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Brief paper

Efficient subset selection for the expected opportunity cost[☆]

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ABSTRACT

A lot of problems in automatic control aim at seeking top designs for discrete-event systems. In many cases, these problems are most suitable to be modeled as simulation optimization problems, and a key question for solving these problems is how to efficiently and accurately select the top designs given a limited simulation budget. This paper considers the generalized problem of selecting the top m designs from a finite set of design alternatives based on simulated outputs, subject to a constraint on the total number of samples available. The quality of the selection is measured by the expected opportunity cost, which penalizes particularly bad choices more than the slightly incorrect selections and is preferred by risk-neutral practitioners and decision makers. An efficient simulation budget allocation procedure, called *EOC-m*, is developed for this problem. The efficiency of the proposed method is illustrated through numerical testing.

1. Introduction

Discrete-event systems (DES) simulation is a powerful tool for analyzing systems and evaluating decision problems, since real situations usually do not satisfy the assumptions of analytical models. DES simulation makes it possible to accurately specify a system through the use of logically complex, and often non-mathematical models. Detailed dynamics of complex, stochastic systems can therefore be modeled (Chen & Lee, 2011). Examples such as queueing systems, inventory control, buffer allocation, pollution control, and portfolio management fall into the applicable areas of DES simulation (Law & Kelton, 2000).

While DES simulation has many advantages for modeling complex systems, efficiency is still a significant concern when conducting simulation experiments (Law & Kelton, 2000). To acquire a good statistical estimate for a design decision, a large number of simulation replications is usually required for each design alternative. An estimate of the mean of a design typically has errors of size $O(1/\sqrt{N})$, the result of averaging independent and identically distributed (i.i.d.) noise, where N is the number of simulation replications. On the other hand, we usually have a budget constraint for simulation in practice, for instance, a decision has to be made within ten hours. Consequently, the problem of how to efficiently allocate the simulation budget to select good designs has drawn great attention.

Most existing research in simulation budget allocation focuses on selecting the best design. Typically there are two main measures of selection quality and three main approaches in this respect. The alignment probability or the probability of correct selection (*PCS*) is one commonly studied measure of performance (Bechhofer, Santner, & Goldsman, 1995). This is defined as the probability of selecting the best design. The other broadly used quality measure is the expected opportunity cost (*EOC*), which is defined as the difference in means between the selected design and the best one (Chick & Inoue, 2001a,b; Chick & Wu, 2005).

The indifference-zone (IZ) approach aims to provide a guaranteed lower bound for *PCS*, assuming that the mean performance of the best design is at least δ^* better than each alternative, where δ^* is the minimum difference worth detecting (Dudewicz & Dalal, 1975; Kim & Nelson, 2001; Nelson, Swann, Goldsman & Song, 2001; Rinott, 1978). The optimal computing budget allocation (OCBA) method allocates the samples sequentially in order to maximize *PCS* or minimize *EOC* under a budget constraint (Chen & Lee, 2011; Chen, Lin, Yücesan & Chick, 2000; He, Chick, & Chen, 2007). The expected value of information (EVI) procedure allocates samples

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to maximize the EVI obtained from sampling in two stages or sequentially using predictive distributions of further samples (Chick & Inoue, 2001a,b).

Another important selection problem in simulation budget allocation is to select the top m designs for $m > 1$ instead of the single best design. The selection of the top m designs is most useful for problems in automation, manufacturing, and other engineering applications when the designs for comparison have multiple dimensions of performance measurements with some qualitative criteria such as environmental consideration or political feasibility. The selection procedure provides the top m designs for the decision maker so that the final decision can be made in a more flexible way by incorporating other qualitative performance measurements than just the quantitative performance measurement. In addition, the problem of selecting an optimal subset is also motivated by recent developments in global simulation optimization algorithms, which require the selection of an elite subset of good candidate solutions in each iteration of the algorithm (Chambers, 1995; Hu, Fu, & Marcus, 2007, 2008; Rubinstein & Kroese, 2004). The information from the elite set is used to guide the search in subsequent iterations for the global optimum. Since the performance of these algorithms depends heavily on the quality of the selected solutions, how to efficiently select an optimal subset then becomes a critical problem for implementation of these algorithms.

For the optimal subset selection problem, a two-stage procedure was established in Koenig and Law (1985) to provide a *PCS* guarantee. However, the number of additional simulation replications for the second stage is computed based on a least favorable configuration and causes the computational cost to be much higher than actually needed. A sequential subset selection procedure was developed in Chen, He, Fu, and Lee (2008), which maximizes *PCS* using the OCBA method and turns out much more efficient than some other selection procedures in the literature.

In this research, we propose to develop an efficient simulation budget allocation procedure to select the top m designs using the *EOC* measure, called *EOC-m*. Compared to *PCS*, *EOC* takes the consequence of incorrect selection into consideration and is particularly useful for risk-neutral practitioners and decision makers (He et al., 2007). For example, suppose there are 100 systems and system i has a cost of i, $i = 1, 2, \ldots, 100$. We want to select three systems that minimize the total cost. Then, the correct selection is systems 1, 2 and 3. For *PCS*, the selections of systems 4, 5, 6 and 98, 99, 100 are treated equally because they are both wrong selections from *PCS* point of view. However, the choice of systems 4, 5 and 6 is much more favorable than 98, 99 and 100 in practice because systems 4, 5 and 6, though not optimal, cost much less than 98, 99 and 100. That is, the opportunity cost of systems 4, 5 and 6 is much less. By using the EOC measure, the selection of 98, 99 and 100 is much more unlikely to appear than the selection of 4, 5, and 6, if a wrong selection is made, and the risk of making a very bad selection is thus avoided. This motivates the use of *EOC* as the quality measure when selecting the top m designs. To the best of our knowledge, this is the first work addressing the subset selection problem for the *EOC* quality measure.

The rest of the paper is organized as follows. In Section 2, we formulate the simulation budget allocation problem for selecting the top m designs. In Section 3, we develop a new and efficient budget allocation strategy. The performance of the proposed method is illustrated with numerical examples in Section 4. Section 5 concludes the paper.

2. Problem statement

This section presents a formulation of the selection problem. In this research, the best design is defined as the design with the smallest mean performance (the largest mean performance could be handled similarly). We introduce the following notation:

T: total number of simulation replications (budget);
k: total number of designs;
$L_{i,j}$: output of the jth simulation replication for design i;
J_i: mean of $L_{i,j}$, i.e., $J_i = E[L_{i,j}]$;
σ_i^2: variance of $L_{i,j}$, i.e., $\sigma_i^2 = Var[L_{i,j}]$;
N_i: number of simulation replications for design i;
$\bar{J}_i$: sample mean of design i, i.e., $\bar{J}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} L_{i,j}$;
S_b: set of m indices indicating the observed top m designs;
S_t: set of m indices indicating the true top m designs;
$\delta_{i,j} = J_i - J_j$;
$\sigma_{i,j}^2 = \sigma_i^2/N_i + \sigma_j^2/N_j$.

We assume no ties in means among the designs in S_t. Let design $b_i \in S_b$ be the observed ith best design and design $t_i \in S_t$ be the true ith best design, i.e., $\bar{J}_{b_1} < \bar{J}_{b_2} < \cdots < \bar{J}_{b_m}$ and $J_{t_1} < J_{t_2} < \cdots < J_{t_m}$. To make the derivation more tractable, we also assume that the simulation output samples are normally distributed and independent from replication to replication, as well as independent across different designs. As a result, $L_{i,j} \sim N(J_i, \sigma_i^2)$ and $\bar{J}_i \sim N(J_i, \frac{\sigma_i^2}{N_i})$. The normality assumption is typically satisfied in simulation, because the output is obtained from an average performance or batch means, so that the Central Limit Theorem holds.

A correct selection occurs when the set of true top m designs S_t is observed, and the expected opportunity cost for the set of observed top m designs S_b is defined as:

$$EOC_m = E\left[\sum_{i\in S_b} J_i - \sum_{i\in S_t} J_i\right]. \tag{1}$$

Consequently, the simulation budget allocation problem is given by

$$\min EOC_m$$
$$s.t. \sum_{i=1}^{k} N_i = T. \tag{2}$$

Here $\sum_{i=1}^{k} N_i$ denotes the total computational cost assuming that the simulation execution times for different designs are the same. This formulation is in a similar format of the OCBA framework. The simulation budget allocation problem given in He et al. (2007) is a special case of (2) with $m = 1$.

3. Efficient simulation budget allocation

In this section we derive a convenient approximation for the EOC_m in optimization problem (2) and design an efficient sequential simulation budget allocation procedure accordingly.

3.1. EOC_m approximation

A major difficulty for problem (2) is that the objective function EOC_m does not have a closed-form expression. Although EOC_m can be estimated using Monte Carlo simulation, it is very time-consuming. Since the purpose of budget allocation is to improve simulation efficiency, we adopt an approximation of EOC_m using an upper bound.

Theorem 1. *Denote Φ as the cumulative distribution function of standard normal distribution. The EOC_m can be bounded as*

$$EOC_m \le \sum_{j=1}^{m}\sum_{i=1, i\notin\{t_1,\ldots,t_j\}}^{k}\sum_{n=1}^{j} \delta_{i,t_n}\Phi\left(\frac{\delta_{t_n,i}}{\sqrt{\frac{\sigma_{t_n}^2}{N_{t_n}}+\frac{\sigma_i^2}{N_i}}}\right). \tag{3}$$