the maximum value of uncertainty $d(k) = d_m$ in (2), we get

$$\begin{aligned} s(k+1) &= f_0 s(k)^{1/n} + (1-f_0)B_d - d_m + d(k) \\ &= s(k)^{1/n} - (1-f_0)(s(k)^{1/n} - B_d) \\ \Rightarrow s(k+1) &< s(k)^{1/n}. \end{aligned} \tag{4}$$

Let us define

$$B_c := 2d_m - B_d. \tag{5}$$

Using (3c) and $f_0 \geq 0$ in the region $s(k) > B_d^n$ and considering the minimum value of uncertainty $d(k) = -d_m$ in (2), we get

$$\begin{aligned} s(k+1) &= f_0 s(k)^{1/n} + (1-f_0)B_d - d_m + d(k) \\ \Rightarrow s(k+1) &> f_0 B_d + (1-f_0)B_d - 2d_m \\ &= -B_c \end{aligned} \tag{6}$$

Now, from (5) and (3a), we get $B_c \in (0, d_m]$. This implies $B_c \leq B_d$. Hence, (5) ensures that $|s(k+1)| < |s(k)|^{1/n}$ even when $s(k)$ crosses zero.

Lemma 2. *Let the relations (3b) and (3c) hold for the reaching law (2). Additionally, suppose*

$$f_0 = 2 - \frac{2d_m}{B_d}. \tag{7}$$

Then the reaching law (2) satisfies $|s(k+1)| < B_d$ whenever $0 < |s_k|^{1/n} \leq B_d$.

Proof. In the region $0 < s(k) \leq B_d^n$, using (3c) and (3b) in (2), we get

$$\begin{aligned} s(k+1) &= f_0 s(k)^{1/n} + (1-f_0)B_d - d_m + d(k) \\ &\leq f_0 B_d + (1-f_0)B_d \\ &= B_d \end{aligned} \tag{8}$$

by taking the maximum bound of the disturbance $d(k) = d_m$.

Let us define

$$B_s := 2d_m - (1-f_0)B_d. \tag{9}$$

In the region $0 < s(k) \leq B_d^n$, using (3c) and taking $s(k) \to 0$ in (2), we get

$$\begin{aligned} s(k+1) &= (1-f_0)B_d - d_m + d(k) \\ &\geq (1-f_0)B_d - 2d_m \\ &= -B_s \end{aligned} \tag{10}$$

by taking the minimum bound of the disturbance $d(k) = -d_m$.

But since f_0 is given by (7), we get $B_s = B_d$. Hence $|s(k+1)| \leq B_d$ even when $\{s(k)\}$ crosses zero.

Theorem 1. *Let us assume the following to hold:*

(1) $B_d \in [d_m, 2d_m)$
(2) $f_0 = 2 - \frac{2d_m}{B_d}$ *and*
(3) $f_3 = (1-f_0)B_d - d_m$.

Then for the cases

(i) $B_d \geq 1, \forall\, n \in \mathbb{I}$

(ii) $n = 1, \forall\, B_d$ *the sequence $\{s_k\}$ following the reaching law (2) will be ultimately bounded by $\delta = B_d\ \forall k \geq K \in \mathbb{N}$, excepting the situation where $|s(0)| > B_d^n$ and the disturbance sequence $\{d(k)\} = d_m\text{sgn}(s(0))\ \forall\, k \geq 0$ with $B_d > d_m$ is chosen. Then $\delta = (1+\rho)B_d$, with $\rho > 0$, is however small.*
For the remaining case

(iii) $B_d < 1, \forall\, n > 1$ *the sequence $\{s_k\}$ following the reaching law (2) will be ultimately bounded by $\delta = f_0 + (1-f_0)B_d\ \forall\, k \geq K \in \mathbb{N}$.*

Proof. Let us first discuss cases (i) and (ii).

From Lemma 2, $|s(k+1)| \leq B_d$ whenever $|s(k)|^{1/n} \leq B_d$. Since $|s(k)| \leq B_d \Rightarrow |s(k)|^{1/n} \leq B_d$ when $B_d \geq 1$, Lemma 2 implies $|s(k+1)| \leq B_d$ whenever $|s(k)| \leq B_d$. Hence B_d can be the ultimate band in this case, i.e., when $B_d \geq 1\ \forall n \in \mathbb{I}$. The same can be said when $n = 1$ for any choice of B_d. However, from Lemma 1,