Automatica 59 (2015) 35–47

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

# Control of uncertain nonlinear systems based on observers and estimators[☆]

Tiantian Jiang[a], Chaodong Huang[b], Lei Guo[c,1]

[a] Beijing Institute of Control Engineering, Beijing, China

[b] Beijing Institute of Space Long March Vehicle, Beijing, China

[c] Institute of Systems Science, AMSS, Chinese Academy of Sciences, Beijing, China

ARTICLE INFO

*Article history:*
Received 5 May 2014
Received in revised form
6 April 2015
Accepted 24 May 2015

*Keywords:*
Nonlinear uncertain systems
Output feedback
Extended state observer (ESO)
Active disturbance rejective control (ADRC)
Projected gradient estimator

ABSTRACT

In this paper, we consider a class of nonlinear dynamical systems with zero dynamics, which is subject to both unknown nonparametric dynamics and external disturbances, and is required to track a given reference signal by using the output feedback. Our controller is designed based on both the extended state observer (ESO) and the projected gradient estimator. While the ESO is used to estimate the total uncertainties, the projected gradient algorithm is used to estimate the nonparametric uncertainties treated as "time-varying parameters". This method overcomes the difficulties that the traditional active disturbance rejection control (ADRC) technique needs to have a "good" prior estimate for the uncertainties in the input channel. The closed-loop system is shown to be semi-globally stable, and at the same time, the tracking error can be made arbitrarily small.



## 1. Introduction

Uncertainties always exist in the modeling of practical dynamical systems due to, e.g., complexity in understanding complex systems, unavoidable changes of systems structures, difficulty in predicting changes of the environment, etc. As a fundamental issue in automatic control, dealing with uncertainties has been the focus of many developments in control theory. Plenty of control methods have been developed for dealing with uncertainties over the past half a century, among which adaptive control (see, e.g., Åström & Wittenmark, 1995, Chen & Guo, 1991 and Krstić, Kanellakopoulos, & Kokotović, 1995) and robust control (see, e.g., Qu, 1998 and Zames, 1981) are two typical approaches. Traditional adaptive control design usually requires that the uncertainties can be expressed linearly in terms of unknown parameters. On the other hand, robust control design usually requires that the uncertainties be bounded in some norm and have certain structural property. What is more, various disturbance estimation techniques have been proposed for rejecting disturbances, such as the unknown input observer (UIO) (Hostetter & Meditch, 1973), the disturbance observer (DOB) (Schrijver & Van Dijk, 2002), the perturbation observer (POB) (Kwon & Chung, 2003), etc. Brief surveys of disturbance observers can be found in Guo, Feng, and Chen (2006) and Radke and Gao (2006). Most estimators, like UIO, DOB and POB, are designed to handle small perturbations, and usually require the model of the plant to reconstruct the disturbances.

Owing to its less dependence on plant information, its capabilities to deal with a wide range of uncertainties, and its simplicity in the control structure, the active disturbance rejection control (ADRC) technique has received much attention in the control community (see, e.g. Gao, 2006, Gao, Huang, & Han, 2001 and Han, 1995, 1998, 2008, 2009). The key of ADRC is to online estimate the total uncertainties that lump unmodeled dynamics and external disturbances by an extended state observer (ESO) (Han, 1995, 2008, 2009). Thus, the uncertainties may then be compensated in real time. Up to now, the idea of ADRC technique has been applied in solving various kinds of engineering problems, e.g., motor control (Feng, Liu, & Huang, 2004; Li & Liu, 2009), flight control (Huang, Xu, Han, & Lam, 2001; Xia, Zhu, Fu, & Wang, 2011), robot control (Su, Ma, Qiu, & Xi, 2004; Talole, Kolhe, & Phadke, 2010), etc. Meanwhile, some progress has also been made in the

☆ This work is supported by National Natural Science Foundation of China under Grants 61227902, 61134008, 61333008 and the key project of the National Center for Mathematics and Interdisciplinary Sciences, CAS. The material in this paper was partially presented at the 10th World Congress on Intelligent Control and Automation, July 6–8, 2012, Beijing, China. This paper was recommended for publication in revised form by Associate Editor Gang Tao under the direction of Editor Miroslav Krstic.

*E-mail addresses:* jiangtt@amss.ac.cn (T. Jiang), chaodonghuang@163.com (C. Huang), lguo@amss.ac.cn (L. Guo).

[1] Tel.: +86 10 62532161.

http://dx.doi.org/10.1016/j.automatica.2015.06.012
0005-1098/

theoretical analysis of the ADRC (Guo & Zhao, 2011; Xue & Huang, 2011a,b; Yang & Huang, 2009; Zheng, Gao, & Gao, 2007). In Zheng et al. (2007), the stability of the closed-loop system based on ADRC is discussed under the condition of bounded uncertainties. The case where there is additional uncertainty in the input channel (denoted by $b(x, t)$) has been considered in Freidovich and Khalil (2008) and Xue and Huang (2011b), with two different control methods proposed to stabilize such uncertain systems. Based on the extended high-gain observer (EHGO), Freidovich and Khalil (2008) proposed an output feedback controller and proved that the closed-loop system was able to recover the performance of the nominal linear model. Xue and Huang (2011b), applying the ADRC method, also demonstrated the stability of the closed-loop system. Nevertheless, a "good" prior estimate of $b(x, t)$, satisfying some algebraic condition, is required for both the above control methods.

So a natural problem is: if it is possible to relax or remove the priori information on $b(x, t)$? Actually, for many practical control systems, there do exist uncertainties in the input channel (e.g. the flight control system Xiao, 1987) and such "good" priori information is usually difficult to obtain. Therefore, it is of great significance, both theoretically and practically, to investigate this problem. To solve it, Huang and Guo (2012) proposed to estimate the uncertainties by combining observers and estimators for the output feedback control of a class of nonlinear systems with the integrators in series structure. Furthermore, in Scheinker and Krstić (2013), a state feedback based on the extremum seeking (ES) design has been developed for semi-global stabilization of unstable and time-varying systems, where the control direction is unknown and is allowed to persistently change signs.

In this paper, we will consider the tracking problem for a class of nonlinear uncertain systems with zero dynamics, which is an extension of the nonlinear systems considered in an earlier paper of Huang and Guo (2012) where no zero dynamics was considered. We remark that many practical plants may be described by nonlinear models with stable zero dynamics (see, e.g. Chen, Yan, & Sun, 2014), and the coupling between internal and external states makes the analyses more complicated, since the stability of both states have to be established simultaneously. By using the method of combining the ESO technique and the projected gradient estimator, we are able to design an output feedback controller, and to show that such controller can ensure the closed-loop stability and make the tracking error arbitrarily small.

In the rest of the paper, we will present the main results in Section 2, and give the detailed proof in Section 3. A numerical example will be given in Section 4. Finally, Section 5 will conclude the paper with some remarks.

## 2. Main results

### 2.1. Problem formulation

We consider the following single-input-single-output (SISO) nonlinear system

$$\begin{cases} \dot{x} = Ax + B[a(x, z, t) + b(x, z, t)u], \\ \dot{z} = f_0(x, z, t), \qquad\qquad t \geq t_0 \\ y = Cx, \end{cases} \tag{1}$$

where $x = [x_1 \quad x_2 \quad \cdots \quad x_n]^T \in \mathbb{R}^n$ and $z = [z_1 \quad z_2 \quad \cdots \quad z_m]^T \in \mathbb{R}^m$are the state variables, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the measured output, $t_0$ is the initial time, and $a(x, z, t)$, $b(x, z, t)$, $f_0(x, z, t)$ are nonlinear time-varying functions which may contain unknown dynamics and external disturbances. In addition, the triple $(A, B, C)$ represents a chain of $n$ integrators, i.e.,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^n,$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{1\times n}.$$

Our control objective is to develop an output feedback controller to make sure that for all initial states in any given compact set, the state signals $(x(t), z(t))$ are bounded, and $x(t)$ tracks the reference trajectory which is generated from the target system

$$\dot{x}^*(t) = A_m x^*(t) + Br(t), \quad t \geq t_0 \tag{2}$$

where $x^*(t) \in \mathbb{R}^n$, the input signal $r(t) \in \mathbb{R}$ satisfying

$$|r(t)| \leq \bar{r}, \qquad |\dot{r}(t)| \leq \bar{r} \tag{3}$$

with $\bar{r} > 0$ a known constant, and

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -k_1 & -k_2 & \cdots & \cdots & -k_n \end{bmatrix} \in \mathbb{R}^{n\times n} \tag{4}$$

is a Hurwitz matrix (i.e., the polynomial $s^n + k_n s^{n-1} + \cdots + k_1$ is Hurwitz), so that there exists a positive definite matrix $P_0 > 0$ such that

$$A_m^T P_0 + P_0 A_m = -I. \tag{5}$$

Throughout the paper, we need the following assumptions:

(A1) $f_0(x, z, t)$ is locally Lipschitz, $a(x, z, t)$ and $b(x, z, t)$ are differentiable with locally Lipschitz derivatives. Moreover, for any constant $\rho \geq 0$, if $\|(x, z)\| \leq \rho$, then

$$\|f_0\| + \|a\| + \|b\| + \|\nabla a\| + \|\nabla b\| \leq \tau(\rho) \tag{6}$$

holds for all $t \geq t_0$, where $\|\cdot\|$ is the Euclidean norm, $\nabla f$ is the gradient of $f$, and $\tau(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is a known finite increasing function.

(A2) The nonlinear function $b(x, z, t)$ is bounded away from zero for all $(x, z, t) \in \mathbb{R}^n \times \mathbb{R}^m \times [t_0, \infty)$, and the sign of $b(x, z, t)$ is known. Without loss of generality, let $b(x, z, t) \geq \underline{b}$ with a known positive number $\underline{b}$.

(A3) There exists a continuously differentiable function $V_0(t, z) : [t_0, \infty) \times \mathbb{R}^m \to \mathbb{R}_+$, such that for all $(x, z, t) \in \mathbb{R}^n \times \mathbb{R}^m \times [t_0, \infty)$,

$$\alpha_1(\|z\|) \leq V_0(t, z) \leq \alpha_2(\|z\|), \tag{7}$$

$$\frac{\partial V_0}{\partial t} + \frac{\partial V_0}{\partial z} f_0(x, z, t) \leq 0, \quad \forall \|z\| \geq \alpha_0(\|x\|), \tag{8}$$

where $\alpha_0(\cdot)$ is a known class $\mathcal{K}$ function and $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ are known class $\mathcal{K}_\infty$ functions (Khalil, 2002).

We remark that Assumption (A3) ensures that the system $\dot{z} = f_0(x, z, t)$, with input $x$, is bounded-input-bounded-state stable (BIBS), which is less restrictive than the input-to-state stability (ISS) because it does not require the origin of $\dot{z} = f_0(0, z, t)$ to be uniformly asymptotically stable (UAS). To our understanding, there is no conclusive assertion that the typical minimum phase condition is weaker than our Assumption (A3), and vice versa. The Assumption (A3) used in this paper is only for the convenience of proof. Of course, the main results in this paper are still true if the zero dynamics $\dot{z} = f_0(0, z, t)$ is (locally uniformly) exponentially stable and Assumptions (A1)–(A2) hold.