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Stability analysis and stabilization of 2-D switched systems under arbitrary and restricted switchings[☆]

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ABSTRACT

In this paper, the problems of stability analysis and stabilization are investigated for discrete-time two-dimensional (2-D) switched systems, which are formulated by the well-known Fornasini–Marchesini local state-space model. Firstly, by using the switched quadratic Lyapunov function approach, a sufficient stability condition is established for such systems under arbitrary switching signal. Then, the extended average dwell time technique combining with the piecewise Lyapunov function approach is developed, and it is utilized for the stability analysis of the 2-D switched systems for the restricted switching case. Based on the stability analysis results, sufficient conditions are presented to stabilize the 2-D switched systems. Finally, two examples are provided to illustrate the effectiveness of the proposed new design techniques.

1. Introduction

In the past few decades, discrete-time two-dimensional (2-D) systems have gained considerable attention since a large number of practical systems can be modeled as 2-D systems, such as those in repetitive processes, image data processing and transmission, thermal processes, gas absorption, water stream heating (Du & Xie, 2002; Dymkov & Dymkou, 2012; Kaczorek, 1985; Marszalek, 1984), and iterative learning control (Cichy, Galkowski, Rogers, & Kummert, 2011; Dabkowski et al., 2013), where one direction of information is from trial-to-trial and the other is along a trial. So far, many important results have been reported in the literature, see for example, Bouagada and Dooren (2013), Gao, Lam, Xu, and Wang (2004), Li and Gao (2012), Paszke, Lam, Gałkowski, Xu, and Lin (2004), Wu, Shi, Gao, and Wang (2008), Wu, Yao, and Zheng (2012), Xiang and Huang (2013), Yang, Xie, and Zhang (2006) and Yeganefar, Yeganefar, Ghamgui, and Moulay (2013) and the references therein. To mention a few, the stability analysis problem for 2-D systems was investigated in Bouagada and Dooren (2013), Fornasini and Marchesini (1980) and Paszke et al. (2004). The controller and filter design problems for 2-D systems were addressed in Du and Xie (2002) and Xie, Du, Zhang, and Soh (2002). The stabilization and $\mathcal{H}_\infty$ control problems for 2-D systems with Markovian jump parameters were investigated in Gao et al. (2004). In addition, the stability analysis and control synthesis were considered in Chen, Lam, Gao, and Zhou (2013) for 2-D fuzzy systems via basis-dependent Lyapunov functions. Recently, in Yeganefar et al. (2013), the definitions of asymptotic and exponential stability were introduced for discrete-time 2-D systems for the first time, and two different Lyapunov theorems were proposed to check the proposed definitions. However, in the literature, when the stability problems are involved, only the so-called "attractivity" is retained without mentioning that the initial conditions have to go to zero at infinity whilst the notion of stability is ignored.

On the other hand, it has been well recognized that many practical systems are subject to abrupt changes (Vesely & Rosinova, 2014; Wang, 2014). A very popular way to characterize these changes is the switching modeling, which assumes that the system under investigation operates at several modes (Shi, Xia, Liu, & Rees, 2006; Wu & Zheng, 2009; Wu, Zheng, & Gao, 2013). A switched

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system is a dynamical system that consists of a finite number of subsystems and a switching signal governing the switching among them. This type of systems can be used to describe many practical applications, such as mechanical systems, chemical processes, electronic circuits, transportation systems, computer controlled systems and communication systems. So far, stability analysis and switching stabilizability are the major concerns in the area of switched systems (Lin & Antsaklis, 2009; Wu, Cui, Shi, & Karimi, 2013). To date, two stability issues have been addressed in the literature, i.e., the stability under arbitrary switching (Vu & Morgansen, 2010) and the stability under restricted switching (Zhang & Gao, 2010). The restrictions on switching signals we are concerned with are time domain restrictions. For the stability analysis problem, the first question is whether the switched system is stable when there is no restriction on the switching signals. This problem is usually called stability analysis under arbitrary switching. On the other hand, switched systems, for example, a closed-loop multiple controller system, may fail to preserve stability under arbitrary switching, but may be stable under restricted switching signals. In practice, a class of controlled switching signals with restrictions on switching instants is frequently encountered, e.g., in the automobile gear switching, particular switching sequence/order (from first gear to the second gear, etc.) must be followed, and considerable attention has been drawn to such a type of switching that we could also call restricted switching.

One way to specify restricted switching is to introduce a scalar τ and restrict the switching signals with a property that the switching times $t_1, t_2, \ldots$ satisfy $t_{i+1} - t_i \geqslant \tau$ for all i belong to the set of positive integers. This scalar τ is regarded as the 'dwell time' in the literature. The dwell time and the 'average dwell time' switching are typical controlled switching signals, which were proposed and developed by Hespanha and Morse (1999) and Morse (1996), respectively. The average dwell time approach means that the average time interval between any two consecutive switching instances is no less than τ, which is determined by two mode-independent parameters, the increase coefficient and decay rate of the Lyapunov-like function (Wu & Zheng, 2009; Wu, Zheng et al., 2013). The average dwell time switching can cover the dwell time switching, and its extreme case is actually the arbitrary switching. Therefore, it is of practical and theoretical significance to investigate the stability of the switched systems under both arbitrary switching and restricted switching with average dwell time approach. Most existing works on switched systems only deal with one-dimensional (1-D) systems. However, many, or even most, physical systems have natural multidimensional characteristics and the switching phenomenon may also occur in practical 2-D systems. It is only for convenience and simplicity, and often to avoid computational complexities, that such features have been neglected.

In many modeling problems of physical processes, a 2-D switching representation is needed. One can cite a 2-D physically based model for advanced power bipolar devices (Igic, Towers, & Mawby, 2004) and heat flux switching and modulating in a thermal transistor (Lo, Wang, & Li, 2008). This class of systems can correspond to 2-D switched systems. Recently, a number of reports on 2-D discrete switched systems appeared, see for example, Benzaouia, Hmamed, Tadeo, and Hajjaji (2011) and Xiang and Huang (2013). The stability analysis was investigated in Benzaouia et al. (2011) for discrete-time 2-D switched systems in Roesser model based on common and multiple Lyapunov functions. The stability analysis and stabilization problems were considered in Xiang and Huang (2013) for 2-D discrete switched systems by a model of Roesser type. Although there are some initial results for 2-D switched systems, it is noted that the aforementioned research efforts have been focused on the Roesser type. It is well known that 2-D systems can be represented by different models such as the Roesser model, Fornasini–Marchesini (FM) model and Attasi model. The Roesser model and the Attasi model can be treated as particular cases of the FM model. Therefore, we choose to tackle the stability analysis and stabilization problem for 2-D switched systems in FM type, which is more general. It is worth mentioning that although the Roesser model can be treated as a particular case of the second FM model, the Roesser model is also studied separately in the literature due to its wide applications and special structure. With the special structure, better results can be achieved. For a complete description of these models and methods to transform the system from one model type to another, the readers can refer to Du and Xie (2002) and Kaczorek (1985).

Stability is one of the fundamental concepts and it plays a most important role in control systems design. Motivated by the discussions above, it is, therefore, our intention in this paper to address the stability analysis and stabilization problems for discrete-time 2-D switched systems represented by Fornasini–Marchesini local state-space (FMLSS) model. To be specific, first of all, attention is focused on the stability analysis for 2-D switched system under both arbitrary and restricted switching signals. Then, the stabilization problem for 2-D switched systems is addressed, with conditions obtained for the existence of stabilizing controllers under both arbitrary and restricted switching signals, respectively. Finally, two examples are provided to demonstrate the effectiveness of the proposed controller design procedures. The main contributions of this paper can be highlighted as follows:

(1) Stability analysis is conducted based on the definition established in Yeganefar et al. (2013). However, in the literature, when it comes to the stability problem, attention is devoted to the so-called "attractivity" whilst the notion of stability is ignored.
(2) Both arbitrary and restricted switchings are considered for the stability analysis and controller design of 2-D switched systems. The average dwell time approach is utilized under the restricted switching signal, which covers the dwell time switching, and its extreme case is actually the arbitrary switching.
(3) Inspired by the work in Fornasini and Marchesini (1980), the notion of "time instant" is introduced for the 2-D switched systems and the concept of average dwell time for 1-D switched systems has been further extended to 2-D switched systems, which makes it possible to investigate the stability analysis and the controller design for the 2-D switched systems under the restricted switching signals based on the extended average dwell time approach.

The rest of this paper is organized as follows. The problem under consideration is formulated in Section 2. The stability analysis under both arbitrary switching and restricted switching signals is provided in Section 3. Based on the results obtained in Section 3, stabilization controller design is presented in Section 4. Two examples are provided in Section 5 to illustrate the effectiveness of the proposed methods, and we conclude this paper in Section 6.

Notations. The notations used throughout the paper are fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m \times n$ and the notation $P > 0$ means that P is real symmetric and positive definite; I and 0 represent identity matrix and zero matrix; $\|\cdot\|$ refers to the Euclidean vector norm; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalues of a real matrix, respectively. Function ceil(a) represents rounding real number a to the nearest integer greater than or equal to a. In symmetric block matrices or long matrix expressions, we use an asterisk '$*$' to represent a term that is induced by symmetry, and diag$\{\ldots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.