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Brief paper

Synchronization of nonlinear systems with communication delays and intermittent information exchange☆

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ABSTRACT

This paper studies the synchronization problem of second-order nonlinear multi-agent systems with intermittent communication in the presence of irregular communication delays and possible information losses. The control objective is to steer all agents' positions to a common position with a prescribed desired velocity available only to some leaders. Based on the small-gain framework, we propose a synchronization scheme relying on an intermittent information exchange protocol in the presence of time delays and possible packet dropouts. We show that our control objectives are achieved with a simple selection of the control gains provided that the directed graph, describing the interconnection between agents, contains a spanning tree. An example of Euler–Lagrange systems is considered to illustrate the application and effectiveness of the proposed approach.

1. Introduction

Motion coordination of nonlinear multi-agent systems has recently received increased interest in the control community due to potential applications involving groups of robotic systems and autonomous vehicles in general (Ren & Cao, 2011). The coordination problem of multi-agent systems can be formulated as a synchronization or a consensus problem, where the goal is to drive the networked subsystems (or agents) to a common state using local information exchange. Other related problems include flocking, swarming, and formation control of mechanical systems. Built around the existing solutions of the consensus problem for linear multi-agent systems, several coordinated control schemes have been recently developed for second-order nonlinear dynamics, which can describe various mechanical systems, with particular interest to leaderless synchronization, cooperative tracking with full access to the reference trajectory, and leader–follower problems (see, for instance, Abdessameud & Tayebi, 2009; Cai & Huang, 2014; Chen & Lewis, 2011; Dimarogonas, Tsiotras, & Kyriakopoulos, 2009; Liu, Xie, Ren, & Wang, 2013; Mei, Ren, Chen, & Ma, 2013; Mei, Ren, & Ma, 2011, 2012; Meng, Dimarogonas, & Johansson, 2014; Su, Chen, Wang, & Lin, 2011; Wang, 2013; Zou, 2014, and references therein). Algebraic graph theory, matrix theory, and the Lyapunov direct method have been shown useful to address various problems related to the systems dynamics, such as uncertainties, and the interconnection topology between the team members.

In addition, various recent papers address the synchronization problem of nonlinear systems by taking into account delays in the information transfer between agents, which is generally performed using communication channels. In Chopra and Spong (2006), it has been shown that output synchronization of nonlinear passive systems is robust with respect to constant communication delays if the interconnection graph is directed, balanced, and strongly connected. In Münz, Papachristodoulou, and Allgöwer (2011), a delay-robust control scheme is proposed for relative-degree two nonlinear systems with nonlinear interconnections. With the same assumption on the delays, adaptive synchronization schemes have been proposed in Nuño, Ortega, Basañez, and Hill (2011) and Wang (2014) for networked robotic systems under a directed graph. In addition to constant delays, a virtual systems approach has been suggested in Abdessameud and Tayebi (2011b, 2013) to account for input saturations and remove the requirements of velocity measurements. Control schemes that consider time-varying communication delays have also been

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proposed for some second-order nonlinear multi-agent systems in Abdessameud and Tayebi (2011a, 2013); Abdessameud, Tayebi, and Polushin (2012); Erdong, Xiaolei, and Zhaowei (2008); Nuño, Sarras, and Basañez (2013), in the case of undirected interconnection graphs, and in Abdessameud, Polushin, and Tayebi (2014a) in the case of more general directed interconnection topologies.

One important problem when dealing with second-order nonlinear systems in the presence of communication delays is to achieve position synchronization, *i.e.,* convergence of all positions to a common value, with some non-zero final velocity. In fact, in most of the above mentioned synchronization laws with communication delays, a static leader or no leader are assumed and position synchronization is achieved with zero final velocity. The only cases where the final velocities match a non-zero value assume a full access to a reference trajectory or to a leader's states (position and velocity). Another issue that can be observed in all the aforementioned results is the assumption that information is transmitted continuously between agents. In fact, it is not clear if these results still apply in situations where agents are allowed to communicate with their neighbors only during some disconnected intervals (or at some instants) of time. This can be induced by environmental constraints, such as communication obstacles, temporary sensor/communication-link failure, or imposed to the communication process to save energy/communication costs in mobile agents. For linear first-order multi-agent systems, Sun and Wang (2009) have proposed a discontinuous algorithm achieving state consensus using intermittent communication in the presence of sufficiently small constant communication delays and bounded packet dropout. Consensus algorithms with intermittent communication have also been proposed for higher-order linear multi-agent systems (Gao & Wang, 2010; Wen, Duan, Ren, & Chen, 2013) and for a class of globally Lipschitz nonlinear systems (Wen, Duan, Li, & Chen, 2012) without communication delays.

In this paper, we consider the synchronization problem of a class of second-order nonlinear systems with intermittent communication in the presence of communication delays and possible packet losses. Here, it is required that all systems achieve position synchronization with some non-zero desired velocity available only to some systems in the group acting as leaders. Based on the small-gain approach, we propose a distributed control algorithm that achieves our control objective in the situation where the agents are allowed to communicate with their neighbors only at some irregular discrete time instants. A discrete-time consensus algorithm is also used to handle the partial access to the desired velocity. In the case where no desired velocity is assigned to the team, the proposed algorithm achieves position synchronization with some constant final velocity agreed upon by all agents. In both cases, it is proved that, under some sufficient conditions, synchronization is achieved in the presence of unknown irregular communication delays and packet losses provided that the interconnection topology between agents is described by a directed graph that contains a spanning tree. The derived conditions impose a maximum allowable interval of time during which a particular agent does not receive information from some or all of its neighbors. This interval, however, can be assigned arbitrarily by a choice of the control gains. To illustrate the applicability of the proposed approach, we derive a solution to the above problems in the case of networked Lagrangian systems, and simulation results that show the effectiveness of the proposed approach are given.

2. Background and problem formulation

2.1. Graph theory

Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be a directed graph, with a set of nodes (or vertices) $\mathcal{N}$, and a set of ordered edges (pairs of nodes) $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$. An edge $(j, i) \in \mathcal{E}$ is represented by a directed link (arc) leaving node j and directed toward node i. A directed graph $\mathcal{G}$ is said to contain a spanning tree if there exists at least one node that has a "directed path" to all the other nodes in the graph; by a directed path (of length q) from j to i is meant a sequence of edges in a directed graph of the form $(j, l_1), (l_1, l_2), \ldots, (l_{q-1}, l_q)$, with $l_q = i$, where for $q > 1$ the nodes $j, l_1, \ldots, l_{q-1} \in \mathcal{N}$ are distinct. Node r is called a root of $\mathcal{G}$ if it is the root of a directed spanning tree of $\mathcal{G}$; in this case, $\mathcal{G}$ is said to be rooted at r.

Given two graphs $\mathcal{G}_1 = (\mathcal{N}, \mathcal{E}_1)$, $\mathcal{G}_2 = (\mathcal{N}, \mathcal{E}_2)$ with the same vertex set $\mathcal{N}$, their composition $\mathcal{G}_3 := \mathcal{G}_1 \circ \mathcal{G}_2 = (\mathcal{N}, \mathcal{E}_3)$ is the graph with the same vertex set $\mathcal{N}$, where $(j, i) \in \mathcal{E}_3$ if and only if $(j, l) \in \mathcal{E}_2$ and $(l, i) \in \mathcal{E}_1$ for some $l \in \mathcal{N}$. Composition of any finite number of graphs is defined by induction. In the case where $\mathcal{G}_1$ and $\mathcal{G}_2$ contain self-links at all nodes, the edges of $\mathcal{G}_1$ and $\mathcal{G}_2$ are also edges of $\mathcal{G}_3$. In this case, the definition above also implies that $\mathcal{G}_3$ contains a path from j to i if and only if $\mathcal{G}_2$ contains a path from j to l and $\mathcal{G}_1$ contains a path from l to i. A finite sequence of directed graphs $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_q$ with the same vertex set is jointly rooted if the composition $\mathcal{G}_q \circ \mathcal{G}_{q-1} \circ \cdots \circ \mathcal{G}_1$ is rooted. An infinite sequence of graphs $\mathcal{G}_0, \mathcal{G}_1, \ldots$ is said to be repeatedly jointly rooted if there exists $k^* \in \mathbb{Z}_+$ such that for any $\sigma \in \mathbb{Z}_+$ the finite sequence $\mathcal{G}_\sigma, \mathcal{G}_{\sigma+1}, \ldots, \mathcal{G}_{\sigma+k^*}$ is jointly rooted (see Cao, Morse, & Anderson, 2008 for more details on graph composition).

A weighted directed graph $\mathcal{G}_w$ consists of the triplet $(\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N}$ and $\mathcal{E}$ are, respectively, the sets of nodes and edges defined as above, and $\mathcal{A}$ is the weighted adjacency matrix defined such that $a_{ii} \triangleq 0$, $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Note that thus defined weighted graph does not contain self-links at any node and will have the same properties as the unweighted graph with the same sets of nodes and edges. The Laplacian matrix $\mathbf{L} := [l_{ij}] \in \mathbb{R}^{n\times n}$ of the weighted directed graph $\mathcal{G}_w$ is defined such that: $l_{ii} = \sum_{j=1}^{n} a_{ij}$, and $l_{ij} = -a_{ij}$ for $i \neq j$.

2.2. System model

Consider n not necessarily identical second-order nonlinear systems (or agents) governed by

$$\begin{aligned} \dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= F_i(p_i(t), v_i(t), u_i(t)), \end{aligned} \qquad i \in \mathcal{N}, \tag{1}$$

where $p_i \in \mathbb{R}^m$ and v_i are the position-like and velocity-like states, respectively, $u_i \in \mathbb{R}^m$ are the inputs, and $\mathcal{N} := \{1, \ldots, n\}$. The functions F_i are assumed to be locally Lipschitz with respect to their arguments. Note that Eqs. (1) may describe the full or partial dynamics of various physical systems.

The systems (1) are interconnected in the sense that some information can be transmitted between agents using communication channels. This interconnection is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of all agents, and an edge $(j, i) \in \mathcal{E}$ indicates that the ith agent *can* receive information from the jth agent; in this case, we say that j and i are neighbors (even though the link between them is directed). While the interconnection graph $\mathcal{G}$ is fixed, the information exchange between agents is not continuous but discrete in time and is subject to communication constraints as described in the next subsection.

2.3. Communication process

In this paper, we consider the case where the communication between agents is intermittent and is subject to time-varying communication delays, information losses, and blackout intervals. Specifically, it is assumed that there exists a strictly increasing and unbounded sequence of time instants $t_k := kT \in \mathbb{R}_+$, $k \in \mathbb{Z}_+ = \{0, 1, \ldots\}$, where $T > 0$ is a fixed sampling period common for all agents, such that each agent is allowed to send its information