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Stabilization for sampled-data systems under noisy sampling interval[☆]

Bo Shen [a,1], Zidong Wang [b], Tingwen Huang [c]

[a] *School of Information Science and Technology, Donghua University, Shanghai 200051, China*

[b] *Department of Computer Science, Brunel University, Uxbridge, Middlesex, UB8 3PH, UK*

[c] *Department of Mathematics, Texas A&M University at Qatar, Doha, P.O.Box 23874, Qatar*

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ABSTRACT

In engineering practice, the sampling interval for a sampled-data system often fluctuates around a nominal/ideal value based on certain probability distributions that can be specified *a priori* through statistical tests. In this paper, a fundamental stabilization problem is investigated for a class of sampled-data systems under noisy sampling interval. The stochastic sampled-data control system under consideration is first converted into a discrete-time system whose system matrix is represented as an equivalent yet tractable form via the matrix exponential computation. Then, by introducing a Vandermonde matrix, the mathematical expectation of the quadratic form of the system matrix is computed. By recurring to the Kronecker product operation, the sampled-data stabilization controller is designed such that the closed-loop system is stochastically stable in the presence of noisy sampling interval. Subsequently, a special case is considered where the sampling interval obeys the continuous uniform distribution and the corresponding stabilization controller is designed. Finally, a numerical simulation example is provided to demonstrate the effectiveness of the proposed design approach.

1. Introduction

The past few decades have witnessed constant research interests on both the analysis and synthesis problems of sampled-data control systems, and a rich body of literature has been available, see e.g. Dai, Hu, Teel, and Zaccarian (2010), Lam (2012), Xu, Su, and Pan (2013), Zhang and Yu (2010) and references therein. Traditionally, the sampling interval has been assumed to be constant in a periodic sampled-data system. Such an assumption is, however, not always true in practical engineering. For example, in networked and embedded control systems, it is often the case that the sampling intervals are uncertain and/or vary with time due largely to unpredictable network-induced phenomena. Recently, the *aperiodic* sampling issue has stirred a great deal of research attention in the design and control of sampled-data systems. In Fridman, Seuret, and Richard (2004) and Suplin, Fridman, and Shaked (2007), an input delay approach has been proposed to deal with the uncertain sampling intervals and the aperiodic sampled-data control problems have been investigated in virtue of the Lyapunov–Krasovskii theorem. In Alexandre (2012), Oishi and Fujioka (2010) and Young (2008), the aperiodic sampled-data control system has been transformed into a discrete-time system by using the exact integration over a sampling interval and less conservative stability criteria have been derived.

It should be pointed out that, in most existing literature, the aperiodic sampling intervals have been implicitly assumed to be deterministic. Nevertheless, it is quite common in practice that the aperiodic samplings often occur in a probabilistic way owing to some undesirable physical constraints such as aperiodic faults in the samplers, fluctuated network loads, intermittent signal quantization/saturation and unwanted changes of some components of the system itself. A typical example is the seismic data extraction where the sampled-data is collected from the random interspersed observation points along the trajectory of the process. Recently, the sampled-data control problem under noisy sampling intervals has begun to receive some initial attention and the corresponding results have been scattered. In Gao, Wu, and Shi (2009), the sampled-data H_∞ control problem has been investigated when sampling intervals randomly switch between two

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E-mail addresses: bo.shen@dhu.edu.cn (B. Shen), Zidong.Wang@brunel.ac.uk (Z. Wang), tingwen.huang@qatar.tamu.edu (T. Huang).

1 Tel.: +86 21 67792371 811; fax: +86 21 67792353.

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given values and, in Kanchanaharuthai and Wongsaisuwan (2002), the sampled-data H_2-optimal controller has been designed with sampling intervals obeying the Erlang distribution. Note that the sampling models in Gao et al. (2009) and Kanchanaharuthai and Wongsaisuwan (2002), though interesting, are quite special and there appears to be a need to cater for more general aperiodic sampling phenomena. As such, it is our first motivation in this paper to establish a fairly comprehensive aperiodic sampling model and examine how the aperiodic sampling interval impacts on the stability and performance of the overall sampled-data system.

In order to obtain the stability criteria for the sampled-data control system with less conservativeness, a preferred approach would be to transform the underlying controlled system into its equivalent discrete-time counterpart. In this case, the resulting discrete-time closed-loop system is highly nonlinear with respect to the sampling interval which, in turn, gives rise to significant difficulties in the analysis and design of the sampled-data control systems. More specifically, three fundamental difficulties are identified as follows: (1) *how can we find an equivalent yet tractable representation for the system matrix which includes a matrix exponential and its integration simultaneously over the sampling interval?* (2) *how can we compute the mathematical expectation of the quadratic form of a matrix exponential involving random scalars? and* (3) *in the design of stochastic sampled-data control system, how can we derive an easy-to-implement design algorithm according to the established stability criteria?* It is, therefore, our second yet primary motivation in our paper to offer satisfactory answers to the three questions.

In response to the above discussion, in this paper, we aim to investigate the stabilization problem for sampled-data systems with noisy sampling interval. The main contributions of this paper are summarized as follows: (1) *a new stabilization control problem is addressed for a class of sampled-data systems where the sampling intervals fluctuate around a nominal/ideal value according to certain probability distributions*; (2) *by employing the matrix theory such as matrix exponential computation, Vandermonde matrix and Kronecker product operation, the stabilization controller is designed; and* (3) *a special case is considered where the sampling intervals obey the continuous uniform distribution*. Finally, a numerical simulation example is presented to show the effectiveness of the proposed sampled-data control scheme.

2. Problem formulation

Consider the following continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, A is the system matrix and B is the input matrix. The initial value is given by x_0.

The control input $u(t)$ is generated by a zero-order hold function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots$

$$u(t) = Kx(t_k), \quad t_k \le t < t_{k+1} \tag{2}$$

where K is the gain matrix to be determined and t_k denotes the sampling instant satisfying $\lim_{k\to\infty} t_k = \infty$.

By substituting (2) into (1), the closed-loop system is obtained as follows:

$$\dot{x}(t) = Ax(t) + BKx(t_k), \quad t_k \le t < t_{k+1}. \tag{3}$$

Integrating the above equation from t_k to t_{k+1}, one has

$$x(t_{k+1}) = \left(e^{A(t_{k+1}-t_k)} + \int_{t_k}^{t_{k+1}} e^{A(t_{k+1}-\theta)} d\theta BK\right) x(t_k). \tag{4}$$

Letting the sampling interval be $T_k = t_{k+1} - t_k$ and denoting $x(t_k)$ by x_k, we come up with a discrete-time system of the following form:

$$x_{k+1} = \left(e^{AT_k} + \int_0^{T_k} e^{As} ds BK\right) x_k. \tag{5}$$

The sampling interval T_k under consideration is subject to noisy perturbations and consists of two parts, i.e., $T_k = T + \upsilon_k$ where T is a constant which stands for the nominal sampling interval and υ_k is a random variable which accounts for the sampling errors/drifts/deviations resulting from unpredictable environmental phenomena. The probability density function of the random variable υ_k is denoted by $f(\upsilon)$ where the argument υ satisfies $T + \upsilon > 0$.

Remark 1. It is worth mentioning that, to date, very little attention has been paid to noisy sampling interval issue despite its practical significance in engineering. In Gao et al. (2009) and Kanchanaharuthai and Wongsaisuwan (2002), the noisy sampling interval has been taken into account where the sampling intervals have been simply assumed to obey Bernoulli or Erlang distribution. In this paper, the probability distributions of the sampling intervals are general that include the above probability distributions as special cases and the aim of this paper is to develop a design approach for the sampled-data control systems with such a general noisy sampling model.

Note that the system (5) is a discrete-time stochastic system due to the random nature of the sampling error, and therefore the following notion of stochastic stability is needed.

Definition 1 (*Dong, Wang, & Gao, 2013*)**.** The discrete-time stochastic system (5) is said to be stochastically stable if

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \|x_k\|^2\right\} < \infty. \tag{6}$$

In this paper, we aim to investigate the sampled-data stabilization problem for the system (1) in presence of the random sampling error, that is, we are interested in finding the controller gain matrix K such that the discrete-time stochastic system (5) is stochastically stable with respect to noisy sampling intervals obeying certain probability distributions.

3. Main results

To start with, a sufficient condition is provided in the following well-known lemma for the stochastic stability of the system (5).

Lemma 1. *Given the controller gain matrices K, the stochastic system (5) is stochastically stable if there exists a positive definite matrix Q such that the following inequality holds:*

$$\mathbb{E}\left\{\left(e^{AT_k} + \int_0^{T_k} e^{As} ds BK\right)^T Q \left(e^{AT_k} + \int_0^{T_k} e^{As} ds BK\right)\right\} - Q < 0. \tag{7}$$

Proof. The proof of this lemma is straightforward and is therefore omitted. □

It can be seen from (7) that the random variable T_k appears in the upper boundary of integral, and this makes it difficult to directly calculate the mathematical expectation of the integral. In what follows, our main efforts will be made towards the computation of this mathematical expectation by recurring to the matrix theory.

First, it follows from Theorem 1 in Van Loan (1978) that

$$e^{CT_k} = \begin{bmatrix} e^{AT_k} & \int_0^{T_k} e^{As} ds B \\ 0 & I \end{bmatrix} \tag{8}$$